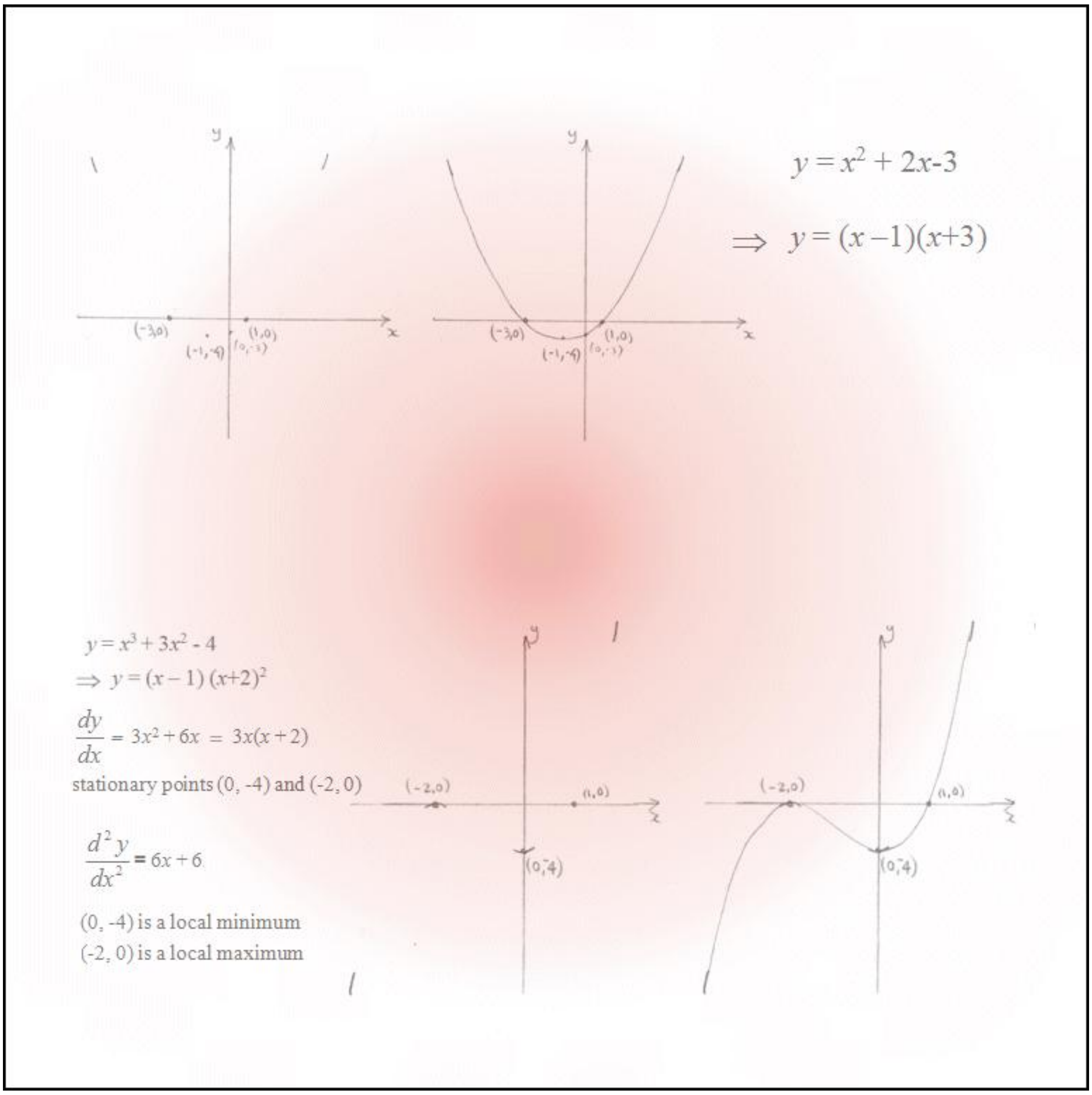
M.K. HOME TUITION

Mathematics Revision Guides

Level: A-Level Year 1 / AS

CURVE SKETCHING



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Sketching Curves.

This topic has been discussed under ‘Quadratics’, but examination questions might also ask for sketches of other types of functions, such as cubics or reciprocals.

Often, one graph can be obtained from another by transformation – many such examples have been shown in the document ‘Transformations of Graphs’.

To sketch the graph of a function in Cartesian form, take the following steps:

Read the question.
The question asks for a sketch graph, and the mark scheme will give a clue about the detail involved. Do not waste time finding complicated turning points if the sketch is only worth 2 marks !

Check if the function can be rewritten or factorised.
A quadratic might factorise easily, or be simplified by completing the square.
Cubics are usually expressed in factor form, but is they are not, the Factor Theorem could be used .

Find the intercepts.
Substitute $x = 0$ to find where the curve meets the y-axis; substitute $y = 0$ to find where it intersects the x-axis.

Check for symmetry.
If the function is even (i.e. it is a sum of even powers of x, including constant terms), then its graph will have reflective symmetry in the y-axis.
If the function is odd, (i.e. it is a sum of odd powers of x only) then its graph will have rotational symmetry of order 2 about the origin.

Remember that the vast majority of functions are neither odd nor even.

Check the behaviour of the function for large values of x and y (both positive and negative).

If the function is of rational or reciprocal , i.e. of the $y = \frac{1}{x}$ type, then its value will tend to a limit without actually approaching it. This limit is known as an asymptote – a line that the curve would approach ever more closely, but never actually meet.

For a polynomial, as x becomes increasingly large, the function takes on the characteristics of the term of the highest power of x. (the highest power of x will tend to swamp the lower ones).

Check for discontinuities, and the behaviour of the function near them.
(Not applicable to polynomials, as they are continuous.)
These will occur in graphs of rational or reciprocal functions, for values of x that make the denominator equal to zero. These discontinuities also give rise to **asymptotes** (see last sentence). Most graphs of this type will occur in two unconnected parts, with the asymptotes as barriers between them.

Check for stationary points if the question asks for them.
If the question doesn’t ask you to find them, it’s either because the question is only worth a few marks, or the calculus becomes too time-consuming and messy.

Find out any points where the gradient is zero (differentiate once), and determine if they are maximum or minimum points (differentiate again). Also note the special case of repeated factors in the polynomial; if a factor (x-a) is repeated, then the x-axis is tangent to the stationary point at $x = a$.

Join up.
Connect all points and arcs with a smooth curve (or curves, if the function is discontinuous).
This part is generally only worth one mark, so don’t lose sleep over the smoothness or accuracy !

Example (1): Sketch the graph of $y = 16 - x^2$ and also mark its turning point.

Find the intercepts.
Firstly we determine where the graph cuts the y-axis; when $x = 0$, $y = 16$
$\therefore$ the graph cuts the y-axis at (0, 16). ***(Plot the point)***

The graph cuts the x-axis when $16 - x^2 = 0$, i.e. when $x^2 = 16$.
$\therefore$ the graph cuts the x-axis at (4, 0) and (-4,0). ***(Plot the points)***

Check for symmetry.
The function $y = 16 - x^2$ is even , therefore its graph is symmetrical about the y-axis.

Check behaviour of function for large x.
When x becomes large, the quadratic term of $-x^2$ dominates the constant of 16, and so the function generally behaves like $-x^2$.

x large and positive $\Rightarrow$ y increasingly large and negative
x large and negative $\Rightarrow$ y increasingly large and negative

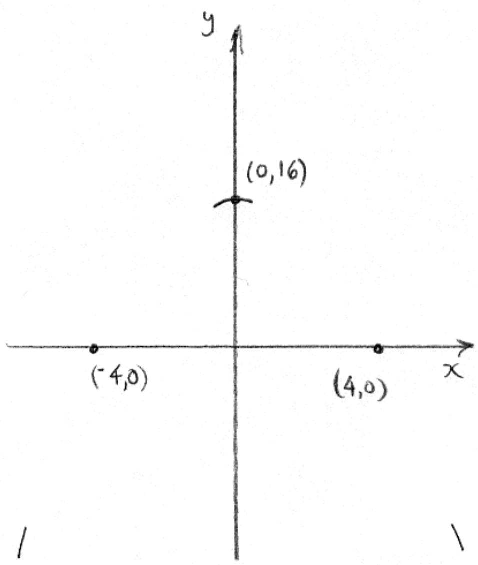
(Draw a short steep upward arc at bottom left of the graph and a short steep downward arc at bottom right.)

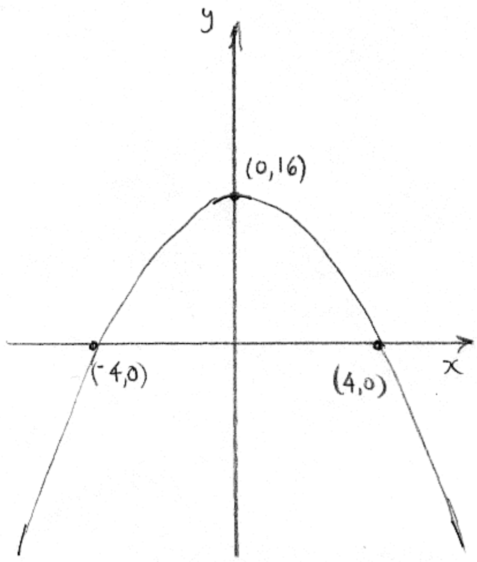
Check for stationary points.
Differentiating the function gives $\frac{dy}{dx} = -2x$.
This value is zero when $x = 0$, therefore (0, 16) is a stationary point.
The coefficient of x^2 is negative, so this stationary point is a maximum.

(We could have found the x-coordinate of the stationary point without calculus, by using the symmetry property and taking the mean of the x-coordinates of the x-intercepts – here it is 0.).

(Draw a short 'crest' arc at (0, 16) to represent the maximum.)

Join up.
(Join the points and arcs plotted so far with a smooth curve.)





Example (2): Sketch the graph of $y = x^2 + 2x - 3$, and also plot its turning point.

<u>Preparation.</u>
The expression can be factorised to $y = (x - 1)(x+3)$.

<u>Find the intercepts.</u>
When $x = 0$, $y = -3$; the graph cuts the y-axis at (0, -3). ***(Plot the point)***
From the factors, the graph cuts the x-axis when $x = -3$ and $x = 1$. ***(Plot the points)***

<u>Check for symmetry.</u>
The function is neither odd nor even, but, being a quadratic, there is a line of symmetry where the x-coordinate is halfway between the two roots. Here it is the line $x = ½(-3 + 1)$ or $x = -1$.

<u>Check behaviour of function for large x.</u>
When x becomes large, the quadratic term of x^2 dominates, so the function generally behaves like x^2

x large and positive $\Rightarrow$ y increasingly large and positive

x large and negative $\Rightarrow$ y increasingly large and positive

(Draw a short steep downward arc at top left of the graph and a short steep upward arc at top right.)

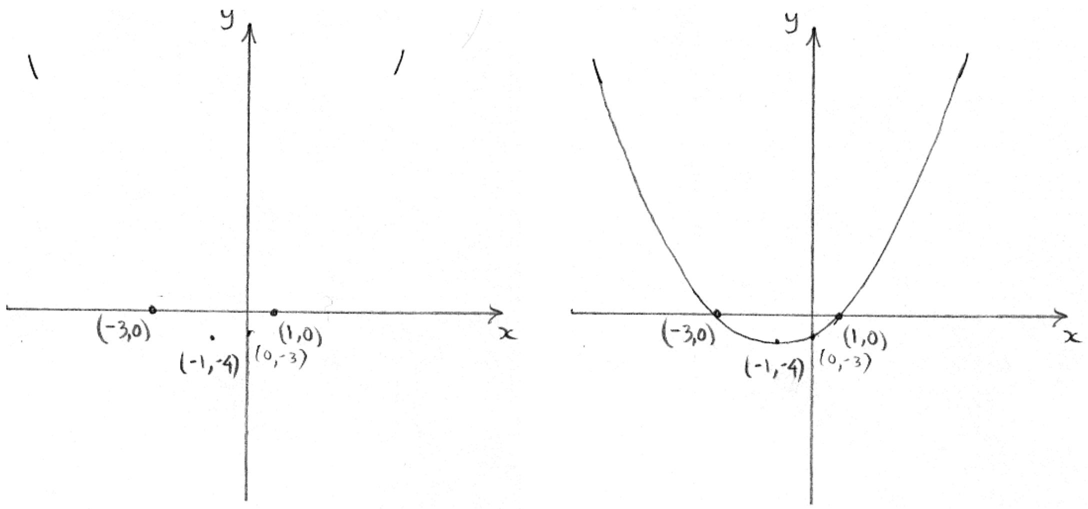
<u>Check for stationary points.</u>

To find the stationary point, the function can be differentiated to give $\frac{dy}{dx} = 2x + 2$.

This value is zero when $x = -1$, therefore (-1, -4) is a stationary point.
The coefficient of x^2 is positive, so this stationary point is a minimum.
(Again, we could have foregone the calculus, using the line of symmetry of $x = -1$.)

There is no need to find a second derivative – the fact that the x^2 term is positive is sufficient to tell us it is a minimum.
(Draw a short 'trough' arc at (-1, -4) to represent the minimum.)

<u>Join up.</u>
(Join the points, lines and curves plotted so far with a smooth curve.)



Example (3): Sketch the graph of $y = x^3 - 9x$, also plotting the turning points.

<u>Preparation.</u>
The expression factorises to $x(x^2 – 9)$, and finally to $x(x+3)(x-3)$.

<u>Find the intercepts.</u>
From the factors, it can be seen that the graph must cut the x–axis at (-3, 0), (0, 0) and (3, 0). It also cuts the y-axis at (0, 0). ***(Plot the points)***.

<u>Check for symmetry.</u>
The function $y = x^3 - 9x$ is odd, and thus the graph has rotational symmetry of order 2 about the origin.

<u>Check behaviour of function for large x.</u>
When x becomes large, the term of x^3 dominates, so the function generally behaves like x^3.

x large and +ve $\Rightarrow$ y increasingly large and +ve

x large and -ve $\Rightarrow$ y increasingly large and -ve

(Draw a short steep downward arc at bottom left and a short steep upward arc at top right.)

<u>Check for stationary points.</u>

This time we need calculus! Differentiating the function gives $\frac{dy}{dx} = 3x^2 – 9$ or $3(x^2 – 3)$.

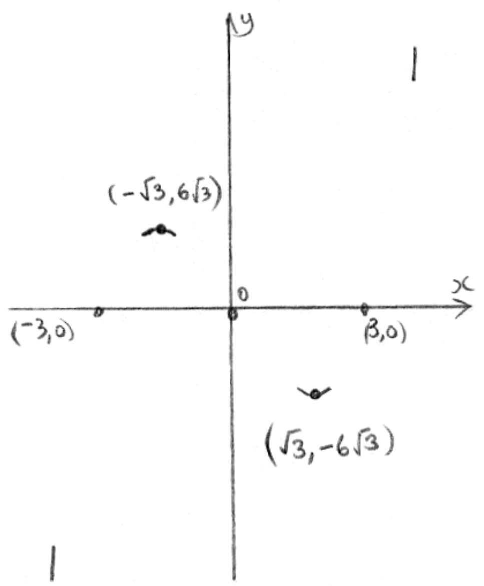
The derivative is zero when $x^2 = 3$, giving stationary points when $x = \pm\sqrt{3}$.
The coordinates of these points are $(-\sqrt{3}, 6\sqrt{3})$ and $(\sqrt{3}, -6\sqrt{3})$. ***(Plot the points).***

The second derivative of y, or $\frac{d^2y}{dx^2}$, is $6x$.

It is $-6\sqrt{3}$ – a negative value- when $x = -\sqrt{3}$, and therefore $(-\sqrt{3}, 6\sqrt{3})$ is a local maximum.
(Draw a short 'crest' arc at ($-\sqrt{3}$, $6\sqrt{3}$) to represent the maximum.)

It is $6\sqrt{3}$ – a positive value- when $x = \sqrt{3}$, and therefore $(\sqrt{3}, -6\sqrt{3})$ is a local minimum.
(Draw a short 'trough' arc at ($\sqrt{3}$, $-6\sqrt{3}$) to represent the minimum.)

<u>Join up.</u>
(Join the points and arcs plotted so far with a smooth curve.)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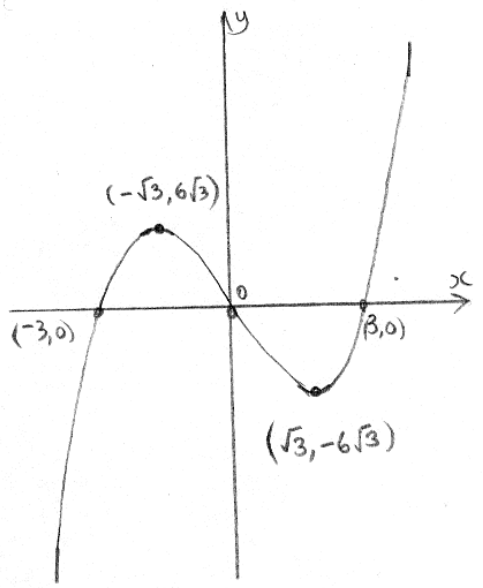

y
x
0
$(-\sqrt{3}, 6\sqrt{3})$
$(-3,0)$
$(3,0)$
$(\sqrt{3}, -6\sqrt{3})$

Example (4): Sketch the graph of $y = -x^3 + 12x + 16$, including turning points.
We are given that the graph passes through (4, 0) and has a factor of the type $(a - x)$.

Preparation.
Per the given data, the expression has a factor of $(4 - x)$ by the factor theorem for polynomials.
Long division (not shown here) gives a quotient of $x^2 + 4x + 4$, which in turn factorises to$(x+2)^2$.
$\therefore$ The full factorised form is $y = (4 - x)(x+2)^2$.

Find the intercepts.
From the last statement, the graph meets the x–axis at (4, 0) and (-2, 0). ***(Plot the points).***
When $x = 0$, $y = 16$, and so the graph meets the y-axis at (0, 16). ***(Plot the point).***

Check for symmetry.
The function is neither odd nor even, so not much help here !

Check behaviour of function for large x.
When x becomes large, the term of $-x^3$ dominates, so the function generally behaves like $-x^3$.
x large and +ve $\Rightarrow$ y increasingly large and -ve
x large and -ve $\Rightarrow$ y increasingly large and +ve
(Draw a short steep downward arc at top left and a short steep upward arc at top right.)

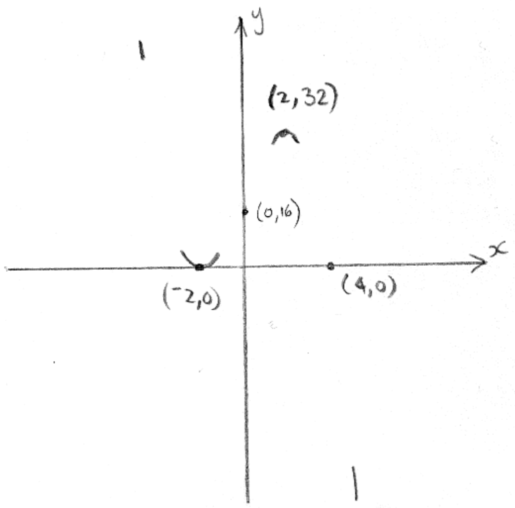
Check for stationary points.
$\frac{dy}{dx} = 12 - 3x^2$ or $3(4 - x^2)$, therefore $\frac{dy}{dx} = 0$ when $x^2 = 4$, $\therefore$ the stationary points occur at $x = \pm 2$.
Their coordinates are (-2, 0) and (2, 32).

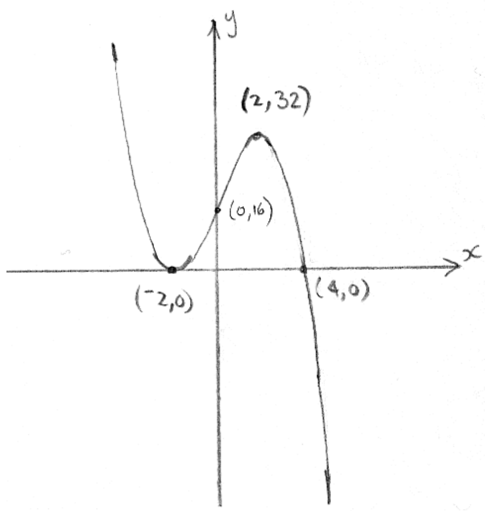
(The presence of the repeated root $(x + 2)$ has also indicated that (-2, 0) is stationary).
(Plot the second point – the first is already there).

The second derivative, $\frac{d^2y}{dx^2} = -6x$.
It is positive (12) when $x = -2$, $\therefore$ (-2, 0) is a local minimum.
(Draw a short 'trough' arc at (-2, 0) to represent the minimum.)

It is negative (-12) when $x = 2$, $\therefore$ (2, 32) is a local maximum.
(Draw a short 'crest' arc at (2, 32) to represent the maximum.)

Join up.
(Join the points and arcs plotted so far with a smooth curve.)





Example (5): Sketch the graph of $y = x^3 + 3x^2 - 4$, including turning points.
Hint: Substitute $x = 1$.

Preparation.
When $x = 1$, $y = 0$, $\therefore$ $(x - 1)$ is a factor by the factor theorem. Long division (not shown here) gives a quotient of $x^2 + 4x + 4$, which in turn factorises to $(x + 2)^2$.
$\therefore$ $y = (x - 1)(x+2)^2$.

Find the intercepts.
From the above, the graph meets the x–axis at (1, 0) and (-2, 0). ***(Plot the points).***
When $x = 0$, $y = -4$, so the graph meets the y-axis at (0, -4). ***(Plot the point)***.

Check for symmetry.
Not much help – the function is neither odd nor even.

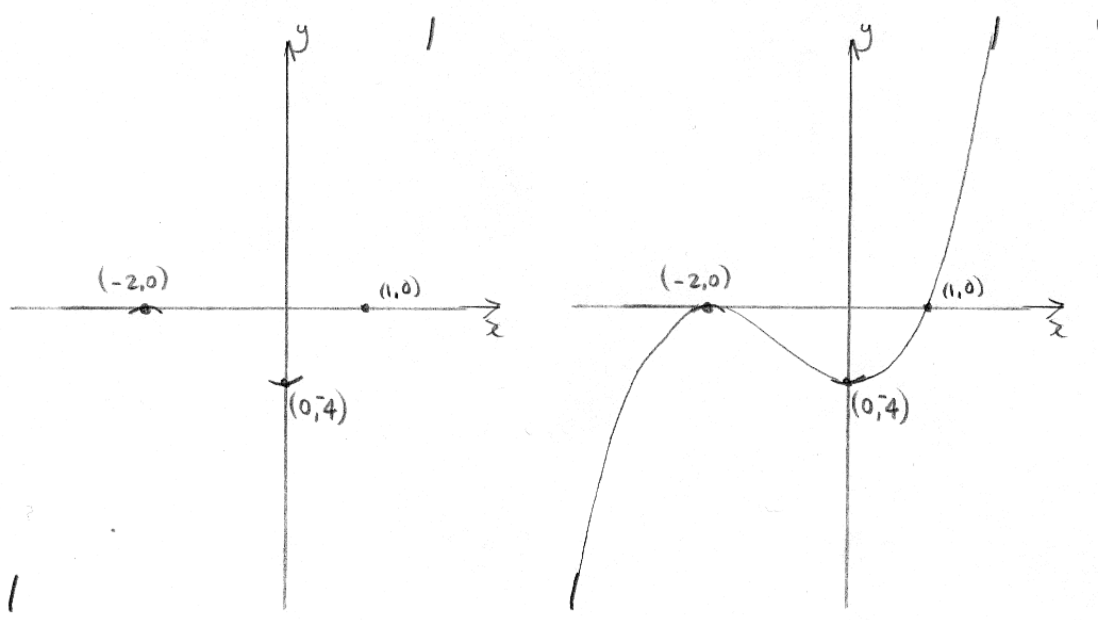
Check behaviour of function for large x.
When x becomes large, the term of x^3 dominates, so the function generally behaves like x^3.
x large and +ve $\Rightarrow$ y increasingly large and +ve
x large and -ve $\Rightarrow$ y increasingly large and -ve
(Draw a short steep downward arc at bottom left and a short steep upward arc at top right.)

Check for stationary points.
$\frac{dy}{dx} = 3x^2 + 6x$ or $3x(x + 2)$, which is zero when $x = 0$ or $x = -2$.
The stationary points of (0, -4) and (-2, 0) are already plotted – the presence of the repeated root $(x+2)$ indicates that the latter is stationary.

The second derivative, $\frac{d^2y}{dx^2} = 6x + 6$.
It is positive (6) when $x = 0$, $\therefore$ (0, -4) is a local minimum.
(Draw a short 'trough' arc at (0, -4) to represent the minimum.)

It is negative (-6) when $x = -2$, $\therefore$ (-2, 0) is a local maximum.
(Draw a short 'crest' arc at (-2,0) to represent the maximum.)

Join up.
(Join the points, lines and curves plotted so far with a smooth curve.)



Example (6): Sketch the graph of $y = x(x-1)(x-3)(x-4)$, showing the intersections with the axes and the maximum turning point. The two minimum turning points have approximate y-coordinates of -2.
(Do not use calculus to find the exact x-coordinates of the minimum points).

Preparation. None required as the polynomial is already in factorised form.

Find the intercepts.
By checking the factors, the graph meets the x–axis at (0,0), (1,0), (3,0) and (4,0). ***(Plot the points).***
(The y-intercept is also 0).

Check for symmetry.
The outer pairs of roots (0 and 1, 3 and 4) are the same distance from each other, so there is a vertical line of symmetry, with equation $x = 2$.

Check behaviour of function for large x.
By checking the factors, we find that the expansion produces a *positive* coefficient of x^4.
When x becomes large, the term of x^4 dominates, so the function generally behaves like x^4.

x large and +ve $\Rightarrow$ y increasingly large and +ve

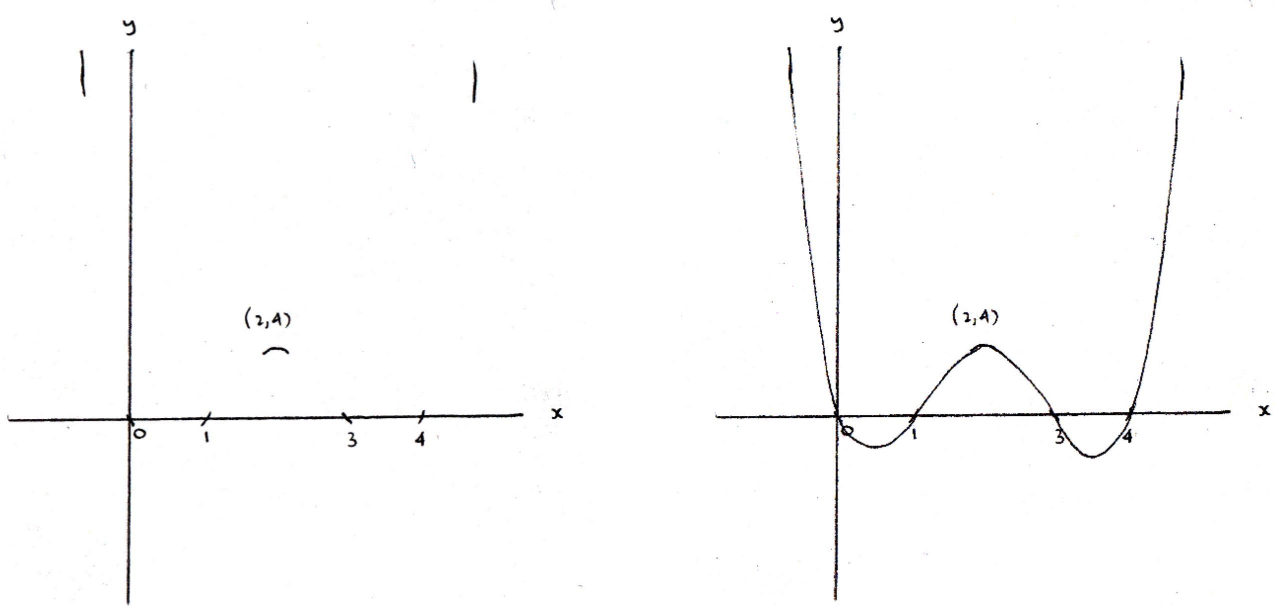
x large and -ve $\Rightarrow$ y increasingly large and +ve

(Draw a short steep downward arc at top left and a short steep upward arc at top right.)

Check for stationary points.
The graph has three stationary points and a W shape, so the order is minimum, maximum, minimum.
We work out the y-coordinate of the maximum but substituting $x = 2$ in the equation.
$y = x(x-1)(x-3)(x-4) = 2(2-1)(2-3)(2-4) = 4$, so the maximum is at exactly (2,4).
(Draw a short 'crest' arc at (2,4) to represent the maximum.)

We can assume that the x-coordinates of the minima are about halfway between 0 and 1, and halfway between 3 and 4.
(Draw short 'trough' arcs at (0.5, -2) and (3.5, -2) to represent the minima.)

Join up.
(Join the points, lines and curves plotted so far with a smooth curve.)



Example (7): Sketch the graph of $y = x(1-x)(x-3)^2$, showing the intersections with the axes. You are given that the coordinates of two of the turning points are very approximately (0.5, 1.5) and (2, -2), so do not use calculus to find them accurately.

Preparation. None required as the polynomial is already in factorised form.

Find the intercepts.
By checking the factors, the graph meets the x–axis at (0,0), (1,0) and (2,0). ***(Plot the points).***
(The y-intercept is also 0).

Check for symmetry.
The roots are unevenly spaced, so there is no line of symmetry.

Check behaviour of function for large x.
By checking the factors, we find that the expansion produces a *negative* coefficient of x^4.
When x becomes large, the term in x^4 dominates, so the function generally behaves like $-x^4$.

x large and +ve $\Rightarrow$ y increasingly large and -ve

x large and -ve $\Rightarrow$ y increasingly large and -ve

(Draw a short steep upward arc at bottom left and a short steep downward arc at bottom right.)

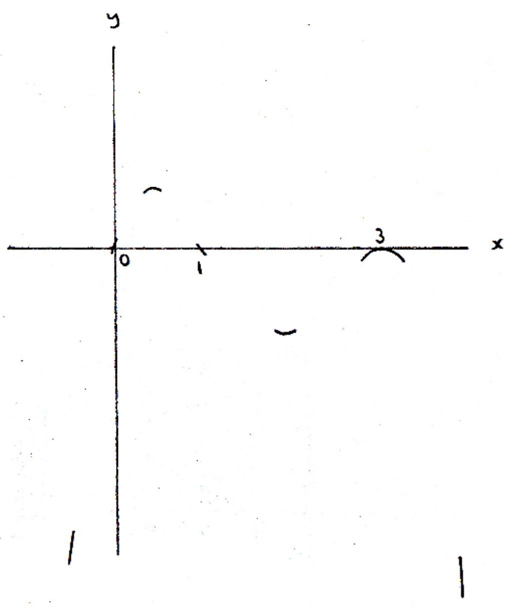
Check for stationary points.
The graph has three stationary points, but because the coefficient of x^4 is negative, the graph has an M rather than a W shape, and the order of the turning points is maximum, minimum, maximum.

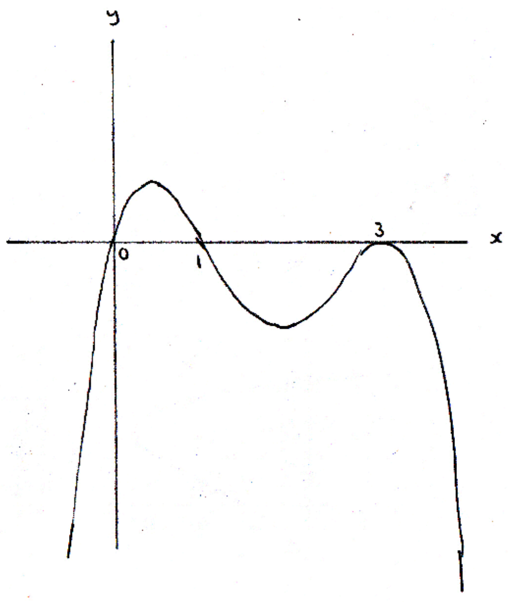
We also have a repeated factor of $(x-3)^2$ in the quartic, corresponding to a repeated root of $x = 3$.
This is the turning point with the highest x-value, making it a maximum, and here the graph touches the x-axis.
(Draw a short 'crest' arc touching the x-axis at (3,0) to represent that maximum.)

The turning point at $\approx$ (0.5, 1.5) is the other maximum, and the one at $\approx$ (2, -2) is the minimum.

(Draw a short 'crest' arc at (0. 1.5) for the 1st maximum and a short 'trough' arc at(2, -2) for the minimum.)

Join up.
(Join the points, lines and curves plotted so far with a smooth curve.)





Example (8): Sketch the graph of $y = (x-4)^2(x-1)^2$, showing the intersections with the axes.
You are given that the y- coordinate of the maximum is approximately 5.
(Do not use calculus to find the exact coordinates of the maximum point).

Preparation. None required as the polynomial is already in factorised form.

Find the intercepts.
By checking the factors, the graph meets the x–axis at (1,0) and (4,0). ***(Plot the points).***
By substituting $x = 0$, the y-intercept is at (0, 16). ***(Plot the point).***

Check for symmetry.
The roots are in two coincident pairs, (1 and 1, 4 and 4) so there is a vertical line of symmetry, with equation $x = 2.5$ (the mean of 1 and 4)..

Check behaviour of function for large x.
By checking the factors, we find that the expansion produces a *positive* coefficient of x^4.
When x becomes large, the term of x^4 dominates, so the function generally behaves like x^4.

x large and +ve $\Rightarrow$ y increasingly large and +ve

x large and -ve $\Rightarrow$ y increasingly large and +ve

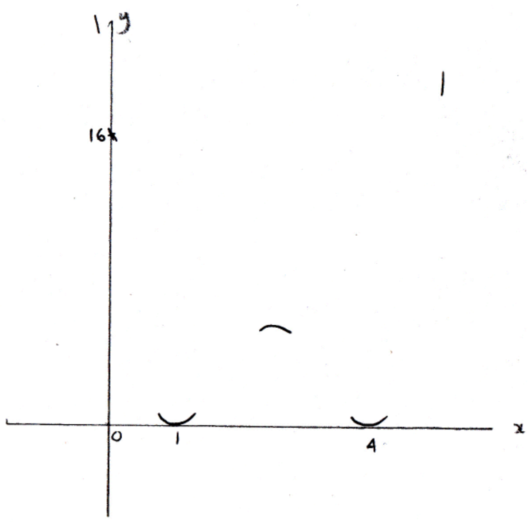
(Draw a short steep downward arc at top left and a short steep upward arc at top right.)

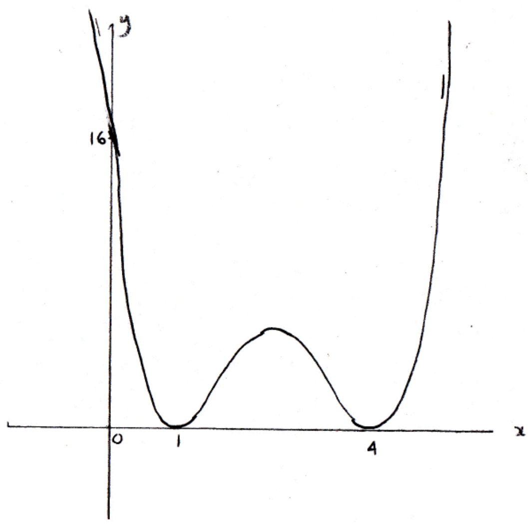
Check for stationary points.
The graph has three stationary points and a W shape, so the order is minimum, maximum, minimum.
We have two repeated factors in the quartic, namely $(x-4)^2$ and $(x-1)^2$, so the graph is tangent to the x-axis at (1,0) and (4,0), and those points are also minima.
The local maximum occurs when $x = 2.5$ (the mean of 1 and 4) and y is approximately 5.

We work out the y-coordinate of the maximum but substituting $x = 2$ in the equation.
$y = x(x-1)(x-3)(x-4) = 2(2-1)(2-3)(2-4) = 4$, so the maximum is at exactly (2,4).

(Draw short 'trough' arcs at (1, 0) and 4, 0) for the minima.)
(Draw a short 'crest' arc at (2.5, 4) for the maximum.)

Join up.
(Join the points, lines and curves plotted so far with a smooth curve.)





Example (9): Sketch the graph of $y = (x-3)(x+2)^3$, showing the intersections with the axes. The coordinates of the minimum point are very approximately (2, -70)
(Do not use calculus to find the exact coordinates of the minimum point).

Preparation. None required as the polynomial is already in factorised form.

Find the intercepts.
By checking the factors, the graph meets the x–axis at (3,0) and (-2,0). ***(Plot the points).***
By substituting $x = 0$, the y-intercept is at (0, -24). ***(Plot the point).***

Check for symmetry.
There is no line of symmetry.

Check behaviour of function for large x.
By checking the factors, we find that the expansion produces a *positive* coefficient of x^4.
When x becomes large, the term of x^4 dominates, so the function generally behaves like x^4.

x large and +ve $\Rightarrow$ y increasingly large and +ve

x large and -ve $\Rightarrow$ y increasingly large and +ve

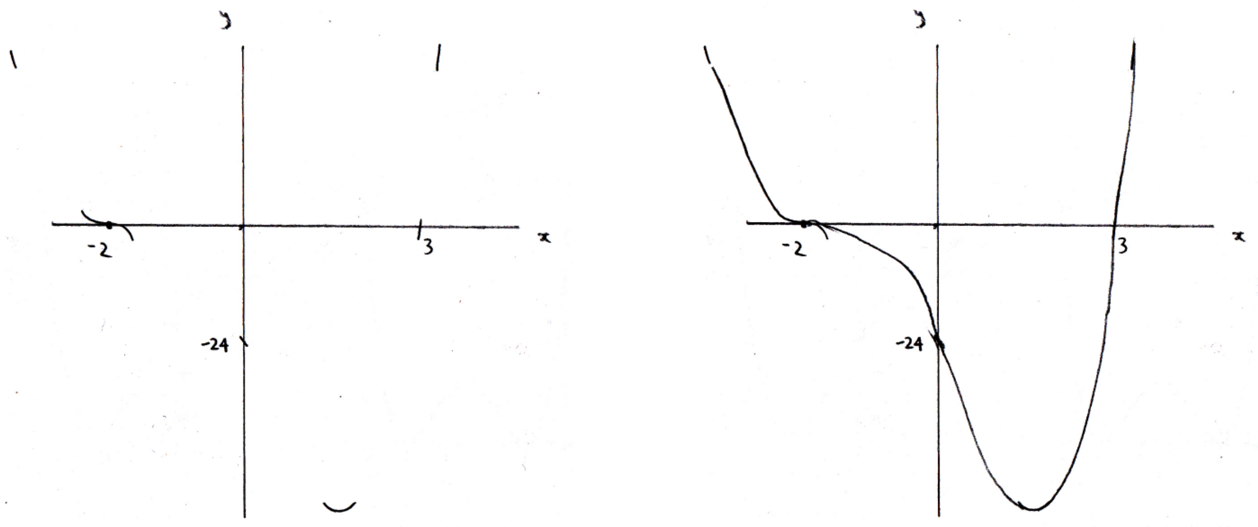
(Draw a short steep downward arc at top left and a short steep upward arc at top right.)

Check for stationary points.
The graph has a thrice-repeated factor of $(x+2)^3$, so the graph of y is quite different from the previous quartic examples. The point (-2, 0) is now a stationary point of inflection and the graph itself resembles a cubic of the form ax^3 around that point.

We have worked out the y-intercept as (0, -24), and also told that the other stationary point is a minimum at T $\approx$ (2, -70). The point of inflection is on the "decreasing" part of the graph, so the graph will resemble that of $y = -x^3$ at rhat point.

(Draw a short portion of a'-x^3' graph at (-2,0) for the point of inflection and a short 'trough' downward arc at (2, -70) for the minimum.)

Join up.
(Join the points, lines and curves plotted so far with a smooth curve.)



So far, the graph sketching examples were of polynomial functions, and therefore their graphs had no discontinuities, but they did have stationary points. The next two examples will have discontinuities and no stationary points, being of reciprocal type.

Example (10): Sketch the graph of $y = \dfrac{5}{x+2}$, including asymptotes.

How is the graph related to that of $y = \dfrac{1}{x}$ by transformations ?

<u>Find the intercepts.</u>
When $x = 0$, $y = 2.5$. There is no value of x which would make y equal to zero, so there is no x-intercept. ***(Plot the point).***

<u>Check for symmetry.</u>
Function neither odd nor even – no help.

<u>Check behaviour of function for large x.</u>
When x becomes large and positive, y approaches zero whilst still remaining positive, e.g. (98, 0.05). ***(Draw a short near-horizontal arc at the right just above the x-axis).***

When x becomes large and negative, y approaches zero whilst still remaining negative e.g. (-102, -0.05). ***(Draw a short near-horizontal arc at the left just below the x-axis).***

<u>Check for discontinuities, and the behaviour of the function near them.</u>
The function is undefined (division by zero error !) when $x + 2 = 0$, i.e. when $x = -2$. The graph is therefore discontinuous, having an asymptote at the line $x = -2$. ***(Draw in the asymptote at x = -2).***

When x is slightly larger than –2 (e.g. –1.99) , we have a "+ve over a +ve", and so y is large and positive, e.g. (-1.99, 500). ***(Draw a short near-vertical arc at the top, just to the right of the asymptote at x = -2).***

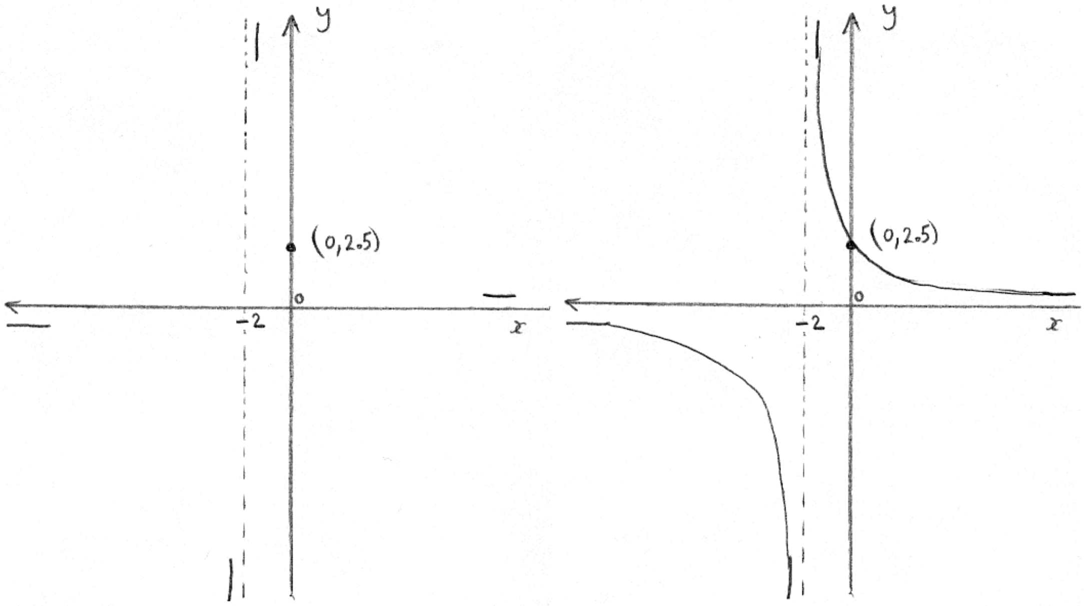
When x is slightly smaller than –2 (e.g. –2.01) , we have a "+ve over a –ve", so y is large and negative, e.g (-2.01, -500). ***(Draw a short near-vertical arc at the bottom of the graph just to the left of the asymptote at x = -2).***

<u>Check for stationary points.</u>
Not applicable here.

Join up.
This graph has an asymptote at x = -2, so there will be two unconnected sections.

(Draw a curve connecting the the two short arcs to the right of the asymptote at x = -2 and passing through the point (0, 2.5)).

(Draw a corresponding curve connecting the the two short arcs to the left of the asymptote at x = -2 and passing through the point (0, 2.5).



The graph above could also have been obtained by transforming $y = \dfrac{1}{x}$ via a translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ to give $y = \dfrac{1}{x-2}$, followed by a y-stretch with scale factor 5 to finally give $y = \dfrac{5}{x+2}$.

Example (11): Sketch the graph of $y = \dfrac{2x-5}{x+3}$, including asymptotes.

Find the intercepts. When x = 0, $y = -\frac{5}{3}$; also when $x = \frac{5}{2}$, y = 0. ***(Plot the points).***

Check for symmetry. Function neither odd nor even – no help.

Check behaviour of function for large x.
When x becomes large (+ve and –ve), then the constant terms become relatively insignificant, and the graph will approach that of $y = \dfrac{2x}{x}$ or merely y = 2.
When x becomes large and positive, y gets closer to 2 from below, e.g. (97, 1.89).
(Draw a short near-horizontal arc at right just below the asymptote x= 2).

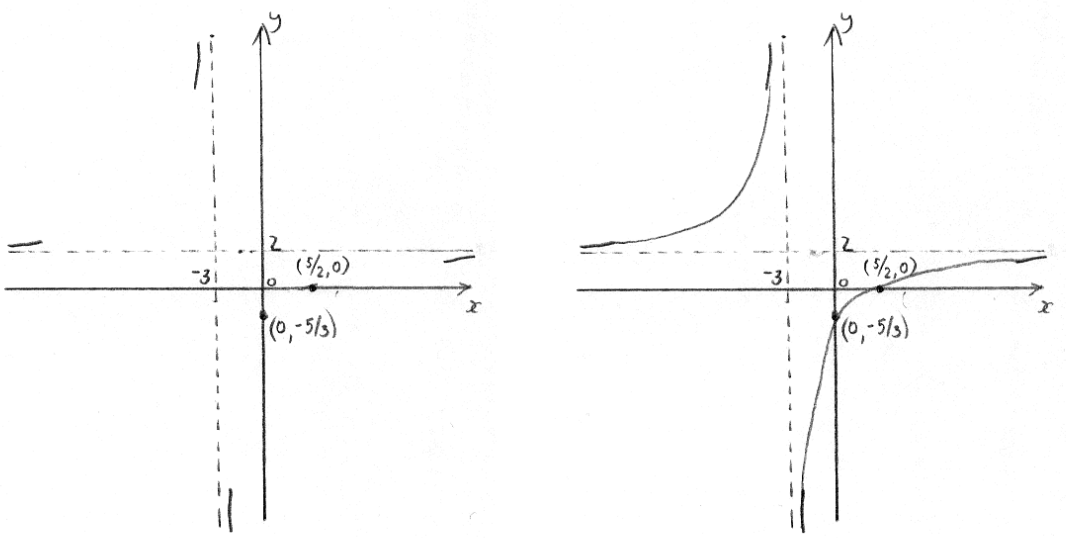
When x becomes large and negative, y approaches 2 from above, e.g. (-103, 2.11). ***(Draw a short near-horizontal arc at left just above the asymptote x = 2).***
(Draw asymptote at y = 2.)

Check for discontinuities, and the behaviour of the function near them.
The function is undefined when x + 3 = 0, i.e. when x = -3. ***(Draw in the asymptote at x = -3).***
When x is slightly larger than –3 (e.g. –2.9), we have a $\frac{'+'}{'-'}$ with a small denominator, so y is large and negative, for example at (-2.9, -108). ***(Draw a short near-vertical line at the bottom of the graph just to the right of the asymptote at x = -3).***
When x is slightly smaller than –3 (e.g. –3.1) , we have a $\frac{'-'}{'-'}$ with a small denominator, and so y is large and positive, for example at (-3.1, 112). ***(Draw a short near-vertical line at the top of the graph just to the left of the asymptote at x = -3).***

Check for stationary points. Not applicable here.

Join up.
(Draw a curve connecting the two short arcs in the lower right of the two asymptotes, passing through points (5/2,0) and (0, -5/3). The curve bends quite sharply as it passes through those points).

(Draw a similar curve connecting the two short arcs to upper left of the two asymptotes).



Example (12): Sketch the graph of $y = \frac{1}{x^2} - 9$. (There are no stationary points.)

Intercepts. When $x = 0$, y is undefined, and so the y-axis is an asymptote to the graph. ***(Mark the y-axis as an asymptote).***

To find the x-intercept(s), we solve $\frac{1}{x^2} = 9$, giving $x^2 = \frac{1}{9}$ or $x = \pm\frac{1}{3}$.

(Plot the points ($\frac{1}{3}$, 0) and (-$\frac{1}{3}$, 0).

Symmetry. Both $\frac{1}{x^2}$ and the constant function –9 are even, therefore $y = \frac{1}{x^2} - 9$ is an even function, with symmetry about the y-axis. (The x-intercepts gave a clue !)

Behaviour for large x. When x is large (both positive and negative, due to symmetry), $\frac{1}{x^2}$ is positive but ever-closer to 0 and thus $\frac{1}{x^2} - 9$ becomes closer to –9 from above. (Asymptote at the line y = -9).
(Draw an asymptote at y = -9.)
(Draw near-horizontal arcs at the left and right of the graph, just above the line y = -9.)

Discontinuities and behaviour near them.
When x is just greater than 0, y is large and positive., and this also holds true when x is just less than 0, by the symmetry of the graph.

(Draw near-vertical arcs at the top of the graph on each side of the y-axis).

Stationary points. Not applicable here.

Join up.
(Connect the arcs and points on each side of the asymptote at the y-axis with a smooth curve).

The graph is actually a y-translation of that of $\frac{1}{x^2}$ by –9 units

For a better sketch, we could also have plotted for x = 1 and x = -1 to get the 'corners' right, but examiners only check the *general* shape.