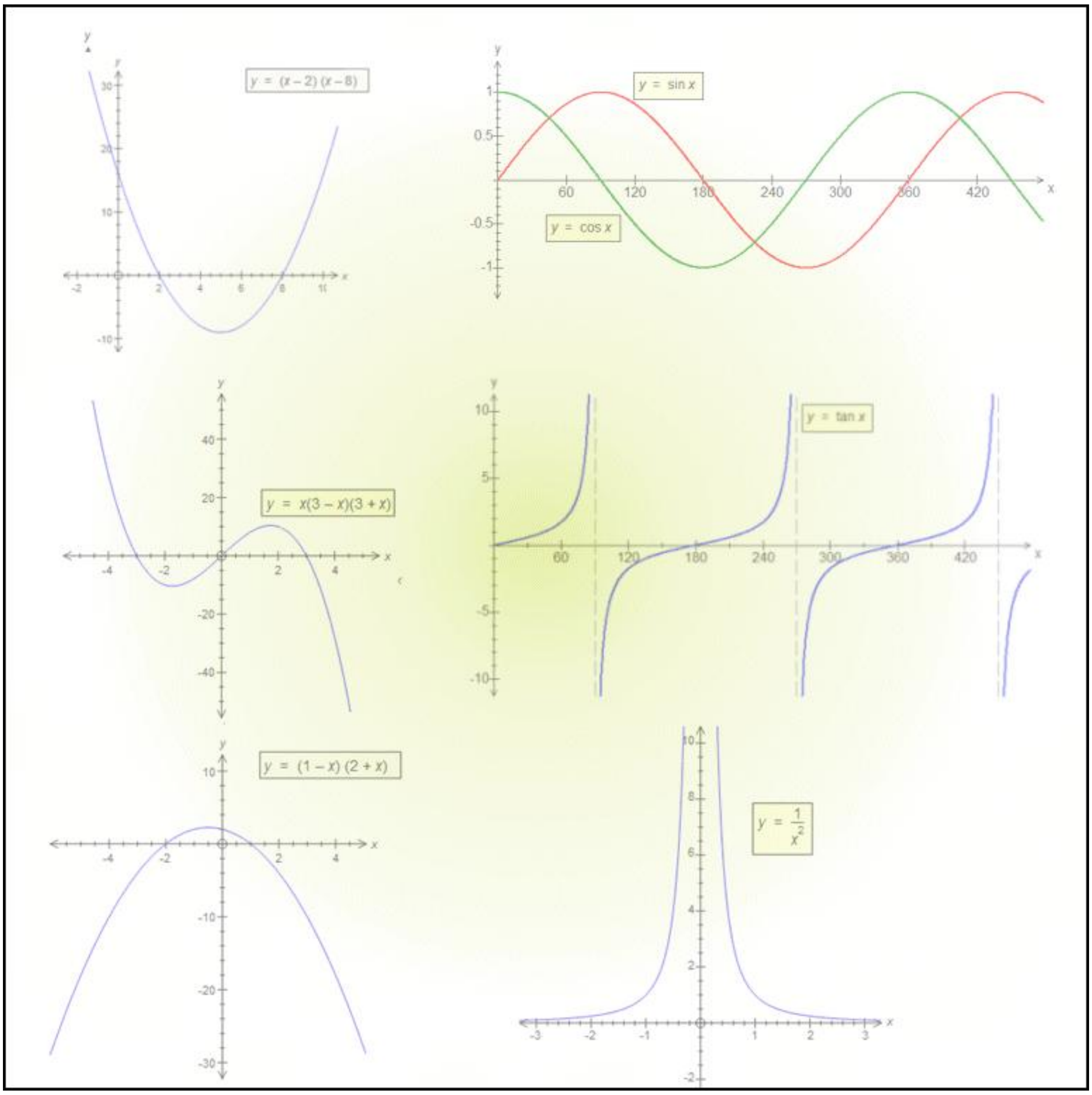
M.K. HOME TUITION

Mathematics Revision Guides

Level: A-Level Year 1 / AS

RECOGNISING GRAPHS OF FUNCTIONS



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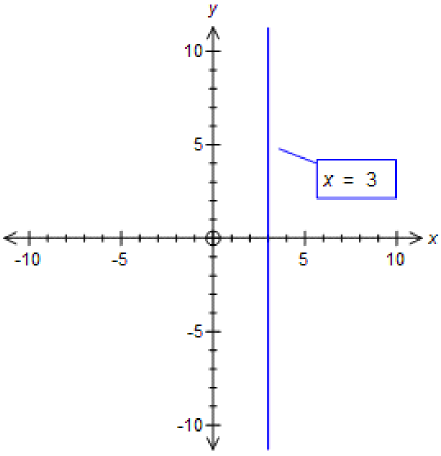
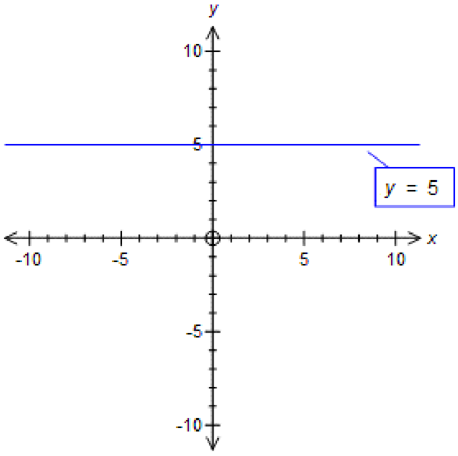
RECOGNISING GRAPHS

Most of the graph types below will be familiar from GCSE, but will be included for the sake of completeness.

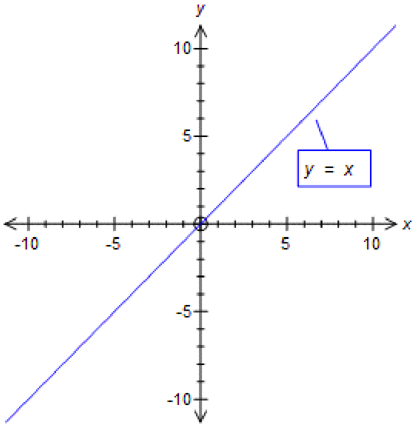
Straight-line graphs.

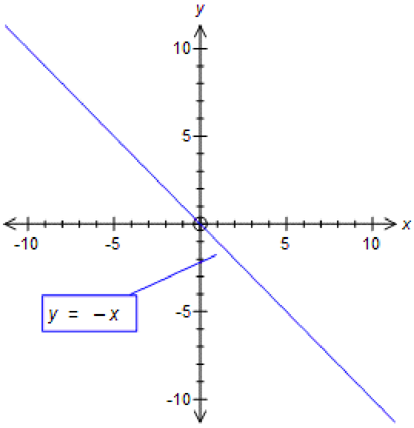
Constant graphs:



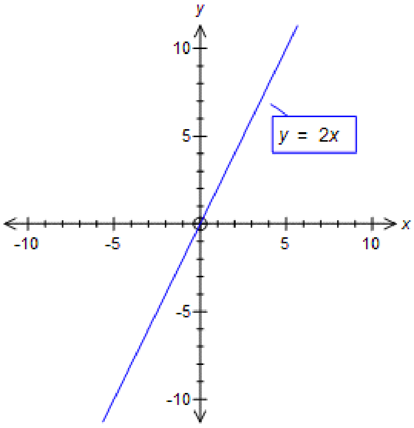


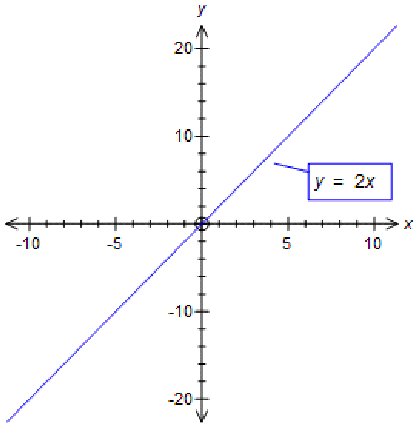
The main diagonals:

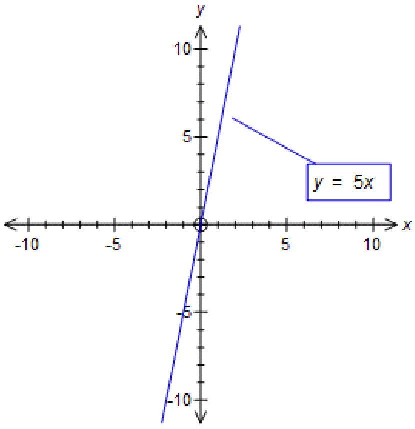


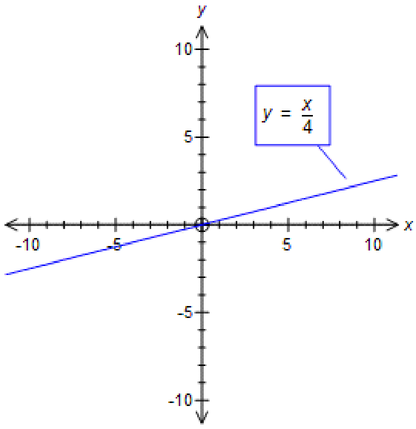


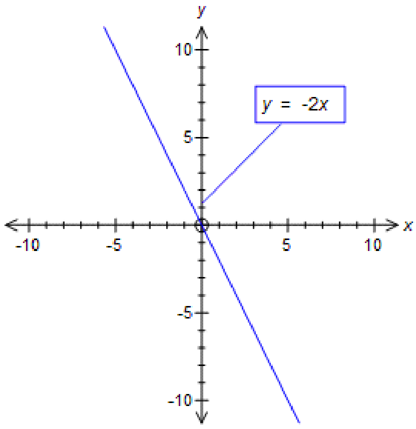
Other straight line graphs passing through the origin:

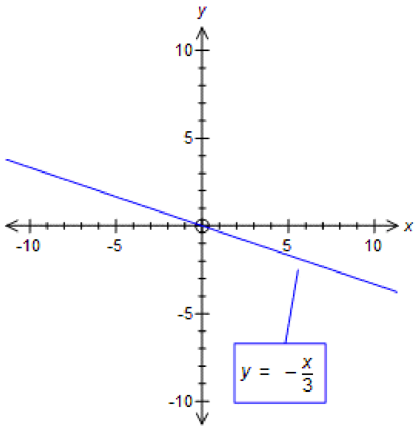




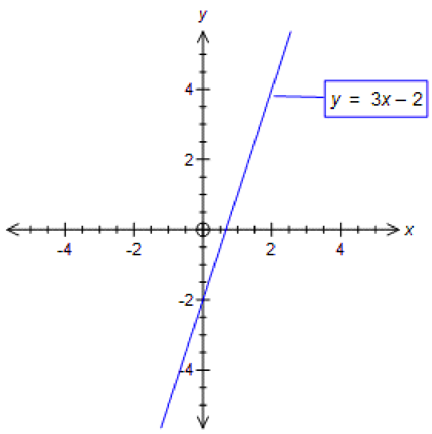


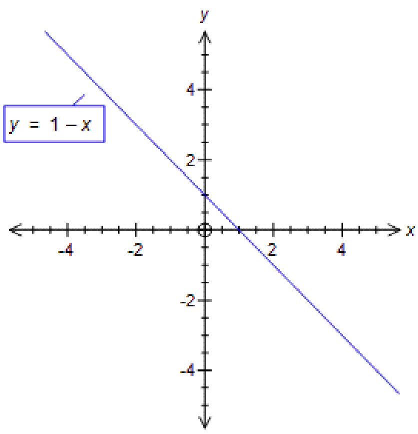






General straight line graphs, not passing through the origin:





Straight-line graphs are just that. Notice also that y is of the form $mx + c$, where m and c are constants, and m is not zero.

1-x is another way of writing (-x+1).

Note that if m is positive, the graph slopes upwards, but when m is negative, the graph slopes downwards. The slope of the graph is known as its **gradient**.

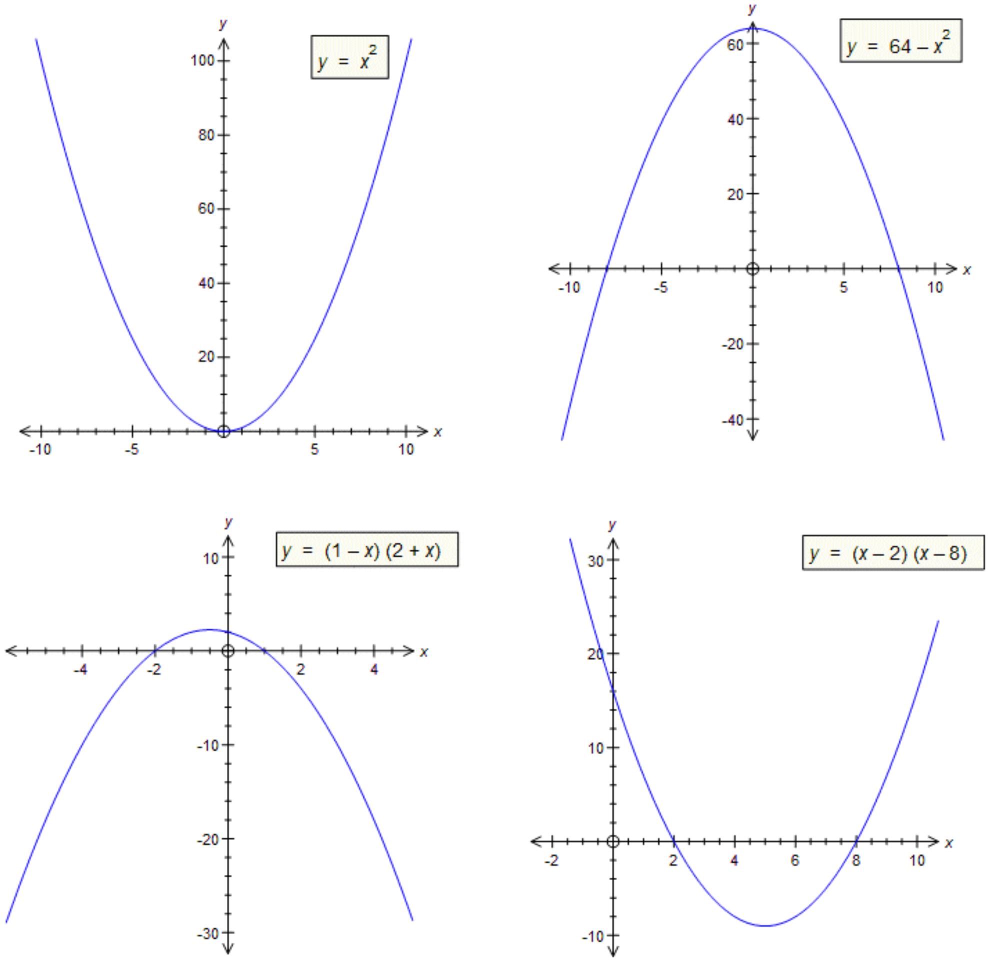
Also note that the graph cuts the y-axis where $y = c$. The equations of the two upper graphs have $c = 0$, and hence they pass through the origin.

The point where the graph cuts the y-axis is called the ***y*-intercept**. (There is also an x-intercept where the graph cuts the x-axis).

Sometimes the x-intercept is called the root, and the y-intercept simply the intercept.

Quadratic graphs.

These graphs are of functions of the form $y = ax^2 + bx + c$ where a, b and c are constants, and a is not zero. The highest power of x is 2 (the square of x). The basic graph of $y = x^2$ is shown upper left.



These graphs are parabolic or 'bucket-shaped'.

When the x^2 term is positive, the graphs point downwards at a trough and the function takes a minimum value. The expansion of $y = (x – 2)(x – 8)$ is $y = x^2 - 10x + 16$.

On the other hand, they point upwards at a crest and have a maximum value when the x^2 term is negative. The expansion of $y = (1 - x)(2 + x)$ is $y = 2 – x - x^2$.

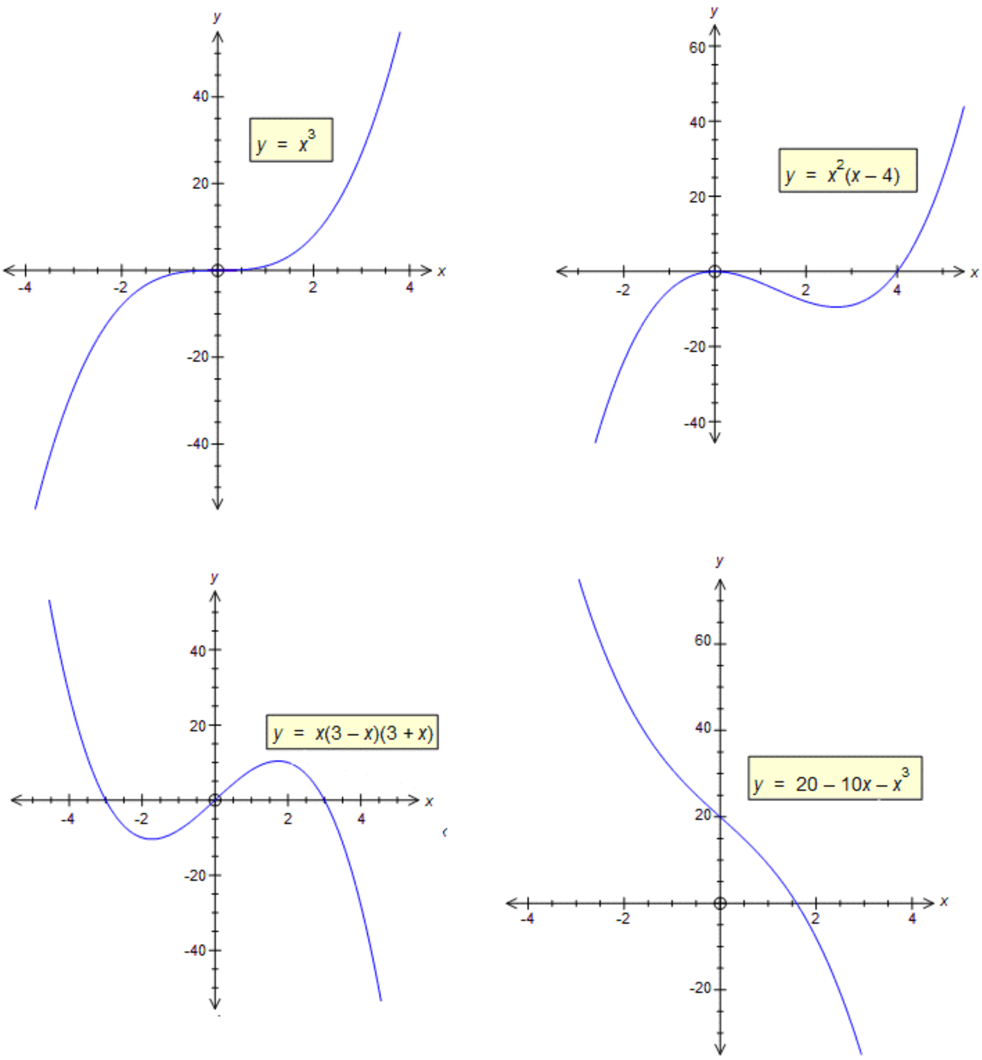
The 'depth' of a parabolic graph can vary, but this is as dependent on the scaling of the graph axes as well as on the actual function.

As discussed in the section of quadratics, a quadratic graph can cut the x-axis at two points, one point or not at all. In other words, there could be two roots, one root, or no roots at all.

Also, the gradient of any non-linear graph is no longer constant, but is dependent on the value of x, as further sections will show.

Cubic graphs.

These are a little more complicated than quadratic graphs. Their functions are of the form $y = ax^3 + bx^2 + cx + d$ where a, b, c and d are constants, and a is not zero.
The basic graph of $y = x^3$ is shown upper left.



These graphs are characterised by a 'double bend' of varying severity, but their general slope is upward if the term in x^3 is positive, and downward if the x^3 term is negative. A cubic graph must cut the x – axis at least once, but there are different cases depending on how the equation can be factorised.

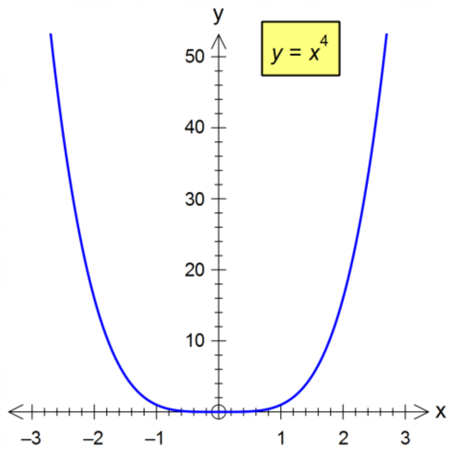
If it can expressed as a product of three distinct factors, as in the example at lower left, then it cuts the x-axis at three points. The example at upper right has a repeated factor, so it cuts the x-axis at two points, with the x-axis as a tangent at the repeated root. The two other examples only cut the x-axis at one point.

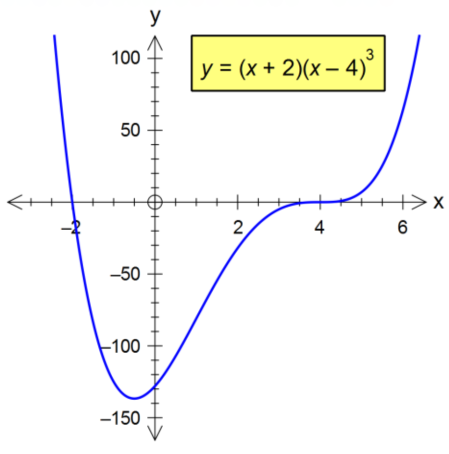
Quartic Graphs.

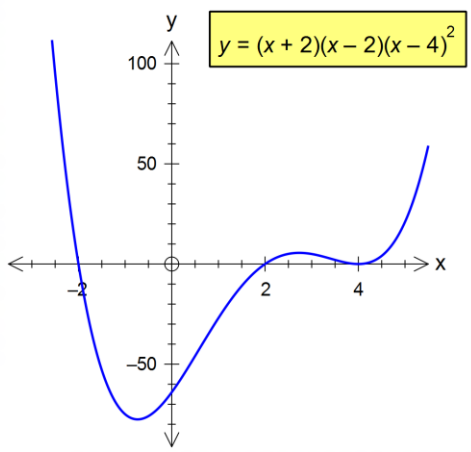
Again, these are more complicated than cubic graphs and can take on a greater variety of basic forms. Their functions are of the form $y = ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are constants, and a is not zero.

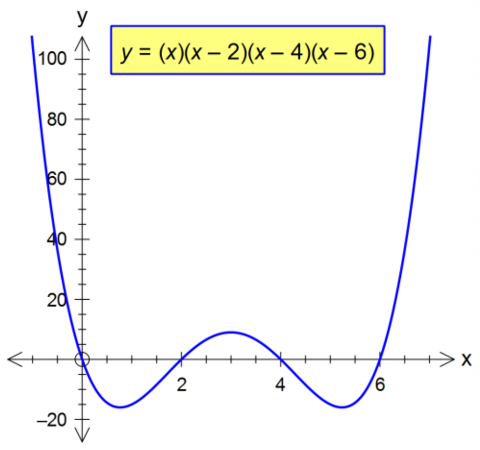
The basic graph of $y = x^4$ is shown upper left, and the graphs below do not exhaust the whole spectrum of graph shapes.

(In each graph, the coefficient of x^4 is positive; for negative coefficients, reflect in the x-axis.





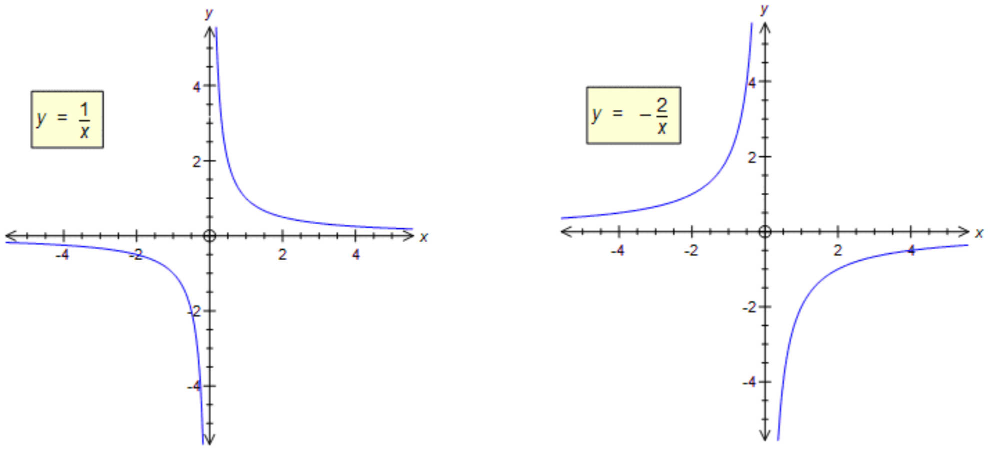




As can be seen, there is far more variation within quartic graphs. From upper left to lower right, they meet the x-axis at one, two, three or four points (they could also not meet the x-axis at all, e.g. for the graph of $y = x^4 + 1$). In other words, a quartic equation can have any number of roots from zero to four.

There is more on quartic graphs in the "Polynomials" and "Curve Sketching" documents.

Reciprocal graphs.



These graphs are shared by functions of the form $y = \frac{k}{x^n}$, where k is a non-zero constant and n is a positive integer. They differ from previous examples in that they seem to be in two unconnected parts, in other words they are discontinuous. If n is odd and k is positive, the two sections are in the upper right and lower left, but if n is odd and k is negative, the sections are in the upper left and lower right.

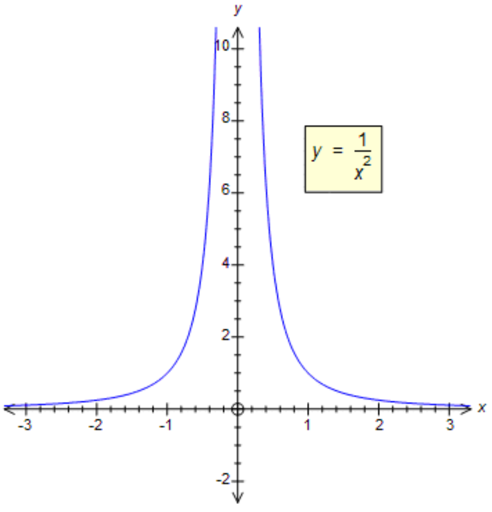
Note the following features of the standard graph $y = \frac{1}{x}$.

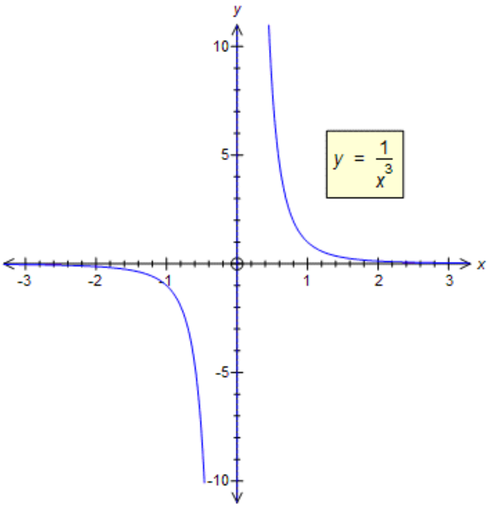
As x becomes large and positive, y stays positive but approaches 0 without actually getting there.
As x becomes large and negative, y stays negative but again approaches 0.
The line $y = 0$ is said to be an **asymptote** of the graph.

As positive x approaches zero, y becomes increasingly large whilst staying positive.
As negative x approaches zero, y becomes increasingly large whilst staying negative.
These values are also said to **tend to infinity.**

Another way of writing this is $\lim_{x \to +0} \frac{1}{x} = +\infty$, similarly, $\lim_{x \to -0} \frac{1}{x} = -\infty$. This brings the idea of limiting values ('lim' is just short for 'limit).

When $x = 0$, y is **undefined**, which is **not** the same as saying $\frac{1}{0} = \infty$. Such a statement is nonsensical, as you cannot divide by zero. The graph therefore also has an asymptote at the line $x = 0$.

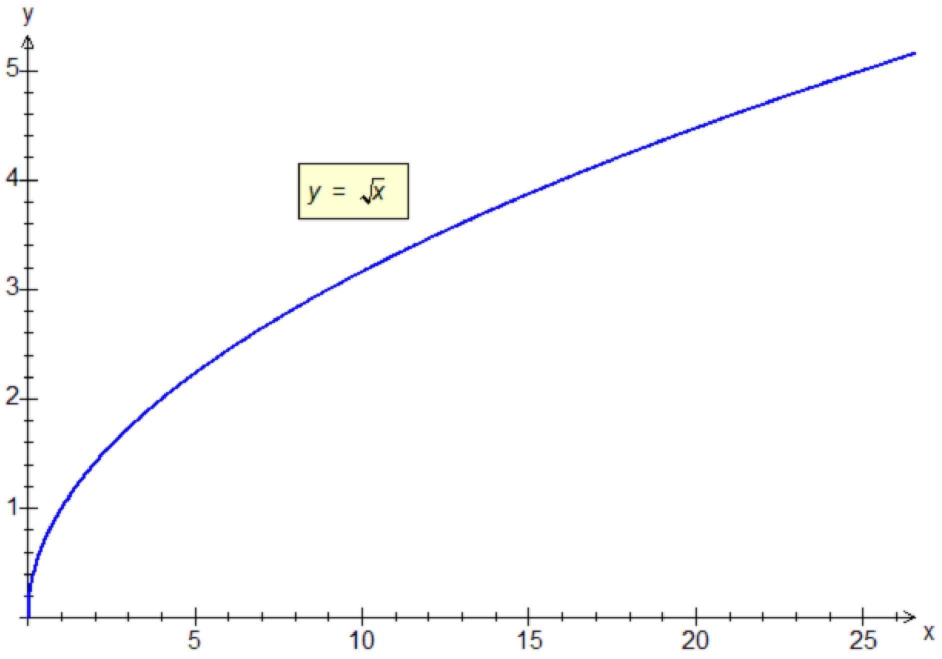




The graph of $y = \frac{1}{x^3}$ resembles that of $y = \frac{1}{x}$, but is steeper on either side of $x = 1$ and $x = -1$. (The graphs of the reciprocals of other odd powers of x will be similar.)

The graph of $y = \frac{1}{x^2}$ is different, as it is always positive for all x, although undefined for $x = 0$. Again, there will be asymptotes at $x = 0$ and $y = 0$. (The graphs of the reciprocals of other even powers of x will be similar.)

The Square Root Graph.



This graph looks like half of the graph of x^2, reflected in the line $y = x$.

This function is defined as the ***positive*** square root of x.

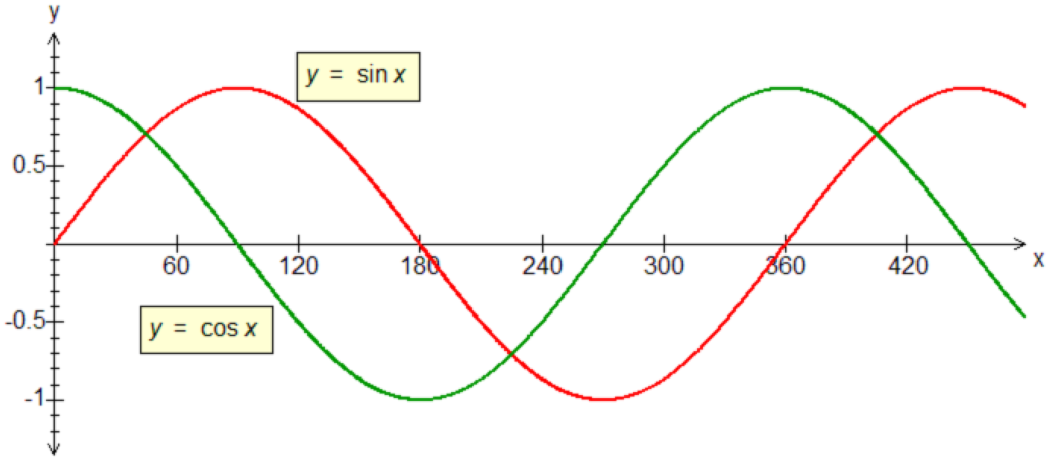
There is a subtle misunderstanding about the square root function. Take the statements below:

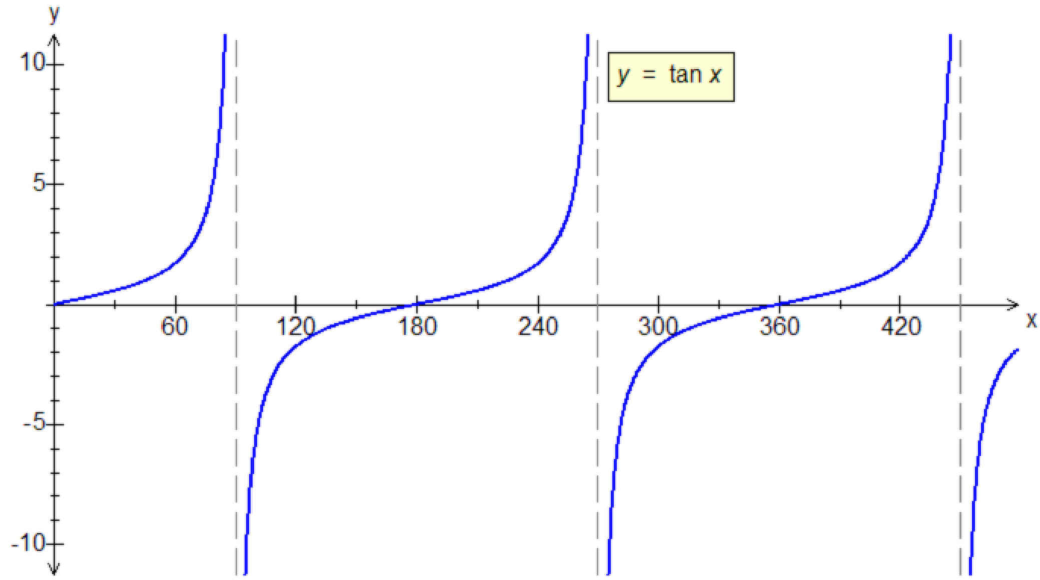
"The number 25 has two square roots, +5 and -5. In other words, $\sqrt{25}$ = +5 or -5."
"The equation $x^2 = 25$ has two solutions, $\sqrt{25}$ or +5, and -($\sqrt{25}$) or -5."

The second statement is the only correct one.

Trigonometric Graphs.

The three main trigonometric functions have the following graphs**:**





The graphs of sin $x°$ and cos $x°$ are similar to each other; in fact they are shown together for comparison. Both functions can only take values in the range -1 to +1, and both repeat themselves every 360°. Indeed, the graph of cos $x°$ is the same as that of sin $x°$ translated 90° to the left, i.e. by the vector $\begin{pmatrix} -90° \\ 0 \end{pmatrix}$.

The graph of tan $x°$ is quite different. It repeats every 180°, and moreover the function is undefined for certain values of x, such as 90°, 270°, and all angles consisting of an odd number of right angles. When x approaches 90° from below, tan $x°$ becomes very large and positive; when x approaches 90° from above, tan $x°$ becomes very large and negative. The tangent graph therefore has asymptotes at 90°, 270°, and all angles 90° + 180n° where n is an integer.

Odd and Even Functions.

The graphs of some functions display certain symmetries.

For example, the graphs of x^2,x^4, cos x and $\frac{1}{x^2}$ all show reflective symmetry about their y-axes, or in other words, $f(-x) = f(x)$.

Because the graphs of even powers of x show this property, such functions are termed **even** functions. This also applies to constant functions $y = k$.

The graphs of certain other functions, such as x, x^3, sin x and $\frac{1}{x}$ show rotational symmetry of order 2 about the origin. For those functions, $f(-x) = -f(x)$. These functions are termed **odd** functions since graphs of odd powers of x show this property.

For an even function, $f(-x) = f(x)$ for all x.
For an odd function, $f(-x) = -f(x)$ for all x.

It must also be added that the vast majority of functions are neither odd nor even.

The sum of two or more odd functions is odd; the sum of two or more even functions is even.
The sum of an odd and an even function is neither.

The product of two odd functions is even; the product of two even functions is even.
The product of an even and odd function is odd.