M.K. HOME TUITION

Mathematics Revision Guides
Level: AS / A Level

AQA : **C3** Edexcel: **C3** OCR: **C3** OCR MEI: **C3**

FURTHER DIFFERENTIATION (TRIG, LOG, EXP FUNCTIONS)

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\ln(2x^2 - 3x + 4)) = \frac{4x-3}{2x^2-3x+4}$$

$$\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cos x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos^2(3x)) = -6 \cos 3x \sin 3x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{(1-x^2)}}$$

Version : 2.2 Date: 29-03-2013

FURTHER DIFFERENTIATION

At AS Level, and when introducing the chain, product and quotient rules, we have dealt with differentiation of functions of type $y = kx^n$.

To recall, any function $y = kx^n$ has a gradient function of $\frac{dy}{dx} = nkx^{n-1}$

In other words, you multiply by the power, and then reduce the power by 1.

Various other techniques allow us to differentiate products, quotients and 'functions of functions'. To recap, they are:

The Chain Rule.

The derivative of a 'function of a function' can be evaluated by using the **chain rule**.
Here y is a function of x and we use an intermediate function u in the working.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx},$$

or, in function notation,

$$g(f(x))' = g'(f(x))\, f'(x)$$

The Product Rule.

If $y = uv$, where u is a function $f(x)$ and v is another function $g(x)$, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

or in function notation, $(f(x)\, g(x))' = f(x)\, g'(x) + f'(x)\, g(x)$.

The Quotient Rule.

If $y = \frac{u}{v}$, where u is a function $f(x)$ and v is another function $g(x)$, then

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

or in function notation, $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$.

Using the result $\frac{dx}{dy} = \frac{1}{dy/dx}$

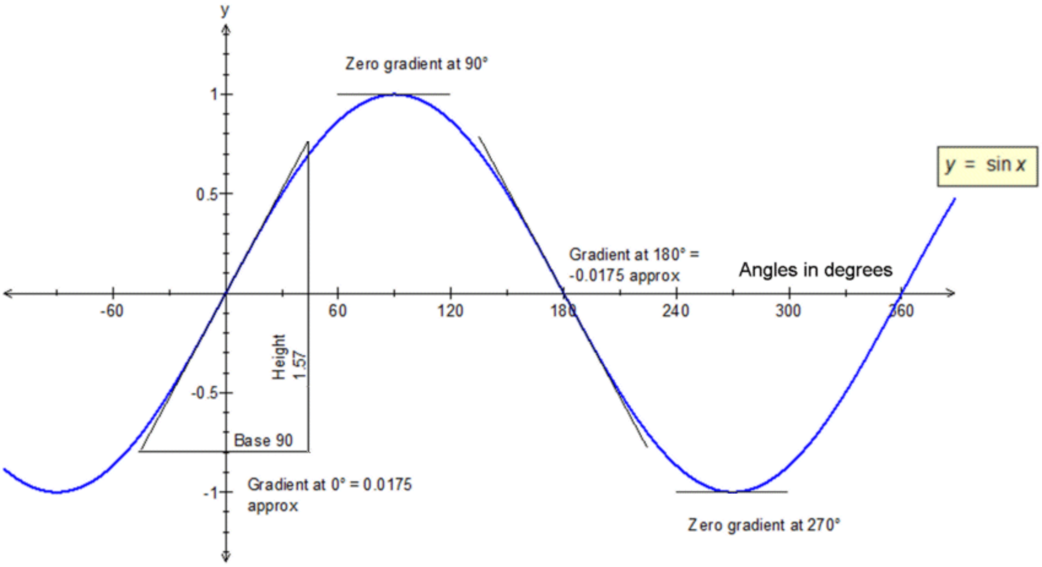
This is another result that is useful for differentiating inverse functions.

Differentiation of Trigonometric Functions.

The two key trigonometric derivatives are:

$$\frac{d}{dx}(\sin x) = \cos x\,; \qquad \frac{d}{dx}(\cos x) = -\sin x$$

This brings to mind an important point. If we were to plot as graph of $y = \sin x°$ and then measure the gradient at various points, we would have the results below:



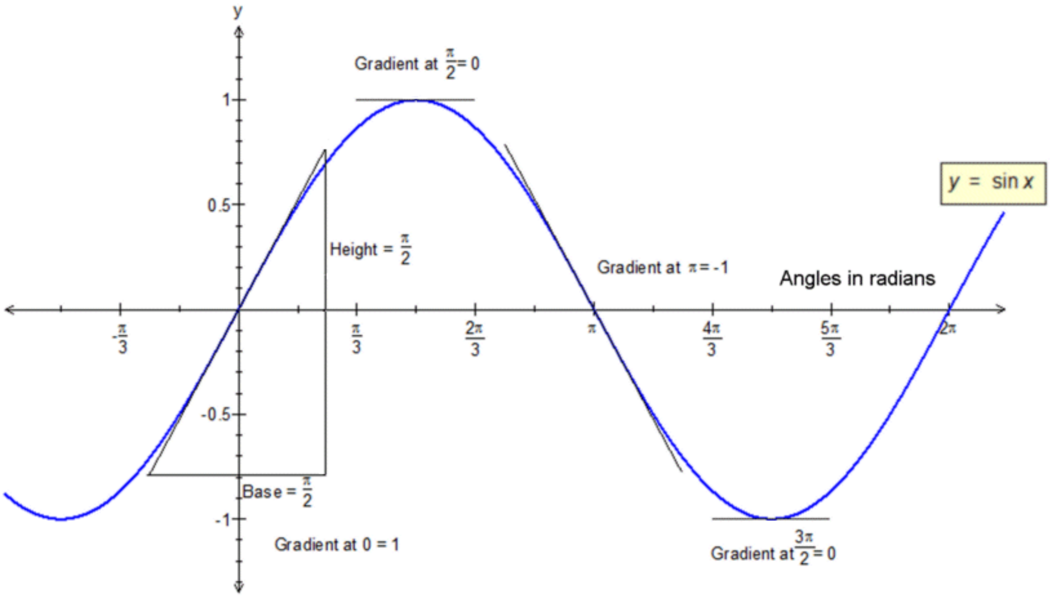
The gradient of $\sin x°$ is equal to zero when x = 90° and 270°, and so is $\cos x°$. But when we come to measure the gradient at x = 0°, we obtain a strange-looking value of 0.0175, whereas $\cos 0° = 1$.
The measured gradient is therefore about 57 times too small.

Similarly, the gradient at x = 180° comes out as -0.0175, but $\cos 180° = -1$.

The gradient seems to be $\cos x° \times 0.0175$, rather than $\cos x°$ itself, so what has gone wrong here ?

The problem is in our choice of units for angle measurement. Whereas degrees are more familiar in everyday use, they are less suitable for calculus.

Now see what happens when we repeat the exercise, but measure our angles in radians.



The gradient of sin x is equal to zero when $x = \pi/2$ and $3\pi/2$, and so is cos x.
But now the gradient at $x = 0$ is equal to 1, which is the same as cos 0.
Also, the gradient at $x = \pi$ is now -1, equal to cos π .

The results have become far more straightforward simply by using radians.

IMPORTANT – Radians are the default units of angle measurement in trig calculus.

The derivatives of the other trigonometric functions can be obtained from the two above:

Example (1): Use the quotient rule to show that $\frac{d}{dx}(\tan x) = \sec^2 x$.

Let $y = \tan x$, $u = \sin x$, $v = \cos x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \Rightarrow \frac{dy}{dx} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos^2 x} = \sec^2 x.$$

(Similar working can be used to show that the derivative of $\cot x$ is $-\mathrm{cosec}^2 x$.)

Example (2): Use the chain rule to show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

Since sec x = 1/cos x, we use u = cos $x \Rightarrow \frac{du}{dx} = -\sin x$

$y = \frac{1}{u} \Rightarrow \frac{dy}{du} = -\frac{1}{u^2} = \frac{-1}{\cos^2 x}$.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{-1}{\cos^2 x} \times (-\sin x) = \frac{\sin x}{\cos^2 x} = \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right).$$

$$\Rightarrow \frac{dy}{dx} = \sec x \tan x.$$

(Similar working can be used to show that the derivative of $\operatorname{cosec} x$ is $-\operatorname{cosec} x \cot x$).

Examples (3): Differentiate (a) cos (4x), (b) sin (π-3x) and (c) tan (2x+ 0.2π).

All these derivatives use the chain rule, where the 'inner' function is linear and the 'outer' function is trigonometric. (Detailed side workings have been omitted).

(a) $f(x)$ = cos (4x) $\Rightarrow$ $f'(x)$ = -4 sin (4x).
(The derivative of cos('thing') is –sin('thing') × derivative of 'thing').

(b) $f(x)$ = sin (π-3x) $\Rightarrow$ $f'(x)$ = -3 cos (π-3x).
(The derivative of sin('thing') is cos('thing') × derivative of 'thing').

(c) $f(x)$ = tan (2x+ 0.2π) $\Rightarrow$ $f'(x)$ = 2 $\sec^2$ (2x+ 0.2π).
(The derivative of tan('thing') is $\sec^2$('thing') × derivative of 'thing').

Examples (4): Differentiate (a) sin $x°$ and (b) tan (5$x°$).

These questions are given in degrees, so the angles must be expressed in radians, where 1° = π/180 radians.

(a) $\sin x° = \sin \frac{\pi x}{180}$ giving a derivative of $\frac{\pi}{180} \cos \frac{\pi x}{180}$.

(b) $\tan 5x° = \tan \frac{\pi x}{36}$ giving a derivative of $\frac{\pi}{36} \sec^2 \frac{\pi x}{36}$.

Examples (5): Differentiate (a) $\sin(x^3)$, (b) $\sin^3 x$, (c) $\sec^4 x$ and (d) $\cos^2(3x)$.

Note that the first two expressions are not the same !
More of the chain rule here:

(a) $f(x) = \sin(x^3) \Rightarrow f'(x) = 3x^2 \cos(x^3)$.
(First take the cube of x, followed by the sine of the result).
End result: derivative of sin('thing') is cos('thing') × derivative of 'thing'.

(b) $f(x) = \sin^3 x \Rightarrow f'(x) = 3\sin^2 x \cos x$.
Remember that $\sin^3 x$ is $(\sin x)^3$, therefore we first take the sine of x, followed by the cube of the result).
End result: derivative of ('thing')^3 is 3('thing')^2 × derivative of 'thing'.

(c) $f(x) = \sec^4 x \Rightarrow f'(x) = (4\sec^3 x)(\sec x \tan x) = 4\sec^4 x \tan x$.
(We first take the secant of x, followed by the fourth power of the result).
End result: derivative of ('thing')^4 is 4('thing')^3 × derivative of 'thing'.

(d) $f(x) = \cos^2(3x) \Rightarrow f'(x) = (2 \cos 3x)(-3 \sin 3x) = -6 \cos 3x \sin 3x$.
This is a double application of the chain rule: first we differentiate $\cos 3x$ as in Examples (1) to obtain $-3 \sin 3x$. We then use this intermediate result to differentiate the square of $\cos 3x$.

Differentiation of Exponential and Logarithmic Functions.

The key exponential derivative is

$\frac{d}{dx}(e^x) = e^x$. The derivative of the function is equal to the function itself.

This property also holds true for e^{x+k}.
This expression can in turn be written as Ae^x, where k is a constant and $A = e^k$.

Example (6): Use $\frac{dx}{dy} = \frac{1}{dy/dx}$ to show that $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

First, rewrite the expression as $x = e^y$
Differentiation gives $\frac{dx}{dy} = e^y$ and therefore $\frac{dy}{dx} = \frac{1}{e^y}$.

Finally, we must rewrite the derivative in terms of x, so substituting x for e^y gives

$\frac{dy}{dx} = \frac{1}{x}$.

Examples (7): Differentiate (a) e^{2x}, (b) $e^{-0.5x}$, (c) e^{x-3}, (d) e^{x^2} (e to the 'x squared')

Use the chain rule: the derivative of 'e to the "thing" ' is just 'e to the "thing" ' × derivative of 'thing'.

The derivative of (a) e^{2x} is therefore $2e^{2x}$; that of (b) $e^{-0.5x}$ is $-0.5e^{-0.5x}$; that of (c) e^{x-3} is e^{x-3} and that of (d) e^{x^2} is $2xe^{x^2}$.

Example (8): Differentiate 2^x.

Because the functions e^x and $\ln x$ are inverses of each other, it follows that $e^{\ln k} = k$ for any positive number k.

We can replace 2^x with $(e^{\ln 2})^x$ or $e^{(x \ln 2)}$, and use the chain rule. (ln 2 is simply a constant.)
The derivative of 2^x is therefore $(e^{(x \ln 2)})(\ln 2)$ or $2^x \ln 2$.

This holds true for any base a: the derivative of a^x is $a^x \ln a$.

Another method is to use $\frac{dy}{dx} = \frac{1}{dx/dy}$.

Let $y = a^x$; taking logs to base e of both sides we have $\ln y = \ln a^x = x \ln a$.

$$x = \frac{1}{\ln a}\ln y \Rightarrow \frac{dx}{dy} = \frac{1}{\ln a}\frac{1}{y} \Rightarrow \frac{dy}{dx} = \frac{1}{dx/dy} = y\ln a = a^x \ln a .$$

Example (9): Differentiate (a) ln(5x), (b) ln(2x^2 - 3x+ 4), (c) $\ln\left(\frac{1}{x^3}\right)$

Using the chain rule, we find that the derivative of 'ln("thing")' is just ' 1 / "thing" ' × derivative of "thing", resulting in a fraction whose top line is the derivative of the bottom line.

(a) The derivative of ln(5x) is thus $\frac{5}{5x}$ or $\frac{1}{x}$.

This rather unexpected result can be explained by the logarithm laws: ln (k(f(x)) = ln k + ln (f(x))

Because ln k is a constant, it disappears during differentiation,

$$\therefore \frac{d}{dx}(\ln k(f(x))) = \frac{d}{dx}(\ln(f(x))).$$

(b) The derivative of ln(2x^2 - 3x+ 4) is similarly $\frac{4x-3}{2x^2-3x+4}$

(top line is the derivative of the bottom line)

(c) The derivative of $\ln\left(\frac{1}{x^3}\right)$ can be found out by either applying the log laws or by the chain rule.

Log laws:

By rewriting $\frac{1}{x^3}$ as x^{-3} we use ln(x^{-3}) = -3 ln x. Differentiation gives $\frac{-3}{x}$.

Chain rule:

Again, rewriting $\frac{1}{x^3}$ as x^{-3} , we obtain the derivative $\frac{-3x^{-4}}{x^{-3}}$ which becomes $\frac{-3}{x}$ when both top and bottom are multiplied by x^4. (The derivative of ln("thing") = derivative of "thing" ÷ "thing").

Differentiation of Inverse Trigonometric Functions.

(Again, the angle *x* <u>must</u> be measured in radians !)

The inverse trigonometric functions can also be differentiated using the rule $\frac{dx}{dy} = \frac{1}{dy/dx}$ and the Pythagorean identities.

Example (10): Differentiate $y = \sin^{-1} x$.

Rewrite the expression as $x = \sin y$.

Differentiation gives $\frac{dx}{dy} = \cos y$ and therefore $\frac{dy}{dx} = \frac{1}{\cos y}$.

Using $\cos^2 y + \sin^2 y = 1$, we can redefine $\cos y$ as $\sqrt{(1-\sin^2 y)}$ or $\sqrt{(1-x^2)}$.

$\therefore$ If $y = \sin^{-1} x$, then $\frac{dy}{dx} = \frac{1}{\sqrt{(1-x^2)}}$.

(Similar working can be used to show that the derivative of $\cos^{-1}x$ is $\frac{-1}{\sqrt{(1-x^2)}}$.).

Example (11): Differentiate $y = \tan^{-1} x$.

Rewrite the expression as $x = \tan y$.

Differentiation gives $\frac{dx}{dy} = \sec^2 y$ and therefore $\frac{dy}{dx} = \frac{1}{\sec^2 y}$.

Using $1 + \tan^2 y = \sec^2 y$, we can redefine $\sec^2 y$ as $1 + \tan^2 y$ or $1+x^2$.

$\therefore$ If $y = \tan^{-1} x$, $\frac{dy}{dx} = \frac{1}{1+x^2}$

(Similar working can be used to show that the derivative of $\cot^{-1}x$ is $\frac{-1}{1+x^2}$).

Example (12): Differentiate $y = \sec^{-1} x$.

Rewrite the expression as $x = \sec y$.

Differentiation gives $\frac{dx}{dy} = \sec y \tan y$ and therefore $\frac{dy}{dx} = \frac{1}{\sec y \tan y}$.

Using $1 + \tan^2 y = \sec^2 y$, we can redefine $\sec y$ as x and $\tan y$ as $\sqrt{(\sec^2 y-1)}$ or $\sqrt{(x^2-1)}$.

$\therefore$ If $y = \sec^{-1} x$, $\frac{dy}{dx} = \frac{1}{x\sqrt{(x^2-1)}}$

(Similar working can be used to show that the derivative of $\text{cosec}^{-1}x$ is $\frac{-1}{x\sqrt{(x^2-1)}}$

Further Miscellaneous Differentiation.

This is a mixed selection of examples bringing together previous methods.

Example (13): Differentiate $x^2 \sin x$.

If $f(x) = x^2 \sin x$, then $f'(x)$ can be obtained using the product rule:
$f'(x) = x^2 \cos x + 2x \sin x$.

Example (14): Differentiate $\dfrac{\sin^3 x}{x}$

The function $f(x) = \dfrac{\sin^3 x}{x}$ can be differentiated using the quotient rule, having used the chain rule (details omitted here) to differentiate the top line first.

$$f'(x) = \frac{x(3\sin^2 x \cos x) - \sin^3 x}{x^2} = \frac{(3x\cos x - \sin x)(\sin^2 x)}{x^2}$$

Example (15): Differentiate $(x^2 + 3x+5) \cos x$.

When $f(x) = (x^2 + 3x+5) \cos x$, then the product rule can again be used to get $f'(x)$.
$f'(x) = (2x+3) \cos x - (x^2 + 3x+5) \sin x$.

Example (16): Differentiate $\dfrac{\sin(x^3)}{\cos x}$.

The quotient and chain rules can again be used to differentiate $f(x) = \dfrac{\sin(x^3)}{\cos x}$

$$f'(x) = \frac{(\cos x)(3x^2 \cos(x^3)) - (\sin(x^3))(-\sin x)}{\cos^2 x}$$

This can be rewritten as $\dfrac{3x^2 \cos(x^3)\cos x + \sin(x^3)\sin x}{\cos^2 x}$ or $\dfrac{3x^2 \cos(x^3) + \sin(x^3)\tan x}{\cos x}$.

Example (17): Differentiate ln (cos x).
A simple application of the chain rule can be used to differentiate $f(x) = \ln(\cos x)$.
Here $f'(x) = \dfrac{-\sin x}{\cos x}$, or $-\tan x$.

Example (18): Differentiate $x^3 e^{2x}$.

The chain and product rules are used to find $f'(x)$ when $f(x) = x^3 e^{2x}$
Here $f'(x) = 3x^2 e^{2x} + (x^3)(2e^{2x})$ or $(3x^2 + 2x^3)(e^{2x})$

or $(x^2)(3 + 2x)(e^{2x})$.

Example (19): Differentiate $2e^x \tan 3x$.

When $f(x) = 2e^x \tan 3x$, again use the chain and product rules

Here $f'(x) = (2e^x)(3\sec^2 3x) + (2e^x \tan 3x) = (2e^x)(3\sec^2 3x + \tan 3x)$.

Example (20): Differentiate $\dfrac{xe^x}{\sin x}$.

Use the chain and quotient rules again to find $f'(x)$ when $f(x) = \dfrac{xe^x}{\sin x}$.

Here $f'(x) = \dfrac{(\sin x)(xe^x + e^x) - (xe^x)(\cos x)}{\sin^2 x}$ which simplifies to

$\dfrac{xe^x(\sin x - \cos x) + e^x \sin x}{\sin^2 x}$ or $\dfrac{xe^x(1 - \cot x) + e^x}{\sin x}$.

Example (21): Differentiate $x^2 \ln(x^3+x)$.

We use the chain and product rules to find the derivative of $f(x) = x^2 \ln(x^3 + x)$.

Here $f'(x) = 2x \ln(x^3 + x) + x^2\left(\dfrac{3x^2+1}{x^3+x}\right)$.

Example (22): Differentiate $x \ln x$, and hence use the logarithm laws to differentiate x^x.

We use the product rule first to find the derivative of $x \ln x$;

it is $x\left(\dfrac{1}{x}\right) + 1.\ln x$ or $1 + \ln x$.

Then, we use the fact that $x = e^{\ln x}$ to rewrite $x^x = (e^{\ln x})^x \Rightarrow x^x = e^{x \ln x}$.

If we differentiate '*e* to the "thing" ' we have just '*e* to the "thing" ' × derivative of 'thing'.

Hence $\dfrac{d}{dx}(x^x) = e^{x\ln x} \times (1 + \ln x) = x^x(1 + \ln x)$.

N.B. Do not try to use $\dfrac{dy}{dx} = nx^{n-1}$ and substitute x for n to get

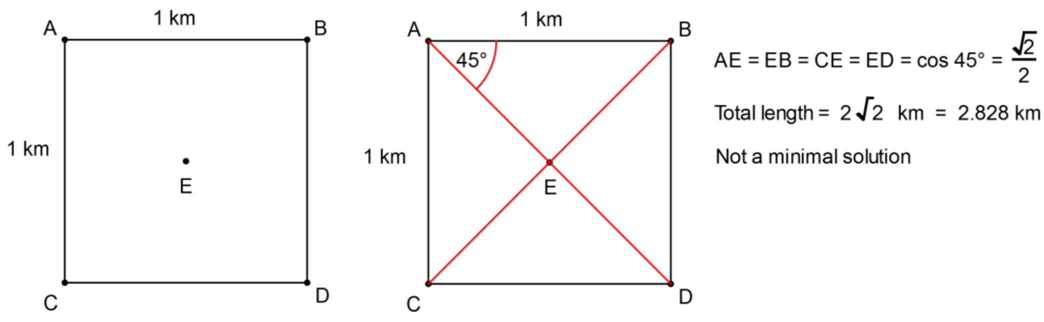
$$y = x^x \Rightarrow \frac{dy}{dx} = x(x^{x-1}) \Rightarrow \frac{dy}{dx} = x^x$$

This is totally incorrect, because we are using a *variable*, x, as if it were a *constant*, n.

Example (23): This is an old problem which can be solved using calculus.

A leisure park has a miniature railway network running around the four vertices *A*, *B*, *C* and *D* of a square of side 1 km, with stops at each vertex.

The park bosses have decided to connect the meeting point at *E*, in the exact centre of the square, to the other four stops. Calculate the minimum length of new railway track needed, disregarding any curves.

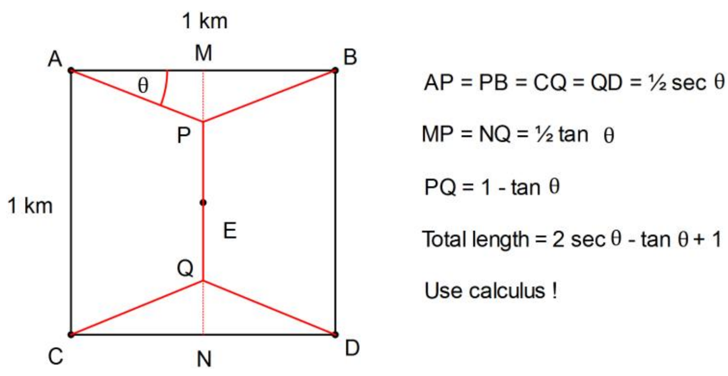


It might be thought that laying the track across the diagonals of the square would provide a minimal solution, but this is not the case !

We need a different approach here, where we use five separate sections of track as shown on the right.

The triangles *APB* and *CQD* are isosceles and congruent, but we are free to vary the angle θ from just over 0° to 45° (the case of the diagonals of the square).

The total length of new track needed, in kilometres, is
$x = 2 \sec \theta - \tan \theta + 1$.



We therefore need to find the value of θ for which *x* takes a minimum value, i.e. $\frac{dx}{d\theta} = 0$.

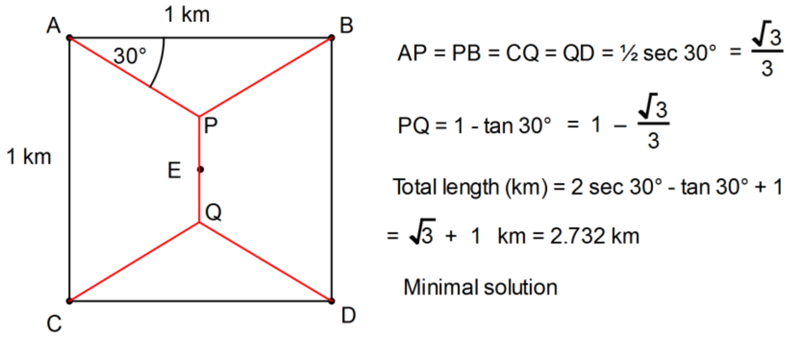
$$x = 2\sec\theta - \tan\theta + 1 \;\Rightarrow\; \frac{dx}{d\theta} = 2\sec\theta\tan\theta - \sec^2\theta$$

We thus solve $2 \sec \theta \tan \theta - \sec^2 \theta = 0 \Rightarrow 2 \sec \theta \tan \theta = \sec^2 \theta$

$\Rightarrow 2 \sec \theta \tan \theta = \sec^2 \theta \;\Rightarrow\; 2 \cos \theta \tan \theta = 1$ (multiply both sides by $\cos^2\theta$)

$\Rightarrow$ $2 \sin \theta = 1 \;\Rightarrow\; \sin \theta = ½ \;\Rightarrow$ **θ = 30°** or **π/6.**

The details of the minimal solution are shown below.



Examples (24): Differentiate (a) $\sin^{-1}(5x)$, (b) $\sin^{-1}\left(\frac{x}{3}\right)$, (c) $\sin^{-1}\left(\frac{4x}{5}\right)$.

Use the result $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{(1-x^2)}}$.

We use the chain rule in each case :

(a) $f(x) = \sin^{-1}(5x) \Rightarrow f'(x) = \frac{1}{\sqrt{1-(5x)^2}} \times 5$

or $\frac{5}{\sqrt{1-25x^2}}$.

(b) $f(x) = \sin^{-1}\left(\frac{x}{3}\right) \Rightarrow f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \times \frac{1}{3}$ or $\frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}x^2}}$.

Multiplying both top and bottom by 3 to get rid of the fractions gives $\frac{1}{\sqrt{9}\sqrt{1-\frac{1}{9}x^2}}$ and finally

$\frac{1}{\sqrt{9-x^2}}$.

(c) $f(x) = \sin^{-1}\left(\frac{4x}{5}\right) \Rightarrow f'(x) = \frac{1}{\sqrt{1-\left(\frac{4x}{5}\right)^2}} \times \frac{4}{5}$ or $\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}x^2}}$.

Multiplying both top and bottom by 5 to get rid of the fractions gives $\frac{4}{\sqrt{25}\sqrt{1-\frac{16}{25}x^2}}$ and finally

$\frac{4}{\sqrt{25-16x^2}}$.

From this, we can establish some general formulae:

If $f(x) = \sin^{-1}(ax)$, then $f'(x) = \frac{a}{\sqrt{1-a^2x^2}}$.

If $f(x) = \sin^{-1}\left(\frac{x}{a}\right)$, then $f'(x) = \frac{1}{\sqrt{b^2-x^2}}$

If $f(x) = \sin^{-1}\left(\frac{ax}{b}\right)$, then $f'(x) = \frac{a}{\sqrt{b^2-a^2x^2}}$

(The corresponding formulae for variants of $\cos^{-1} x$ are very similar, except that the top line of the fraction has the sign reversed).

Examples (25): Differentiate (a) $\tan^{-1}(3x)$, (b) $\tan^{-1}\left(\frac{x}{4}\right)$, (c) $\tan^{-1}\left(\frac{5x}{2}\right)$.

Use the result $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$.

Again we use the chain rule in each case :

(a) $f(x) = \tan^{-1}(5x) \Rightarrow f'(x) = \frac{1}{1+(5x)^2} \times 5$

or $\frac{5}{1+25x^2}$.

(b) $f(x) = \tan^{-1}\left(\frac{x}{4}\right) \Rightarrow f'(x) = \frac{1}{1+\left(\frac{x}{4}\right)^2} \times \frac{1}{4}$ or $\frac{\frac{1}{4}}{1+\frac{1}{16}x^2}$.

Multiplying both top and bottom by 4^2 or 16 to get rid of the fractions gives $\frac{4}{16+x^2}$.

(c) $f(x) = \tan^{-1}\left(\frac{5x}{2}\right) \Rightarrow f'(x) = \frac{1}{1+\left(\frac{5x}{2}\right)^2} \times \frac{5}{2}$ or $\frac{\frac{5}{2}}{1+\frac{25}{4}x^2}$.

Multiplying both top and bottom by 2^2 or 4 to get rid of the fractions gives $\frac{10}{4+25x^2}$.

The above results can be used to derive the general formulae:

If $f(x) = \tan^{-1}(ax)$, then $f'(x) = \frac{a}{1+a^2x^2}$

If $f(x) = \tan^{-1}\left(\frac{x}{a}\right)$, then $f'(x) = \frac{a}{a^2+x^2}$

If $f(x) = \tan^{-1}\left(\frac{ax}{b}\right)$, then $f'(x) = \frac{ab}{b^2+a^2x^2}$