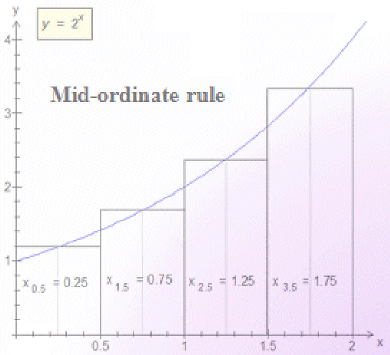
M.K. HOME TUITION

Mathematics Revision Guides
Level: AS / A Level

AQA : **C3** OCR: **C3**

METHODS OF NUMERICAL INTEGRATION –MID-ORDINATE AND SIMPSON'S RULES



$I = h(y_{0.5} + y_{1.5} + y_{2.5} + \ldots\ldots y_{n-0.5})$

n	x_n	$x_{n+0.5}$	$y_{n+0.5}$
0	0		
		0.25	1.189
1	0.5		
		0.75	1.682
2	1		
		1.25	2.378
3	1.5		
		1.75	3.364
4	2		
TOTAL			**8.613**
Multiply by *h* (here 0.5)			**4.31**

Mid-ordinate rule

$\int_0^2 2^x\,dx \approx 4.31$

Simpson's Rule $I = \frac{1}{3}h\left\{(y_0 + y_n) + 4(y_1 + y_3 + \ldots\ldots y_{n-1}) + 2(y_2 + y_4 + \ldots\ldots y_{n-2})\right\}$

$y = 2^x \quad b\text{-}a = 2 \quad h = 0.5 \quad n = 4$

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	1	1	1
1	0.5	$\sqrt{2}$	4	$4\sqrt{2}$
2	1	2	2	4
3	1.5	$2\sqrt{2}$	4	$8\sqrt{2}$
4	2	4	1	4
TOTAL				**25.9706**
Multiply by $\frac{1}{3}h$ (here $h = 0.5$)				**4.3284**

$\int_0^2 2^x\,dx \approx 4.328$

$y = \dfrac{4}{1+x^2} \quad b\text{-}a = 1 \quad h = 0.25 \quad n = 4$

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	4	1	4
1	0.25	$64/17$	4	$256/17$
2	0.5	$16/5$	2	$32/5$
3	0.75	$64/25$	4	$256/25$
4	1	2	1	2
TOTAL				**37.69882**
Multiply by $\frac{1}{3}h$ (here $h = 0.25$)				**3.14157**

$\int_0^1 \frac{4}{1+x^2}\,dx \approx 3.1416$

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METHODS OF NUMERICAL INTEGRATION.

In Core Unit 2 we used the trapezium rule to provide numerical approximations to definite integrals.

The trapezium rule – revision.

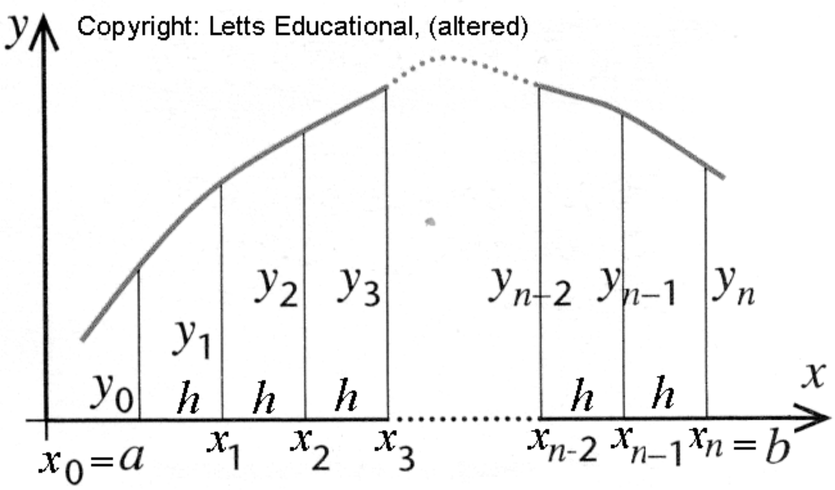
In the diagram below, the value of $I = \int_a^b f(x)dx$ represents the area under the graph below of $y = f(x)$ between the points $x = a$ and $x = b$.

The area is divided into strips of equal width , $h = \frac{b-a}{n}$ where n is the number of strips.

Moving from left to right, the first strip has an area of $\frac{1}{2}h(y_0 + y_1)$, the second one an area of $\frac{1}{2}h(y_1 + y_2)$, the third $\frac{1}{2}h(y_2 + y_3)$ and so on until the last strip whose area is $\frac{1}{2}h(y_{n-1} + y_n)$.

Note how each y-value is counted double except the first and the last.
Taking out $\frac{1}{2}h$ as a factor, the total area I can be given by the trapezium rule as

$$I = \tfrac{1}{2}h(y_0 + 2(y_1 + y_2 + ...y_{n-1}) + y_n)$$



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Example (1): Use the trapezium rule with 8 strips to estimate $\int_0^2 2^x\,dx$ to two decimal places.

We will use the rule $I = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + ...y_{n-1}) + y_n)$

The number of strips n = 8, the interval is $b - a$ = 2, and so the width of a single strip, h, is 0.25.

Using $I = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + ...y_{n-1}) + y_n)$, the results are:

y **= 2*x*** ***b-a*** **= 2** ***h*** **= 0.25** ***n*** **= 8**

(**Short form of table** – other *y*-values doubled *after* summing).

n	x_n	y_n (first & last)	y_n (other values)
0	0	1	
1	0.25		1.189
2	0.5		1.414
3	0.75		1.682
4	1		2
5	1.25		2.378
6	1.5		2.828
7	1.75		3.364
8	2	4	
Sub-totals (1):		**5**	**14.855**
			(× 2)
Sub-totals (2):		**5**	**29.711**
TOTAL			**34.711**
Multiply by ½*h* (here ½ × 0.25)			**4.339**

The value of $\int_0^2 2^x\,dx$ is therefore estimated at 4.34 to two decimal places .
The true value for the integral is 4.3281 square units, and therefore the error has been reduced to just one unit in the second decimal place, or about 0.2%.

We have used decimal approximations here using 3 decimal places but again we could have used exact calculator values.

We could have also written the sum out in linear form:
$\int_0^2 2^x\,dx$ ≈ ½ (0.25(1 + 2(1.189 + 1.414 + 1.682 + 2 + 2.378 + 2.828 + 3.364) + 4)) or **4.34**.

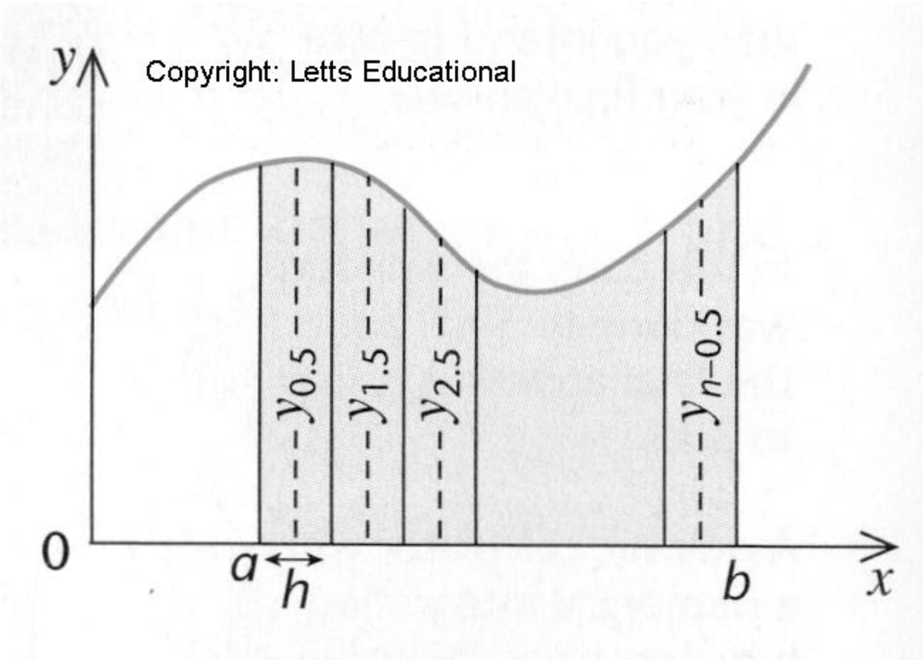
Because the trapezium rule is only of moderate accuracy, we shall look at two other methods of numerical evaluation of definite integrals.

The mid-ordinate rule (AQA only).

In this method, the area to be integrated is to be split into rectangles of equal width, rather than into trapezia.

In the diagram below, the value of $I = \int_a^b f(x)dx$ represents the area under the graph below of $y = f(x)$ between the points $x = a$ and $x = b$.

The area is divided into strips of equal width , $h = \frac{b-a}{n}$ where n is the number of strips.



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This time, the height of each rectangle corresponds to the y-value **halfway along** each rectangle while all the rectangles have a width of h. Thus the first rectangle will have a height corresponding to the y-value for x halfway between x_0 and x_1 - namely a height of $y_{0.5}$. The height of the second rectangle will correspond to the y-value for x halfway between x_1 and x_2 - namely a height of $y_{1.5}$. This continues until we reach the mean of the final x-values, x_{n-1} and x_n and the final y-value, $y_{n-0..5.}$

The formula can be expressed as $I = h\,(y_{0.5} + y_{1.5} + y_{2.5} + ...y_{n-0.5})$

where $h = \frac{b-a}{n}$.

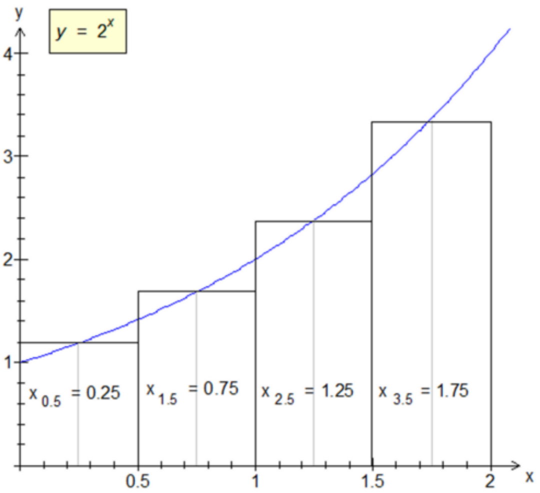
Example (2): Use the mid-ordinate rule with 4 strips to estimate the value of $\int_0^2 2^x \, dx$ to two decimal places.

We will use the rule

$$I = h(y_{0.5} + y_{1.5} + y_{2.5} + ...y_{n-0.5})$$

The number of strips n = 4, the interval is $b - a$ = 2, and so the width of a single strip, h, is 0.5.

It is best to tabulate the values for ease of checking; also, work to one more decimal place than is required for the final answer (or keep values stored in multiple calculator memories).



y* = 2*^x^ ***b-a* = 2** ***h* = 0.5**
***n* = 4**

n	x_n	$x_{n+0.5}$	$y_{n+0.5}$
0	0		
		0.25	1.189
1	0.5		
		0.75	1.682
2	1		
		1.25	2.378
3	1.5		
		1.75	3.364
4	2		
TOTAL			**8.613**
Multiply by *h* (here 0.5)			**4.31**

The value of $\int_0^2 2^x \, dx$ is therefore estimated at 4.31 to two decimal places .

The working is simpler than in the trapezium rule in that all of the y-values are summed singly.

The true value is 4.3281, and therefore the estimate is in error by about 0.5%.

In linear form:

$\int_0^2 2^x \, dx$ ≈ 0. 5(1.189 + 1.682 + 2.378 + 3.364) or **4.31**.

Again, the accuracy can be improved by increasing the number of strips.

Example (3): Repeat Example 2, but this time use 8 strips to estimate the value of $\int_0^2 2^x \, dx$ to two decimal places.

We will use the rule $I = h(y_{0.5} + y_{1.5} + y_{2.5} + \ldots\ldots y_{n-0.5})$
The number of strips n = 8, the interval is $b - a$ = 2, and so the width of a single strip, h, is 0.25.
This time, the table of values will look like this:

y* = 2*ˣ → $\boldsymbol{y = 2^x}$ **b-a = 2** **h = 0.25** **n = 8**

n	x_n	$x_{n+0.5}$	$y_{n+0.5}$
0	0		
		0.125	1.091
1	0.25		
		0.375	1.297
2	0.5		
		0.625	1.542
3	0.75		
		0.875	1.834
4	1		
		1.125	2.181
1	1.25		
		1.375	2.594
2	1.5		
		1.625	3.084
3	1.75		
		1.875	3.668
4	2		
TOTAL			**17.291**
Multiply by *h* (here 0.25)			**4.323**

The value of $\int_0^2 2^x \, dx$ is therefore estimated at 4.32 to two decimal places .

The error has been reduced to 0.005 compared to the true value of 4.3281, or about 0.1%. The estimate to two decimal places is still out by a digit in the second place, but it is an improvement upon the trapezium rule.

Example (4): Use the mid-ordinate rule with 4 strips to estimate the value of $\int_0^1 \frac{1}{1+x^2}\,dx$ to three decimal places.

The number of strips n = 4, the interval is $b - a$ = 1, and so the width of a single strip, h, is 0.25.
Using $I = h(y_{0.5} + y_{1.5} + y_{2.5} + ...y_{n-0.5})$, the results are:

$y = \frac{1}{1+x^2}$ ***b-a* = 1** ***h* = 0.25** ***n* = 4**

(Fractions have been used here, as modern calculators can display such results in exact form.)

n	x_n	$x_{n+0.5}$	$y_{n+0.5}$
0	0		
		0.125	$64/65$
1	0.25		
		0.375	$64/73$
2	0.5		
		0.625	$64/89$
3	0.75		
		0.875	$64/113$
4	1		
TOTAL			**3.147**
Multiply by *h* (here 0.25)			**0.787**

The value of $\int_0^1 \frac{1}{1+x^2}\,dx$ is approximately 0.787 to three decimal places.

In linear form:

$\int_0^1 \frac{1}{1+x^2}\,dx \approx 0.25\left(\frac{64}{65} + \frac{64}{73} + \frac{64}{89} + \frac{64}{113}\right)$ or **0.787**.

(The true value is $\frac{\pi}{4}$ or 0.7854 to four decimal places – the total above it is a rough value for π).

Example (5): Use the mid-ordinate rule with 4 strips to estimate the value of $\int_0^4 x\sqrt{(x^2+9)}\,dx$ to one decimal place.

The number of strips n = 4, the interval is $b - a$ = 4, and so the width of a single strip, h, is 1.

y **=** $x\sqrt{(x^2+9)}$ ***b-a*** **= 4** ***h*** **= 1** ***n*** **= 4**

n	x_n	$x_{n+0.5}$	$y_{n+0.5}$
0	0		
		0.5	1.52
1	1		
		1.5	5.03
2	2		
		2.5	9.76
3	3		
		3.5	16.13
4	4		
TOTAL			**32.44**
Multiply by *h* (here 1)			**32.4**

The value of $\int_0^4 x\sqrt{(x^2+9)}\,dx$ is approximately 32.4 to one decimal place.

(The true value is $32\frac{2}{3}$, suggesting an error of just under 1%).

In linear form:

$\int_0^4 x\sqrt{(x^2+9)}\,dx \approx$ 1(1.52 + 5.03 + 9.76 + 16.13) or **32.4**.

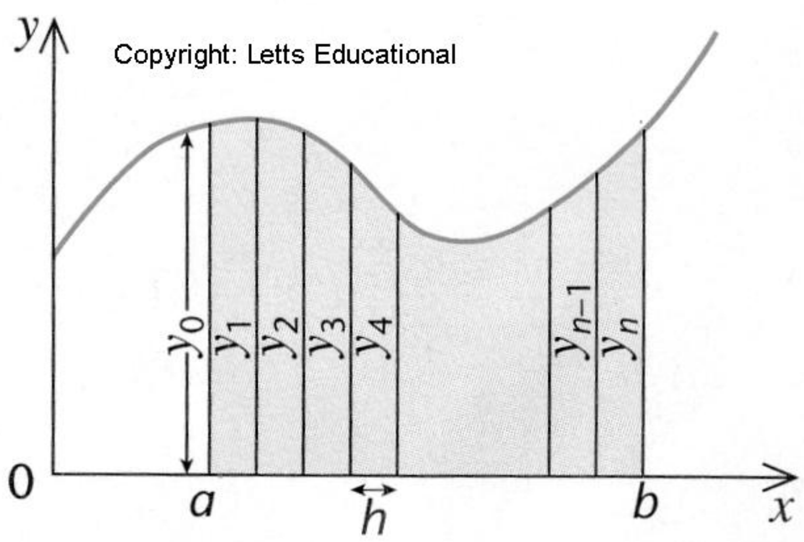
Simpson's Rule. (AQA, OCR only)

The trapezium and mid-ordinate rule provided reasonable numerical approximations to definite integrals, but Simpson's rule is considerably more accurate.

This method divides the area to be integrated into an **even** number of parallel strips. The theory of the process is outside the scope of the syllabus, but the idea is to take three ordinates at a time and join them by quadratic arcs rather than by joining two points at a time as in the trapezium rule.

In the diagram below, the value of $I = \int_a^b f(x)\,dx$ represents the area under the graph below of $y = f(x)$ between the points $x = a$ and $x = b$.

The area is divided into strips of equal width , $h = \frac{b-a}{n}$ where n is an **even** number of strips.



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Moving from left to right, the area covered by the first **two** strips is given as $\frac{1}{3}h(y_0 + 4y_1 + y_2)$, the area covered by the next two strips is given as $\frac{1}{3}h(y_2 + 4y_3 + y_4)$, and so on until the last two strips whose area is $\frac{1}{3}h(y_{n-2} + 4y_{n-1} + y_n)$.

Notice the pattern in which the ordinates (y-values) are repeated before multiplying by $\frac{1}{3}h$. The first and last ones (y_0 and y_n) are counted once, the 'odd' ones (y_1 , y_3 and so forth) are counted 4 times, and the other 'even' ones are counted twice.

This pattern of multipliers for the y-values can be visualised as :

1, 4, 1 for three ordinates (2 strips)
1, 4, 2, 4, 1 for five ordinates (4 strips)
1, 4, 2, 4, 2, 4, 1 for seven ordinates (6 strips)
1, 4, 2, 4, 2, 4, 2, 4, 1 for nine ordinates (8 strips), and so forth.

Simpson's rule can therefore be given as

$$I = \frac{1}{3}h\left\{(y_0 + y_n) + 4(y_1 + y_3 + ...y_{n-1}) + 2(y_2 + y_4 + ...y_{n-2})\right\}$$

This is easier than it looks, as worked examples will show later.

With the simplest case (two strips), the approximation of the integral is $\frac{1}{3}h(y_0 + 4y_1 + y_2)$.

Example (6): Use Simpson's rule with 4 strips to estimate $\int_0^2 2^x \, dx$ to three decimal places.

The number of strips n = 4, the interval is $b - a$ = 2, and so the width of a single strip, h, is 0.5.

It is best to tabulate the values for ease of checking; also, work to one more decimal place than is required for the final answer (or keep values stored in multiple calculator memories).
Examination questions will usually be limited to 2 or 4 strips due to time issues.

There are various ways of tabulating the values – choose the one which suits you best !

Table style 1.

This style uses a separate column for the multipliers. Note the pattern: × 1 for the first and last ordinates, × 4 for the 'odd' ordinates, and × 2 for other 'even' ordinates. We then add another column for the multiplied y-values, and finish by multiplying the total by $\frac{1}{3}h$.

$\boldsymbol{y = 2^x}$ $\boldsymbol{b\text{-}a = 2}$ $\boldsymbol{h = 0.5}$ $\boldsymbol{n = 4}$

n	x_n	y_n	Multiplier	y_n × Multiplier
0	0	1	1	1
1	0.5	√2	4	4√2
2	1	2	2	4
3	1.5	2√2	4	8√2
4	2	4	1	4
TOTAL				**25.9706**
Multiply by $\frac{1}{3}h$ **(here** h **= 0.5)**				**4.3284**

(Surds have been used here up to the final totalling.)

Table style 2.

the first and last ordinates (y-values) are added in one column, the 'odd' ordinates added in another, and the remaining 'even' ordinates added in a third. The final multiplications are performed at the end.

$\boldsymbol{y = 2^x}$ $\boldsymbol{b\text{-}a = 2}$ $\boldsymbol{h = 0.5}$ $\boldsymbol{n = 4}$

n	x_n	y_n (first & last)	y_n ('odd' n)	y_n (other 'even' n)
0	0	1		
1	0.5		√2	
2	1			2
3	1.5		2√2	
4	2	4		
Sub-totals (1):		**5**	**3√2**	**2**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**5**	**12√2**	**4**

TOTAL	**25.9706**
Multiply by $\frac{1}{3}h$ **(here** h **= 0.5)**	**4.3284**

The value of $\int_0^2 2^x \, dx$ is therefore estimated at 4.328 to three decimal places .
This is far closer to the true value of 4.3281, demonstrating the superiority of Simpson's rule. By comparison, the trapezium rule had an error in the **second** decimal place using twice as many strips.

It is possible to show a Simpson's rule working in linear form, but it is best restricted to the simplest case, namely with two strips.

Example (6): Use Simpson's rule with 2 strips to show that the value of $\int_2^4 3\ln x\,dx$ is approximately ln 648.

The number of strips n = 2, the interval is $b - a$ = 2, and so the width of a single strip, h, is 1.

Also $x_0 = 2$, $x_1 = 3$ and $x_2 = 4$.

The approximate value of the integral is $\frac{1}{3}h(y_0 + 4y_1 + y_2)$ or
$\frac{1}{3}(1)(3\ln 2 + 12\ln 3 + 3\ln 4)$.

This simplifies to ln 2 + 4 ln 3 + ln 4 , or ln($2 \times 3^4 \times 4$) = ln (2 × 81 × 4) = ln 648.

The tabular layout would look like this:

y **=** $3\ln x$ ***b-a*** **= 2** ***h*** **= 0.5** ***n*** **= 2**

Table style 1.

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	2	3 ln 2	1	3 ln 2
1	3	3 ln 3	4	12 ln 3
2	4	3 ln 4	1	3 ln 4 (= 6 ln 2)
TOTAL				**9 ln 2 + 12 ln 3**
Multiply by $\frac{1}{3}h$ **(here *h* =** 1)				**3 ln 2 + 4 ln 3 = ln 648**

Example (8): Use Simpson's rule with 4 strips to estimate the value of $\int_0^1 \frac{4}{1+x^2}\,dx$ to four decimal places.

The number of strips n = 4, the interval is $b - a$ = 1, and so the width of a single strip, h, is 0.25.

$y = \frac{4}{1+x^2}$ **b-a = 1** **h = 0.25** **n = 4**

(Exact fractions used here.)

Table style 1.

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	4	1	4
1	0.25	$64/17$	4	$256/17$
2	0.5	$16/5$	2	$32/5$
3	0.75	$64/25$	4	$256/25$
4	1	2	1	2
TOTAL				**37.69882**
Multiply by $\frac{1}{3}h$ (here h = 0.25)				**3.14157**

Table style 2.

n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	4		
1	0.25		$64/17$	
2	0.5			$16/5$
3	0.75		$64/25$	
4	1	2		
Sub-totals (1):		**6**	**6.32471**	**3.20000**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**6**	**25.29882**	**6.40000**

TOTAL	**37.69882**
Multiply by $\frac{1}{3}h$ (here h = 0.25)	**3.14157**

The value of $\int_0^1 \frac{4}{1+x^2}\,dx$ is approximately 3.1416 to four decimal places.

(The true value is π itself – this value agrees to four decimal places.)

Once again, increasing the number of strips leads to an increase in the accuracy.

Example (9): Use Simpson's rule with 8 strips to estimate the value of $\int_0^1 \frac{4}{1+x^2}\,dx$. Use 7 decimal places throughout. How accurate is the calculated value of π ?

The number of strips n = 8, the interval is $b - a$ = 1, and so the width of a single strip, h, is 0.125.

$y = \frac{4}{1+x^2}$ **b-a = 1** **h = 0.125** **n = 8**

Table style 1.

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	4.0000000	1	4.0000000
1	0.125	3.9384615	4	15.7538462
2	0.25	3.7647059	2	7.5294118
3	0.375	3.5068493	4	14.0273973
4	0.5	3.2000000	2	6.4000000
5	0.625	2.8764045	4	11.5056180
6	0.75	2.5600000	2	5.1200000
7	0.875	2.2654867	4	9.0619469
8	1	2.0000000	1	2.0000000
TOTAL				**75.3982201**
Multiply by $\frac{1}{3}h$ (here h = 0.125)				**3.1415925**

Table style 2.

n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	4.0000000		
1	0.125		3.9384616	
2	0.25			3.7647059
3	0.375		3.5068493	
4	0.5			3.2000000
5	0.625		2.8764045	
6	0.75			2.5600000
7	0.875		2.2654867	
8	1	2.0000000		
Sub-totals (1):		**6.00000**	**12.5872021**	**9.5247059**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**6.00000**	**50.3488083**	**19.0494118**

TOTAL	**75.3982201**
Multiply by $\frac{1}{3}h$ (here h = 0.125)	**3.1415925**

$\int_0^1 \frac{4}{1+x^2}\,dx \approx 3.1415925$ to 7 decimal places, using Simpson's rule with 8 strips.

This agrees with the true value of π to 6 decimal places and is only 1 digit out in the seventh.

Example (10): Use Simpson's rule with 4 strips to estimate the value of $\int_0^4 x\sqrt{(x^2+9)}\,dx$ to two decimal places.

The number of strips n = 4, the interval is $b - a$ = 4, and so the width of a single strip, h, is 1.

$y = x\sqrt{(x^2+9)}$ **b-a = 4** **h = 1** **n = 4**

(Exact surds used throughout)

Table style 1.

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	0	1	0
1	1	√10	4	4 √10
2	2	2√13	2	4√13
3	3	3√18	4	12√18
4	4	20	1	20
TOTAL				**97.983**
Multiply by $\frac{1}{3}h$ (here h = 1)				**32.661**

Table style 2.

n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	0		
1	1		√10	
2	2			2√13
3	3		3√18	
4	4	20		
Sub-totals (1):		**20**	**√10 + 3√18**	**2√13**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**20**	**4√10 + 12√18**	**4√13**

TOTAL	**97.983**
Multiply by $\frac{1}{3}h$ (here h = 1)	**32.661**

$\int_0^4 x\sqrt{(x^2+9)}\,dx \approx$ 32.66 to 2 decimal places.

The true value of the integral is $32\frac{2}{3}$, or 32.67 to 2 decimal places – an error of just one digit in the second decimal place.

Example (11): The function *f* is defined as:

$f(x) = 1$ for $x = 0$; $\dfrac{\sin x}{x}$ for all other x.

Use Simpson's rule with 4 strips to estimate the value of $\int_0^{\pi/3} f(x)\,dx$ to 4 decimal places.

The number of strips $n = 4$, the interval is $b - a = \pi/3$, and so the width of a single strip, h, is $\pi/12$.

$y = 1$ for $x = 0$; $\dfrac{\sin x}{x}$ for all other x $\quad$ $b\text{-}a = \pi/3$ $\quad$ $h = \pi/12$ $\quad$ $n = 4$

Table style 1.

n	x_n	y_n	Multiplier	$y_n \times$ Multiplier
0	0	1	1	1.00000
1	π/12	0.98862	4	3.95446
2	π /6	0.95493	2	1.90986
3	π /4	0.90032	4	3.60127
4	π /3	0.82699	1	0.82699
TOTAL				**11.29258**
Multiply by $\frac{1}{3}h$ (here $h = \pi/12$)				**0.98546**

Table style 2.

n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	1		
1	π/12		0.98862	
2	π /6			0.95493
3	π /4		0.90032	
4	π /3	0.82699		
Sub-totals (1):		**1.82699**	**1.88893**	**0.95493**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**1.82699**	**7.55573**	**1.90986**

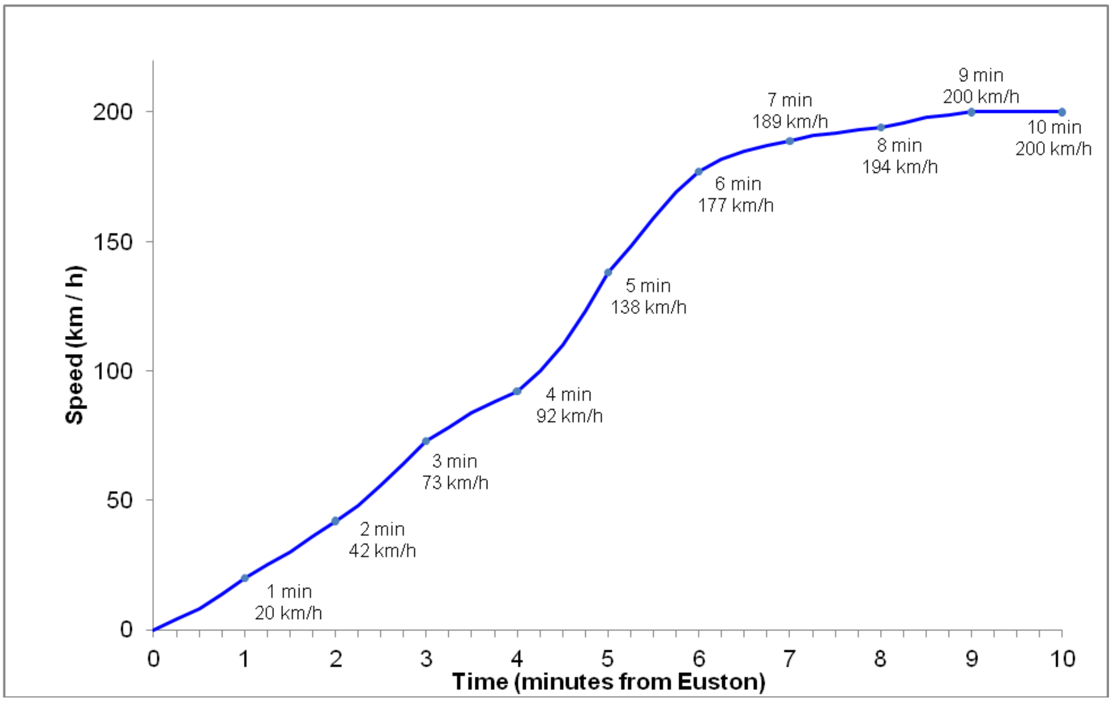
TOTAL	**11.29258**
Multiply by $\frac{1}{3}h$ (here $h = \pi/12$)	**0.98546**

Hence $\int_0^{\pi/3} f(x)\,dx \approx 0.9855$ to 4 decimal places.

(The actual function cannot be integrated analytically).

The previous examples were purely mathematical in nature, but the method can equally be applied in real-life situations. Two such examples follow from here.

Example (12): The travel graph below shows the speed (in km/h) of a train leaving London's Euston station, over a ten-minute time interval.



Given that the area under the curve is equal to the distance travelled, use Simpson's rule with 10 strips to find the distance covered by the train during this 10-minute interval. (Do not forget to divide *h* by 60 due to the use of km/h as the unit of speed).

The number of strips n = 10, and the width of a single strip, h, is $\frac{1}{60}$ after adjustment.

Table style 2.

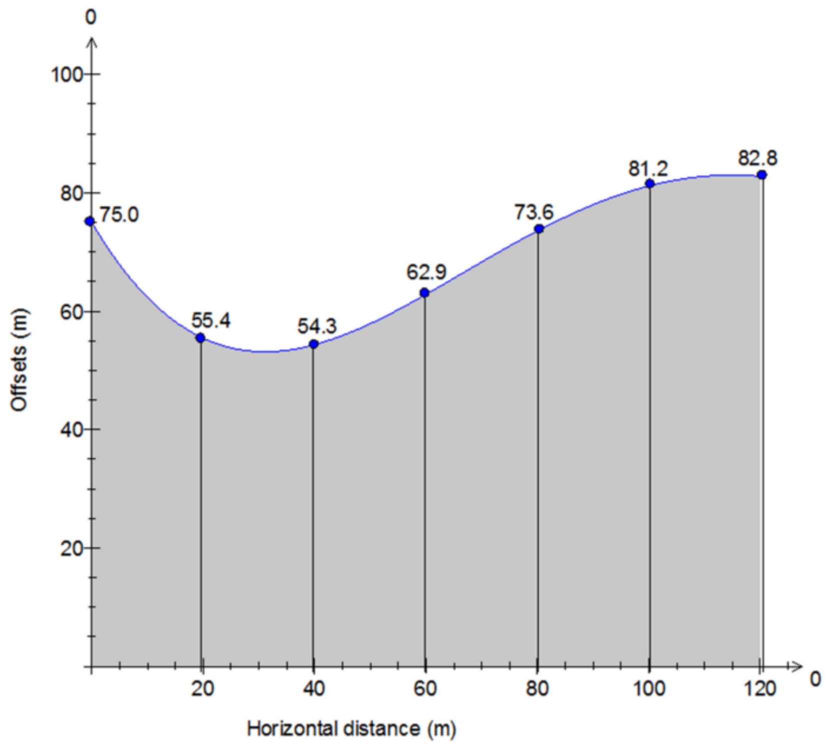
n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	0		
1	1		20	
2	2			42
3	3		73	
4	4			92
5	5		138	
6	6			177
7	7		189	
8	8			194
9	9		200	
10	10	200		
Sub-totals (1):		**200**	**620**	**505**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**200**	**2480**	**1010**

TOTAL	**3690**
Multiply by $\frac{1}{3}h$ **(here** $h = \frac{1}{60}$ **)**	**20.50**

∴ The train has covered **20.50 km** in 10 minutes, using Simpson's rule with 10 strips.

Example (13): Estimate the area of the plot of land below, divided into 20m-wide strips. The perpendicular offset distances from the baseline are also shown here.

Use Simpson's rule with 6 strips to estimate the area in hectares to 3 significant figures.
(1 hectare = 10,000 m^2.)



Here, h = 20 and the y-values are the offsets.

Table style 2.

n	x_n	y_n (first & last)	y_n ('odd' y)	y_n (other 'even' y)
0	0	75.0		
1	20		55.4	
2	40			54.3
3	60		62.9	
4	80			73.6
5	100		81.2	
6	120	82.8		
Sub-totals (1):		**157.8**	**199.5**	**127.9**
(After multiplication)			(× 4)	(× 2)
Sub-totals (2):		**157.8**	**798.0**	**255.8**

TOTAL	**1211.6**
Multiply by $\frac{1}{3}h$ **(here** h **= 20)**	**8077**

∴ Estimated area of the plot = **8080 m^2, or 0.808 hectares**, using Simpson's rule with 6 strips.