M.K. HOME TUITION

Mathematics Revision Guides

Level: A-Level Year 2

INTEGRATION TECHNIQUES (TRIG, LOG, EXP FUNCTIONS)

$$\int \frac{1}{x}\,dx = \ln x + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\frac{d}{dx}(1-3x)^5 = -15\,(1-3x)^4$$

$$\therefore \int (1-3x)^4\,dx = -\tfrac{1}{15}(1-3x)^5 + c$$

$$\int 6e^{2x}\,dx = 3e^{2x} + c$$

$$\int e^x\,dx = e^x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int_{\pi/6}^{\pi/2} 2\sin^4 x\,\cos x\,dx = \left[\tfrac{2}{5}\sin^5 x\right]_{\pi/6}^{\pi/2} = \tfrac{2}{5}\left(1-\tfrac{1}{32}\right) = \tfrac{31}{80}$$

$$\int_2^3 \frac{10-8x}{2x^2-5x+4}\,dx = \left[-2\ln\left(2x^2-5x+4\right)\right]_2^3 = -2\,(\ln 7 - \ln 2) = \ln\left(\tfrac{4}{49}\right)$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int_2^3 (2x-3)^5\,dx = \left[\frac{1}{2\times 6}(2x-3)^6\right]_2^3 = \left[\frac{1}{12}(2x-3)^6\right]_2^3 = \frac{1}{12}\left(3^6-1^6\right) = \frac{728}{12} = 60\frac{2}{3}$$

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INTEGRATION TECHNIQUES.

Polynomial Integrals – a recap.

The list below recalls the general results for differentiation of polynomial functions. Those highlighted are results obtained by the chain rule.

(We shall include fractional and negative powers of x in this section, though they are not polynomials in the true sense of the word).

(General rules)	
$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
$kf(x)$	$kf'(x)$
(Polynomial)	
x^n	nx^{n-1}
$(ax+b)^n$	$an(ax+b)^{n-1}$
$(f(x))^n$	$nf'(x)(f(x))^{n-1}$

From this table, we can produce a similar list of rules for integration.

(General rules)	
$\int f(x) \pm g(x)dx$	$\int f(x)dx \pm \int g(x)dx$
$\int kf(x)dx$	$k\int f(x)dx$

$y = f(x)$	$\int f(x)dx$
x^n (provided $n \neq -1$)	$\frac{x^{n+1}}{n+1} + c$
$(ax+b)^n$ (provided $n \neq -1$)	$\frac{1}{a(n+1)}(ax+b)^{n+1} + c$
$f'(x)(f(x))^n$ (provided $n \neq -1$)	$\frac{1}{n+1}(f(x))^{n+1} + c$

Note the patterns in the results obtained by the chain rule.

When we differentiate a power of a linear expression in x, we have to reduce the power by 1, multiply by the original power, and then **multiply** by the coefficient of x.

When integrating a power of a linear expression, we add 1 to the power, divide by the updated power, and finally **divide** by the coefficient of x.

Examples (1): Integrate with respect to *x:* i) x^2 ; ii) x +5; iii) $8x^3$; iv) $x^4 + 6x^2$; v) $\sqrt{x}$; vi) $\frac{1}{x^2}$

i) $\int x^2 dx = \frac{x^3}{3} + c$; ii) $\int x + 5 \; dx = \frac{x^2}{2} + 5x + c$; iii) $\int 8x^3 dx = 2x^4 + c$

iv) $\int x^4 + 6x^2 dx = \frac{x^5}{5} + 2x^3 + c$; v) $\sqrt{x} = x^{\frac{1}{2}}$, so $\int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$ or $\frac{2x\sqrt{x}}{3} + c$

vi) $\frac{1}{x^2} = x^{-2}$, so $\int x^{-2} dx = \frac{x^{-1}}{-1} + c$ or $\frac{-1}{x} + c$

Example (2): Find the value of $\int_3^4 3x^2 \, dx$.

$$\int_3^4 3x^2 \, dx = \left[x^3\right]_3^4 = 4^3 - 3^3 = 64 - 27 = 37.$$

Example (3): Find the value of $\int_0^3 x^2 - 4x + 5 dx$

$$\int_0^3 x^2 - 4x + 5 dx = \left[\frac{x^3}{3} - 2x^2 + 5x\right]_0^3$$

= (9 - 18 + 15) – (0) = 6.

Reversing the Chain Rule – Integration by inspection.

The next examples all make use of integration by inspection, by using the chain rule in reverse.
This method can be used if we can spot a product of a function and some multiple of its derivative.

Example (4): Differentiate $(1-3x)^5$ and hence find $\int (1-3x)^4\,dx$.

Using the chain rule, we have $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ where $u = (1-3x)$, $\frac{du}{dx} = -3$ and

$\frac{dy}{du} = 5u^4 = 5(1-3x)^4$.

$\therefore \frac{dy}{dx} = -15(1-3x)^4$ and hence $\int -15(1-3x)^4\,dx = (1-3x)^5 + c$.

Our required integrand, $\int (1-3x)^4\,dx$, is -15 times smaller than the result obtained by differentiating $(1-3x)^5$, so we divide by -15 to obtain $\int (1-3x)^4\,dx = -\tfrac{1}{15}(1-3x)^5 + c$.

Alternatively, we could have used the result $\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c$

to give $\frac{1}{((-3)\times(5))}(1-3x)^5 + c = -\frac{1}{15}(1-3x)^5 + c$.

Example (5): Find $\int_2^3 (2x-3)^5\,dx$.

We can use the same formula as in the final part of Example (4) to obtain:

$$\int_2^3 (2x-3)^5\,dx = \left[\frac{1}{2\times 6}(2x-3)^6\right]_2^3 = \left[\frac{1}{12}(2x-3)^6\right]_2^3 = \frac{1}{12}\left(3^6 - 1^6\right) = \frac{728}{12} = 60\frac{2}{3}.$$

We could also make an 'educated guess' and test the result by differentiation using the chain rule.
Since the power of an expression is raised by 1 by integration, we can guess that the integral will be some multiple of $\left[(2x-3)^6\right]_2^3$.

Differentiating $(2x - 3)^6$ by the chain rule would give $12(2x - 3)^5$, which is too large by a factor of 12.

We therefore adjust the guess to $\left[\frac{1}{12}(2x-3)^6\right]_2^3$.

Example (6): Find $\int x(3x^2-4)^3\,dx$.

Here we can use the formula $\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^n$

since we can spot the cube of the function $f(x) = 3x^2 - 4$, multiplied by x, which is one-sixth of its derivative, $f'(x) = 6x$.

Applying the formula as it stands gives $\frac{1}{4}\left(3x^2-4\right)^4 + c$, but because we have x and not $6x$ in this integrand, we must divide the result by 6 to get the true answer, $\frac{1}{24}\left(3x^2-4\right)^4 + c$.

Alternatively, we could have made a 'guess' of $(3x^2 - 4)^4$ and checked it by differentiation. The derivative works out as $24x(3x^2 - 4)^3$ – a result 24 times higher than the required integrand.

Dividing the guess by 24 will give $\int x(3x^2-4)^3\,dx = \frac{1}{24}\left(3x^2-4\right)^4 + c$.

Example (7): Find $\int_0^1 \frac{x+1}{\left(2x^2+4x+5\right)^2}\,dx$.

Again we can use the formula $\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^n$

since we can spot the reciprocal of the square of the function $f(x) = 2x^2 + 4x + 5$, multiplied by the expression $x + 1$, which is one-quarter of its derivative, $f'(x) = 4x + 4$.

Applying the formula as it stands gives $\left[-1\left(2x^2+4x+5\right)^{-1}\right]_0^1$, but because we have $x+1$ and not $4x + 4$ in this integrand, we must divide the result by 4 to get the true answer,

$$\left[-\frac{1}{4}\left(2x^2+4x+5\right)^{-1}\right]_0^1 = \left[\frac{-1}{4\left(2x^2+4x+5\right)}\right]_0^1 = \left[-\frac{1}{44}\right]-\left[\frac{-1}{20}\right] = \frac{3}{110}.$$

Alternatively, we could have made a 'guess' of $\frac{1}{2x^2+4x+5}$ and checked it by differentiation. The derivative works out as $\frac{-4x-4}{\left(2x^2+4x+5\right)^2}$, which is -4 times as large as the required integrand.

Adjusting the 'guess' and adding the limits gives the correct integral of $\left[\frac{-1}{4\left(2x^2+4x+5\right)}\right]_0^1$.

There is a formal method corresponding to the last four examples, namely **integration by substitution**. This will be illustrated in the relevant document.

Trigonometric Integrals - introduction.

Suppose we were to estimate the area under the graph of cos $x°$ in the interval $0° < x < 90°$, using the trapezium rule with 9 strips.

$y = \cos x°$ $b\text{-}a = 90$ $h = 10$ $n = 9$

The width of each strip is therefore 10°.

n	x_n	y_n (first & last)	y_n (other values)
0	0	1	
1	10		0.9848
2	20		0.9397
3	30		0.8660
4	40		0.7660
5	50		0.6428
6	60		0.5000
7	70		0.3420
8	80		0.1736
9	90	0	
Sub-totals (1):		**1**	**5.215**
			(× 2)
Sub-totals (2):		**1**	**10.43**
TOTAL			**11.43**
Multiply by ½*h* (here ½ × 10)			**57.15**

The value of $\int_0^{90} \cos x°\, dx$ is therefore estimated at 57.2 to three significant figures.

However, we learned earlier that the derivative of $f(x) = \sin x$ is $f'(x) = \cos x$.

The value of about 57 seems at odds with the result $\left[\sin x\right]_0^{90}$ or 1-0, or 1.

The reason for the discrepancy is again due to the use of degrees instead of radians.

Using radians, $h = \frac{\pi}{18}$ and not 10°, and $\frac{1}{2}h = \frac{\pi}{36}$, or approximately 0.0873.

The true integrand is therefore $\int_0^{\frac{\pi}{2}} \cos x\, dx$.

Its estimated value from the previous result is 0.0873 × 11.43, or approximately 0.998, which is close to the true value of 1.

IMPORTANT – Radians are the default units of angle measurement in trig calculus.

Trigonometric Integrals.

Standard Trig Derivatives (plus some chain rule examples).
(Those for sin, cos and tan are the most important.)

The angle x must also be measured in **radians**, not degrees.

$y = f(x)$	$y' = \frac{dy}{dx} = f'(x)$
$\sin x$	$\cos x$
$\sin(ax+b)$	$a\cos(ax+b)$
$\sin^n x$	$n\sin^{n-1} x \cos x$
$\cos x$	$-\sin x$
$\cos(ax+b)$	$-a\sin(ax+b)$
$\cos^n x$	$-n\cos^{n-1} x \sin x$
$\tan x$	$\sec^2 x$
$\tan(ax+b)$	$a\sec^2(ax+b)$
$\tan^n x$	$n\tan^{n-1} x \sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\operatorname{cosec}(ax+b)$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$
$\operatorname{cosec}^n x$	$-n\operatorname{cosec}^n x \cot x$
$\sec x$	$\sec x \tan x$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$
$\sec^n x$	$n\sec^n x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\cot(ax+b)$	$-a\operatorname{cosec}^2(ax+b)$
$\cot^n x$	$-n\cot^{n-1} x \operatorname{cosec}^2 x$

Note how the highlighted functions behave on differentiation by the chain rule.

The derivative of sin $5x$, for instance, is 5 cos $5x$; a constant multiplier of 5 has appeared.
Similarly, the derivative of cos $4x$ is –4 sin $4x$, and that of tan ½x is ½ $\sec^2$ (½x).

Another example is that of $\cos^4 x$; its derivative is $-4\cos^3 x \sin x$.
Here the power of cos x has been reduced by 1, and a multiplier of the original power (4) and the derivative of cos x, i.e. –sin x, have also appeared .

Similarly, differentiating $\sin^3 x$ gives $3\sin^2 x \cos x$. The power of sin x has been reduced by 1, and a multiplier of the original power (3) and the derivative of sin x, i.e. cos x , have also appeared.

From this table, we can produce a similar list of standard trig integrals.
Again, angles must be measured in radians !

$y = f(x)$	$\int f(x)dx$
$\sin x$	$-\cos x + c$
$\sin (ax + b)$	$-\frac{1}{a}\cos(ax + b) + c$
$\sin^n x \cos x$	$\frac{1}{n+1}\sin^{n+1} x + c$
$\cos x$	$\sin x + c$
$\cos (ax + b)$	$\frac{1}{a}\sin (ax + b) + c$
$\cos^n x \sin x$	$-\frac{1}{n+1}\cos^{n+1} x + c$
$\sec^2 x$	$\tan x + c$
$\sec^2 (ax + b)$	$\frac{1}{a}\tan (ax + b) + c$
$\tan^n x \sec^2 x$	$\frac{1}{n+1}\tan^{n+1} x + c$
$\text{cosec}\, x \cot x$	$-\text{cosec}\, x + c$
$\text{cosec}(ax + b) \cot (ax + b)$	$-\frac{1}{a}\text{cosec}\,(ax + b) + c$
$\text{cosec}^n x \cot x$	$-\frac{1}{n}\text{cosec}^n x + c$
$\sec x \tan x$	$\sec x + c$
$\sec(ax + b) \tan (ax + b)$	$\frac{1}{a}\sec (ax + b) + c$
$\sec^n x \tan x$	$\frac{1}{n}\sec^n x + c$
$\text{cosec}^2 x$	$-\cot x + c$
$\text{cosec}^2 (ax + b)$	$-\frac{1}{a}\cot (ax + b) + c$
$\cot^n x\, \text{cosec}^2 x$	$-\frac{1}{n+1}\cot^{n+1} x + c$

Note how the highlighted functions behave on integration, by reversing the process of differentiation by the chain rule.

The derivative of sin 5*x*, for instance, was 5 cos 5*x*; a **multiplier** of 5 had appeared.
Conversely, integrating cos 5*x* would produce $\frac{1}{5}\sin 5x$; this time we had to **divide** by 5 instead.

Another example is that of $\cos^3 x \sin x$; its integral is $-\frac{1}{4}\cos^4 x$.
Here the power of cos *x* has been increased by 1 to 4, and a multiple of the **reciprocal** of that new power (4) has appeared. In addition, we have divided the result by the derivative of cos *x*, i.e. –sin *x*.

To solve integrals of these types, we can use the general formulae above, or use the reverse chain rule to make an 'educated guess', differentiate the 'guess', and adjust it if necessary.

Example (8): Find $\int 15\sin 5x\,dx$.

We can either use the tabled result $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$

to obtain the answer $15 \times -\frac{1}{5}\cos 5x + c$ = -3 cos 5*x*+ *c*, or we can make a guess.

We know that the integral of sin *x* is –cos *x* + *c* , so we make and initial guess of -cos 5*x*.
Differentiation of that 'guess' gives a result of 5 sin 5*x*, which is 3 times too small. We must therefore adjust the 'guess' of 5 sin 5*x* by multiplying it by 3, giving –3 cos 5*x* + *c*.

Example (9): Find $\int 6\sec^2 3x\,dx$.

We can either use the tabled result $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+c$

to obtain the answer $6 \times \frac{1}{3}\tan 3x + c = 2\tan 3x + c$, or we can make an educated guess.

We know that the integral of $\sec^2 x$ is $\tan x + c$, so we can guess $\tan 3x + c$.

Differentiating $\tan 3x$ by the chain rule gives us a result of $3\sec^2 3x$ which is of the right type, but too small by a factor of 2. We must therefore adjust the 'guess' of $\tan 3x$ by multiplying it by 2.

Again, this gives us $2\tan 3x + c$.

Example (10): Find $\int_{\pi/6}^{\pi/2} 2\sin^4 x \cos x\,dx$.
(Remember – radian measure must be used !)

We can use the tabled result $\int \sin^n x\cos x = \frac{1}{n+1}\sin^{n+1} x + c$

and obtain the answer $2 \times \frac{1}{5}\sin^5 x + c = \frac{2}{5}\sin^5 x + c$.

Alternatively, we can look at the integral and notice that it includes a power of $\sin x$ (the fourth power) multiplied by its derivative – a reverse chain rule result.

We can thus guess that the integral will be something like $\sin^5 x$ (compare integrating x^4 to get $\frac{1}{5}x^5$).

Differentiating $\sin^5 x$ gives $5\sin^4 x\cos x$, but our original integral was $2\sin^4 x\cos x$. The guess is too large by a factor of $\frac{5}{2}$, so we need to multiply it by $\frac{2}{5}$ to bring it to scale, again giving $\frac{2}{5}\sin^5 x + c$.

This is a definite integral, so its value is $\left[\frac{2}{5}\sin^5 x\right]_{\pi/6}^{\pi/2} = \frac{2}{5}(1-\frac{1}{32}) = \frac{31}{80}$.
(Remember: $\sin(\pi/2) = 1$; $\sin(\pi/6) = ½$).

Example (11): Find $\int \sec^4 x \tan x\,dx$.

We can use the tabled result $\int \sec^n x\tan x = \frac{1}{n}\sec^n x + c$

and obtain the answer $\frac{1}{4}\sec^4 x + c$.

Alternatively, we can rewrite the integrand as $\sec^3 x \sec x \tan x$, thus showing the product of the cube of $\sec x$ and its derivative, $\sec x \tan x$, more clearly. This suggests an answer of the form $\sec^4 x$.

Differentiating $\sec^4 x$ gives $4\sec^3 x \sec x\tan x = 4\sec^4 x\tan x$. This result is too large by a factor of 4, therefore the true integral is $\frac{1}{4}\sec^4 x + c$ as above.

Example (12): Find $\int -42\cos^6 2x \sin 2x\,dx$.

This type of integral is not shown in the table, but looking at it reveals a product of a power of $\cos 2x$ ($\cos^6 2x$) and its derivative, $-2\sin 2x$.

The integral therefore looks as it if was obtained by differentiating some multiple of $\cos^7 2x$
We will therefore make 'first guess' of $\cos^7 2x$ and differentiate it.

Applying the chain rule twice gives a derivative of $-14\cos^6 2x\sin 2x$. This guess is too small by a factor of 3, and therefore the true integral is $3\cos^7 2x + c$.

Examples 8-12 above can also be evaluated by 'substitution' (see **Integration by Substitution**)

Other trigonometric integrals can be evaluated by using identities and compound angle formulae to simplify complicated integrals into forms which are easier to integrate. These will be discussed in a separate document.

Exponential and Logarithmic Integrals.

Standard Exponential and Logarithmic Derivatives (plus some chain rule examples).

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$y = f(x)$	$y' = \frac{dy}{dx} = f'(x)$
e^x	e^x
e^{ax+b}	ae^{ax+b}
$e^{f(x)}$	$f'(x)e^{f(x)}$
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\ln (x^n)$	$\frac{n}{x}$
$\ln (f(x))$	$\frac{f'(x)}{f(x)}$

From this table, we can produce a similar list of standard exponential and logarithmic integrals.

$y = f(x)$	$\int f(x)dx$
e^x	$e^x + c$
e^{ax+b}	$\frac{1}{a}e^{ax+b} + c$
$f'(x)e^{f(x)}$	$e^{f(x)} + c$
a^x	$\frac{a^x}{\ln a} + c$
$\frac{1}{x}$	$\ln \lvert x\rvert + c$ Alternative form: $\ln \lvert Ax\rvert$
$\frac{n}{x}$	$\ln \lvert(x^n)\rvert + c = n \ln \lvert x\rvert + c$ Alternative form: $\ln \lvert(Ax^n)\rvert$
$\frac{f'(x)}{f(x)}$	$\ln \lvert(f(x))\rvert + c$ Alternative form: $\ln \lvert A(f(x))\rvert$

Note the patterns in the results obtained here.

When differentiating e raised to a function , then the result is the original function multiplied by its derivative.

For example, if e is raised to an expression $(ax + b)$, then the derivative is the original function **multiplied** by a. Conversely, when integrating e raised to an expression $(ax + b)$, then the integral is the original function **divided** by a.

When differentiating a^x we multiply by $\ln a$: when integrating a^x we divide by $\ln a$.

The list also shows how to integrate $\frac{1}{x}$, remembering that the standard rule $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ cannot be used for *n* = -1.

The logarithm laws also give rise to alternative ways of denoting the constant of integration, i.e. ln *x* + *c* = ln (*Ax*) where $A = e^c$.

Also, since there can be *no* logarithm of a negative number, the modulus function | *f* (*x*) | must strictly be included in integrands leading to a logarithmic function, unless we are sure that the function whose logarithm is being taken cannot itself take a negative or zero value.

Finally, note the neat result of the chain rule when applied to differentiating a logarithmic expression: the result is a fraction whose top line is the derivative of the bottom line.

Examples (13): Find i) $\int 6e^{2x} dx$ ii) $\int_1^2 xe^{x^2} dx$

In i) we take the factor of 6 outside the integral and use the result $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$ to obtain $6 \times \frac{1}{2} e^{2x}$ or $3e^{2x} + c$.
Alternatively, we could have made a 'first guess' of e^{2x} and differentiated that to give $2e^{2x}$. This result is too small by a factor of 3, so therefore the correct integral is $3e^{2x} + c$ as before.

In ii) we recognise that *x* is one-half the derivative of x^2 , so we can take ½ out as a factor and use the result $\int f'(x) e^{f(x)} = e^{f(x)} + c$ to obtain $\left[\frac{1}{2} e^{x^2}\right]_1^2 = \frac{1}{2}(e^4 - e) = \frac{1}{2} e(e^3 - 1)$.
Alternatively, we could have spotted a reversed chain rule result and guessed at e^{x^2} . Differentiation would give $2xe^{x^2}$, which is too large by a factor of 2, so we would adjust the integral to $\frac{1}{2} e^{x^2}$.

(Both examples could also have been worked out by substitution (see document 'Integration by Substitution').

Examples (14): Find i) $\int_0^2 2^x dx$ ii) $\int_1^2 \frac{1}{x} dx$ iii) $\int_{-10}^{-2} \frac{2}{x} dx$ iv) $\int_{-2}^{2} \frac{1}{x} dx$

For i) we use $\int a^x dx = \frac{a^x}{\ln a} + c$, to give a required integral of $\left[\frac{2^x}{\ln 2}\right]_0^2 = \frac{4-1}{\ln 2} = \frac{3}{\ln 2}$.

In ii), we use the standard result to obtain $[\ln x]_1^2$ = ln 2 – ln 1 = ln 2. Because the bottom line of the integrand is positive within the range being integrated, there is no need to include the modulus sign.

Part iii) makes use of the standard result again to give $[2 \ln |x|]_{-10}^{-2}$ = 2 (ln 2 – ln 10) $= 2\ln(\frac{1}{5}) = \ln(\frac{1}{25})$.

This time, we had to use the modulus function , since the logarithmic function is not defined for negative numbers.

The integral in part iv) cannot be evaluated, because the function $\frac{1}{x}$ is not defined for **all** values of *x* within the range of the integral (-2 to 2). The problem value here is *x* = 0; $\frac{1}{0}$ is undefined.

This is a general condition for all definite integrals: if the function is undefined for any value of *x* between the limits, the integral cannot be evaluated (at least using techniques learned at A-level).

Examples (15): Find i) $\int \frac{2x+3}{x^2+3x-5}\,dx$ ii) $\int_2^3 \frac{10-8x}{2x^2-5x+4}\,dx$

In i) the top line is the exact derivative of the bottom line, therefore the integral is

$$\ln\left(|\,x^2+3x-5\,|\right)+c \text{ or } \ln\left(|\,A\left(x^2+3x-5\right)|\right).$$

We do need the modulus sign here, because the quadratic within the logarithmic expression can take negative values, e.g. -5 when $x = 0$.

In ii) the top line is the derivative of the bottom line multiplied by –2.

The integral is therefore $\left[-2\ln\left(2x^2-5x+4\right)\right]_2^3 \rightarrow$ -2 (ln 7 – ln 2) .
There is no need to include the modulus sign around the logarithm, since the quadratic has no real roots ($b^2 - 4ac < 0$) and is thus > 0 for all x.

This can be rewritten as a single logarithm using log laws:
$\ln 7 - \ln 2 = \ln \frac{7}{2}$; $-2(\ln \frac{7}{2}) = \ln\left(\frac{7}{2}\right)^{-2} = \ln\left(\frac{4}{49}\right)$.