

M.K. HOME TUITION

Mathematics Revision Guides

Level: A-Level Year 1 / AS

A COMEDY OF ERRORS
(and how to avoid them)

Version : 1.2 Date: 20-12-2013

A COMEDY OF ERRORS

The main purpose of this section is to prevent unnecessary loss of marks in examinations.

Errors fall into various categories, namely

- Misreading the question
- Miscopying during the working
- Incorrect use of mathematics

Many of these errors can be avoided by reading the question carefully, managing time and checking over your work.

In each example, correct working is shown by a tick symbol and errors shown by a cross. An exclamation mark represents a possibility of some 'follow-through' credit despite an error in an earlier stage.

The written examples use the single arrow for implication, i.e.

$5x = 20 \rightarrow x = 4$ i.e. ' if $5x = 20$, then $x = 4$' , although the expression $5x = 20 \Rightarrow x = 4$ is the more correct form.

Errors caused by misreading the question.

Example (1): Differentiate i); $9x^2 + 6x$; ii) $\sqrt{x}$; iii) $\dfrac{1}{x^2}$

(i) $\int 9x^2 + 6x \, dx = 3x^3 + 3x^2 + c$

(ii) $\int \sqrt{x} \, dx = \int x^{1/2} \, dx = \dfrac{x^{3/2}}{3/2} = \dfrac{2}{3} x^{3/2} + c$.

iii) $\int \dfrac{1}{x^2} \, dx = \int x^{-2} \, dx = \dfrac{x^{-1}}{-1} = -\dfrac{1}{x} + c$.

This is a big misread of the original question: the functions should have been *differentiated*, but the pupil *integrated* them.

Correct:

(i) $\dfrac{d}{dx}\left(9x^2 + 6x\right) = 18x + 6$

(ii) $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{d}{dx}\left(x^{1/2}\right) = \dfrac{1}{2} x^{-1/2} = \dfrac{1}{2\sqrt{x}}$.

iii) $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = \dfrac{d}{dx}\left(x^{-2}\right) = -2x^{-3} = \dfrac{-2}{x^3}$.

Example (2):

i) Show by the Factor Theorem that $(x + 4)$ is a factor of $P(x) = x^3 - 2x^2 - 9x + 60$.
ii) Factorise $P(x)$ and hence show that $P(x)$ has only one root.

$P(x) = x^3 - 2x^2 - 9x + 60$

$P(-4) = -64 - 32 + 36 + 60 = 0 \quad \therefore (x+4)$ is factor of $P(x)$.

$$\begin{array}{r|l} & x^2 - 6x + 15 \\ \hline x+4 & x^3 - 2x^2 - 9x + 60 \\ & \underline{x^3 + 4x^2} \\ & \quad -6x^2 - 9x \\ & \quad \underline{-6x^2 - 24x} \\ & \qquad\quad 15x + 60 \\ & \qquad\quad \underline{15x + 60} \\ & \qquad\qquad\quad 0 \end{array}$$

$P(x) = (x+4)(x^2 - 6x + 15)$

x	$P(x)$
-5	-70
-4	0
-3	42
-2	62
-1	66
0	60
1	50
2	42
3	42
4	56

Doesn't look as if it will = 0 again!

The working of most of the question was correct, but the pupil must have wasted a lot of time in calculating the values of $P(x)$ to convince the examiner that $P(x)$ had only one root.

The pupil should have inspected the quadratic factor and calculated the discriminant instead.

$P(x) = x^3 - 2x^2 - 9x + 60$

$P(-4) = -64 - 32 + 36 + 60 = 0 \quad \therefore (x+4)$ is factor of $P(x)$.

$$\begin{array}{r|l} & x^2 - 6x + 15 \\ \hline x+4 & x^3 - 2x^2 - 9x + 60 \\ & \underline{x^3 + 4x^2} \\ & \quad -6x^2 - 9x \\ & \quad \underline{-6x^2 - 24x} \\ & \qquad\quad 15x + 60 \\ & \qquad\quad \underline{15x + 60} \\ & \qquad\qquad\quad 0 \end{array}$$

$P(x) = (x+4)(x^2 - 6x + 15)$

Quadratic factor,

$x^2 - 6x + 15$ has discriminant

"$b^2 - 4ac$" $= (-6)^2 - 4(1)(15) = 36 - 60 = -24$

Since $x^2 - 6x + 15$ has no real roots, $(b^2 - 4ac < 0)$

there are no more roots of $P(x)$

other than $x = -4$.

Example (3): Find the equation of the normal to the curve $y = 8 - \sqrt{x}$ at the point (4, 6).

$y = 8 - \sqrt{x} \rightarrow \frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$ at (4, 6), $\frac{dy}{dx} = \frac{-1}{4}$

Gradient of tangent at (4, 6) = $-\frac{1}{4}$, so its equation is

$y - 6 = -\frac{1}{4}(x - 4) \rightarrow 4y - 24 = 4 - x$

$\rightarrow x + 4y - 28 = 0$

The pupil had misread the question and found the equation of the *tangent* to the curve instead of the equation of the *normal* to it.

Correct version:

$y = 8 - \sqrt{x} \rightarrow \frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$ at (4, 6), $\frac{dy}{dx} = \frac{-1}{4}$

$\therefore$ Gradient of normal at (4, 6) = 4, so its equation is

$y - 6 = 4(x - 4) \rightarrow 4x - 16 - y + 6 = 0$

$\rightarrow 4x - y - 10 = 0$

Example (4): An arithmetic series has 26 for its first term, a common difference of 4 and a sum of 378 to n terms. Find n.

The pupil had submitted a near-perfect answer, marred only by their failing to reject the negative value of n as being inadmissible in the context of the question.

The correct working is below.

$\frac{1}{2}n\left(2a + (n-1)d\right) = 378$

$a = 26\ ,\ d = 4$

$\rightarrow n\left(52 + 4(n-1)\right) = 756$

$\rightarrow n\left(52 + 4n - 4\right) = 756$

$\rightarrow 4n^2 + 48n - 756 = 0$

$\rightarrow n^2 + 12n - 189 = 0$

$\rightarrow (n+21)(n-9) = 0$

$\therefore n = 9$ or -21.

$\frac{1}{2}n\left(2a + (n-1)d\right) = 378$

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$\rightarrow (n+21)(n-9) = 0$

$\therefore n = 9$ or -21.

Only $n = 9$ is valid in scope of question, so A.P. has 9 terms.

Errors caused by miscopying during the working.

Example (5): Evaluate $\sqrt{a^2+b^2+11}$ where a = 3 and b = 4.

$$a = 3,\ b = 4 \rightarrow \sqrt{a^2+b^2+11} = \sqrt{3^2+4^2+11}$$

$$= \sqrt{25} + 11 = 16.$$

The pupil had blundered on the last step: the 11 was *inside* the square root sign up until then, but was incorrectly brought *outside* the square root sign, giving an incorrect final result. Be careful with expressions within square roots; if in doubt, use brackets to avoid confusion.

The correct working is shown below:

$$a = 3,\ b = 4 \rightarrow \sqrt{a^2+b^2+11} = \sqrt{3^2+4^2+11}$$

$$= \sqrt{25+11} = \sqrt{36} = 6$$

Example (6): Solve the equation $2x^2 - 3x - 1 = 0$, giving your answers correct to 2 decimal places.

$$y = 2x^2 - 3x - 1 = 0 \rightarrow a, b, c = 2, -3, -1$$

$$\text{Roots are } \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}$$

$$= 3 + \frac{\sqrt{17}}{4},\ 3 - \frac{\sqrt{17}}{4} \text{ or } 4.03,\ 1.97 \text{ to 2 dp.}$$

The pupil had substituted the correct values into the general quadratic formula, but the fraction $\frac{3 \pm \sqrt{17}}{4}$ was miscopied as $3 \pm \frac{\sqrt{17}}{4}$ because the fraction bar was not written clearly below the 3.

Correct working :

$$y = 2x^2 - 3x - 1 = 0 \rightarrow a, b, c = 2, -3, -1$$

$$\text{Roots are } \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3 \pm \sqrt{17}}{4}$$

$$= \frac{3+\sqrt{17}}{4},\ \frac{3-\sqrt{17}}{4} \text{ or } 1.78,\ -0.28 \text{ to 2 dp.}$$

Always write fractional expressions with the bar clearly shown !

Example (7): A curve has equation $y = x^3 - 6x^2 + 8x$. Find the x-coordinates of the turning points in an exact form, and determine their nature.

$$y = x^3 - 6x^2 + 8x \rightarrow \frac{dy}{dx} = 3x^2 - 12x + 8 \rightarrow \frac{d^2y}{dx^2} = 6x - 12$$

$$\frac{dy}{dx} = x^3 - 6x^2 + 8x = 0 \text{ at turning points} \rightarrow x(x^2 - 6x + 8) = 0 \rightarrow x(x-2)(x-4) = 0$$

$\therefore$ Turning points at $x = 0$, $x = 2$, $x = 4$

$\frac{d^2y}{dx^2} = 6x - 12$, so

at $x = 0$, $\frac{d^2y}{dx^2} = -12 \rightarrow$ maximum

at $x = 2$, $\frac{d^2y}{dx^2} = 0 \rightarrow$ neither max nor min

at $x = 4$, $\frac{d^2y}{dx^2} = 12 \rightarrow$ minimum

The pupil started the question correctly by working out the first and second derivatives, but then miscopied the first derivative as the original function $x^3 - 6x^2 + 8x$ instead of $3x^2 - 12x + 8$, and carried the error throughout the rest of the question.

Correct working:

$$y = x^3 - 6x^2 + 8x \rightarrow \frac{dy}{dx} = 3x^2 - 12x + 8 \rightarrow \frac{d^2y}{dx^2} = 6x - 12$$

$$\frac{dy}{dx} = 0 \text{ when } 3x^2 - 12x + 8 = 0 \rightarrow x = \frac{12 \pm \sqrt{144 - 96}}{6} \rightarrow x = \frac{12 \pm \sqrt{48}}{6}$$

$$\rightarrow x = \frac{6 \pm 2\sqrt{3}}{3} \quad \therefore x = 2 + \frac{2\sqrt{3}}{3}, \; 2 - \frac{2\sqrt{3}}{3}$$

$\frac{d^2y}{dx^2} = 6x - 12$, so at $x = 2 + \frac{2\sqrt{3}}{3}$, $\frac{d^2y}{dx^2} = 12 + 4\sqrt{3} - 12 = 4\sqrt{3}$ > 0, so local minimum

at $x = 2 - \frac{2\sqrt{3}}{3}$, $\frac{d^2y}{dx^2} = 12 - 4\sqrt{3} - 12 = -4\sqrt{3}$ < 0, so local maximum

Example (8): A line L_1 has the equation $y = 2x + 7$.

Line L_2 is perpendicular to L_1 and passes through the point (0, 1).

Find the coordinates of the point of intersection of the two lines.

Line $y = 2x + 7$ has a gradient of 2

$\therefore$ Perpendicular line has gradient of $-1/2$

Perpendicular line passes through (0, -1),

so its equation is $y = -\frac{1}{2}x - 1$

Lines intersect when $2x + 7 = -\frac{1}{2}x - 1 \rightarrow 2\frac{1}{2}x + 8 = 0$

$\rightarrow x = \frac{-16}{5}$. Sub. in $y = 2x+7$, $y = \frac{-32}{5} + 7 = \frac{3}{5}$

$\therefore$ Lines meet at $\left(\frac{-16}{5}, \frac{3}{5}\right)$.

The pupil had started well but miscopied the y-intercept of line L_2 as (0, -1) instead of (0, 1), carrying the error into the latter part of the question.

Correct answer:

Line $y = 2x + 7$ has a gradient of 2

$\therefore$ Perpendicular line has gradient of $-1/2$

Perpendicular line passes through (0, 1),

so its equation is $y = -\frac{1}{2}x + 1$ or $y = 1 - \frac{1}{2}x$

Lines intersect when $2x + 7 = 1 - \frac{1}{2}x \rightarrow 2\frac{1}{2}x + 6 = 0$

$\rightarrow x = \frac{-12}{5}$. Sub in $y = 2x+7$, $y = \frac{-24}{5} + 7 = \frac{11}{5}$

$\therefore$ Lines meet at $\left(\frac{-12}{5}, \frac{11}{5}\right)$.

Example (9): Expand $\left(2+\frac{1}{2}x\right)^3$ by the binomial theorem.

$$\left(2+\tfrac{1}{2}x\right)^3 = 2^3 + \binom{3}{1}2^2\left(\tfrac{1}{2}x\right) + \binom{3}{2}2\left(\tfrac{1}{2}x\right)^2 + \left(\tfrac{1}{2}x\right)^3$$

$$= 8 + 12\left(\tfrac{1}{2}x\right) + 6\left(\tfrac{1}{4}x^2\right) + \tfrac{1}{8}x^3$$

$$= 8 + 6x + \tfrac{3}{2}x^2 + \frac{1}{8x^3}$$

The pupil's working was correct, but the final term, $\frac{1}{8}x^3$,was miscopied as $\frac{1}{8x^3}$.

Always write expressions involving fractions in such a way that there can be no confusion.

Correct is:

$$\left(2+\tfrac{1}{2}x\right)^3 = 2^3 + \binom{3}{1}2^2\left(\tfrac{1}{2}x\right) + \binom{3}{2}2\left(\tfrac{1}{2}x\right)^2 + \left(\tfrac{1}{2}x\right)^3$$

$$= 8 + 12\left(\tfrac{1}{2}x\right) + 6\left(\tfrac{1}{4}x^2\right) + \tfrac{1}{8}x^3$$

$$= 8 + 6x + \tfrac{3}{2}x^2 + \tfrac{1}{8}x^3$$

Errors caused by incorrect mathematics.

The previous errors were all caused by carelessness in reading the question, or in miscopying at some stage of the working. This section deals with actual errors in the mathematics.

Dodgy Distributivity.

We know that $(a + b)\,x = ax + bx$ where x is any non-zero quantity, .e.g. $5 \times (3 + 4) = (5 \times 3) + (5 \times 4)$.

In other words, multiplication is distributive over addition.

Since division by a non-zero number is the same as multiplication by its reciprocal, the same rule holds, e.g. $\dfrac{3+2}{7} = \dfrac{3}{7} + \dfrac{2}{7}$

The same thing **cannot** be said of most other functions.

Example (10):
i) Work out $\sqrt{98} + \sqrt{2}$, simplifying your answer.
ii) Write down the value of lg 7 + lg 5.
iii) Work out the value of sin 60°, given sin 30° = 0.5, and using a diagram if you wish.

(i) $\sqrt{98} + \sqrt{2} = \sqrt{100} = 10$ ✗

(ii) $\lg(7) + \lg(5) = \lg(12)$ ✗

(iii) $\sin 60^\circ = \sin(30^\circ + 30^\circ)$
$= \sin 30^\circ + \sin 30^\circ = 0.5 + 0.5 = 1$ ✗

In general,

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \;;\; \sqrt{16+9} \neq \sqrt{16} + \sqrt{9}$$

$\lg(a+b) \neq \lg(a) + \lg(b)$; by the log laws, $\lg(a) + \lg(b) = \lg(ab)$

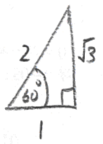
$\sin(a+b) \neq \sin(a) + \sin(b)$; let $a = b = 45°$, $\sin(a + b) = \sin 90° = 1$, but $\sin 45° + \sin 45° = \sqrt{2}$.

The correct answers are shown below:

(i) $\sqrt{98} + \sqrt{2} = \sqrt{49}\sqrt{2} + \sqrt{2}$ ✓
$= 7\sqrt{2} + \sqrt{2} = 8\sqrt{2}$. ✓

(ii) $\lg(7) + \lg(5) = \lg(7 \times 5) = \lg 35$. ✓

(iii) $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$ ✓



Other examples of non-distributivity are:

$(a+b)^2 \neq a^2 + b^2$; proved by expanding, $(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 \neq a^2 - b^2$; proved by expanding, $(a-b)^2 = a^2 - 2ab + b^2$, and besides,

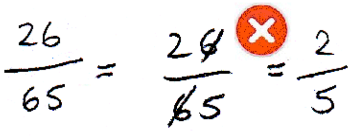
$a^2 - b^2 = (a+b)(a-b)$.

$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$; simplifying the RHS gives $\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$.

$x^{a+b} \neq x^a + x^b$; by the power laws, $x^{a+b} = x^a \times x^b$

Crazy Cancellation.

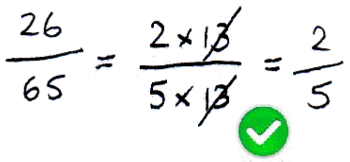
Example (11): Express the fraction $\frac{26}{65}$ in its lowest terms, showing your working.



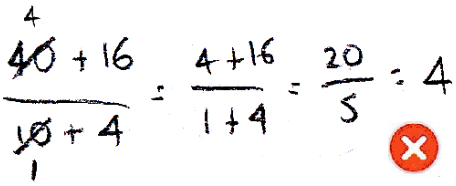
This is a ridiculous, but amusing, instance of the pupil obtaining the correct result, but using a totally incorrect method. The 6's were 'cancelled out' as if they were factors !

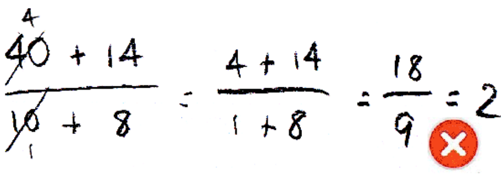
As a matter of interest, $\frac{16}{64}$ and $\frac{19}{95}$ can also be reduced to their lowest terms using this "method".

The correct method is to factorise the top and bottom lines.



Example (12) (intro): Here is another strange example, using numbers :





The starting expression in the first case is equal to $\frac{56}{14}$. We then tried cancelling a factor of 10 from one of the terms on the top and bottom, and ended up with a result of $\frac{20}{5}$. Both results happened to be equal to 4, but this method does not work in general.

In the second case, $\frac{54}{18}$, or 3, is certainly not equal to $\frac{18}{9}$, or 2.

We can only cancel out *factors* and not *individual terms* when simplifying expressions.

This example was somewhat contrived, but it is surprising how many pupils quite happily cancel out expressions 'term by term', as the next example will show.

Example (12): Simplify $\dfrac{2x^2+7x-4}{2x^2+9x+4}$.

$$\frac{2x^2+7x-4}{2x^2+9x+4} = \frac{\cancelto{1}{2x^2}+7x-\cancelto{1}{4}}{\cancelto{1}{2x^2}+9x+\cancelto{1}{4}} = \frac{7x}{9x+2}$$

The pupil spotted the common terms of $2x^2$ on the top and bottom, so they were cancelled out to 1. The same method was applied when the pupil cancelled out the 4's from top and bottom.

This is totally wrong - we **cannot** cancel out terms !

The correct working is :

$$\frac{2x^2+7x-4}{2x^2+9x+4} = \frac{(2x-1)(x+4)}{(2x+1)(x+4)}$$

$$= \frac{(2x-1)\cancel{(x+4)}}{(2x+1)\cancel{(x+4)}} = \frac{2x-1}{2x+1}$$

We *factorise* the quadratics on both the top and bottom lines, find the common factor of $(x + 4)$, and finally cancel it out.

Another false 'method' is the following:

$\frac{3}{2}+\frac{9}{4}=\frac{3+9}{2+4} \Rightarrow \frac{12}{6}=2$, which shows serious ignorance of fraction arithmetic.

Example (13): Simplify $\frac{4x^2-18}{x-3}$.

$$\frac{4x^2-18}{x-3} = \frac{4x^2}{x} - \frac{18}{3}$$

$$= 4x - 6 = 2(x-3)$$

The pupil had 'decoupled' the terms from top and bottom to form two separate fractions – the inverse of the wrong method shown in the previous sentence. The correct method is:

$$\frac{4x^2-18}{x-3} = \frac{2(x+3)\cancel{(x-3)}}{\cancel{x-3}} = 2(x+3)$$

.

Simplification of equations can also lead to incorrect conclusions and 'lost' solutions.

Example (14): Solve the equation $2x^2 = 7x$.

$$2x^2 = 7x \rightarrow 2x = 7$$

$$\rightarrow x = 7/2$$

We have divided both sides of the equation by the variable x , but in so doing, we have lost a solution.
.
The correct working is:

$$2x^2 = 7x \rightarrow 2x^2 - 7x = 0$$

$$\rightarrow x(2x-7) = 0$$

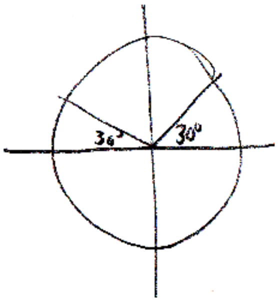
$$\rightarrow x = 0,\ x = 7/2$$

We had lost the solution $x = 0$ in the incorrect working.

Here is another example of thoughtless cancelling leading to a ‘lossy’ solution.

Example (15): Solve the equation $2 \sin^2 x = \sin x$ giving all solutions in the range $0° \le x < 360°$.

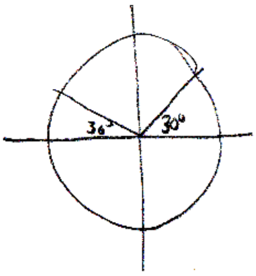
$$2 \sin^2 x = \sin x$$
$$\rightarrow 2 \sin x = 1$$
$$\rightarrow \sin x = \tfrac{1}{2}$$
$$\rightarrow x = 30°, 150°$$



The pupil had made the error of cancelling out a factor of sin *x* from both sides of the equation, and losing some solutions in so doing. The only factors we can cancel out safely are non-zero constants, although expressions can be cancelled out if they cannot take a value of zero in the context of the question.

The correct working is shown below – notice that there are four solutions to the original equation, not two. The solutions corresponding to $\sin x = 0$ were ‘lost’ in the incorrect version.

$$2 \sin^2 x - \sin x = 0$$
$$\sin x (2 \sin x - 1) = 0$$
$$\sin x = 0 \rightarrow x = 0°, 180°$$
$$\sin x = \tfrac{1}{2} \rightarrow x = 30°, 150°$$



Careless cancellation can lead to more than just missing solutions.

Example (16):

Here we have managed to ‘prove’ that 2 = 1.

i) Take two numbers *a* and *b* such that *a* = *b*.

ii) Multiply both sides by *a* and then subtract b^2 from both sides.

$$a = b \rightarrow a^2 = ab \rightarrow a^2 - b^2 = ab - b^2$$
$$\rightarrow (a-b)(a+b) = b(a-b)$$

iii) Factorise, cancel and simplify.

$$\rightarrow (a+b) = b$$
$$\rightarrow b+b = b$$
$$\rightarrow 2b = b$$

iv) 2 = 1.

$$\rightarrow 2 = 1$$

Something has gone wrong here, but what ?

The next brevity is even sillier.

Example (17): Solve the equation 3*x* = 2*x*.

Divide both sides by *x*, and we have this weird result:

$$3x = 2x$$
$$\therefore 3 = 2$$

The correct result below holds the key to both this example and the last one !

$$3x = 2x \rightarrow 3x - 2x = 0 \rightarrow x = 0$$

Recall the previous example:

Everything is tickety-boo so far...

$$a = b \rightarrow a^2 = ab \rightarrow a^2 - b^2 = ab - b^2$$

Factorising ....

$$\rightarrow (a-b)(a+b) = b(a-b)$$

Dividing both sides by *a* – *b*, but at the start of the sum we assumed *a* = *b*.

$$\rightarrow (a+b) = b$$
$$\rightarrow b+b = b$$
$$\rightarrow 2b = b$$
$$\rightarrow 2 = 1$$

Hence we are dividing by zero.

In this example, the only solution to the equation 3*x* = 2*x* is *x* = 0 when done correctly. When we factored out *x* in the faulty working, we divided by zero.

Look at the following deduction:

If $x = ab$, then $a = \frac{x}{b}$. Let *b* = 0, and $a = \frac{x}{0}$ hence *x* = *a* × 0 = 0.

∴ if *x* = 0 and *b* = 0, then $a = \frac{0}{0}$. This means that $\frac{0}{0}$ can take any value we give for *a* .

$\frac{0}{0}$ is not equal to 1 – in fact it is **undefined**.

We will finish on this one :

If $f(x) = \dfrac{1}{x}$, what happens to $f(x)$ when x approaches zero ?

As positive x approaches zero, $f(x)$ becomes increasingly large and positive.

As negative x approaches zero, $f(x)$ becomes increasingly large and negative.

These values are also said to **tend to infinity.**

Another way of writing this is $\lim\limits_{x \to +0} \frac{1}{x} = +\infty$, similarly, $\lim\limits_{x \to -0} \frac{1}{x} = -\infty$.

This brings the idea of limiting values ('lim' is just short for 'limit).

Notice how the limits go off in different directions depending if x is positive or negative.
When $x = 0$, $f(x)$ is **undefined**, which is **not** the same as saying $\frac{1}{0} = \infty$, which is nonsensical.

Quite simply – **division by zero is undefined**.

Quadratic Quandaries.

Example (18): Show that the quadratic equation $x^2 - 6x + 15 = 0$ has no real roots.

$x^2 - 6x + 15$ has discriminant

"$b^2 - 4ac$" $= -6^2 - 4(1)(15) = -36 - 60 = -96$

$\therefore$ $x^2 - 6x + 15$ has no real roots $(b^2 - 4ac < 0)$

This explanation is correct – or is it ? There is an arithmetic error lurking there, but it does not appear to affect the answer.

The correct answer is

$x^2 - 6x + 15$ has discriminant

"$b^2 - 4ac$" $= (-6)^2 - 4(1)(15) = 36 - 60 = -24$

$\therefore$ $x^2 - 6x + 15$ has no real roots $(b^2 - 4ac < 0)$

The error in the first answer was in calculating the square of -6, not helped by the notation -6^2.
The pupil had interpreted -6^2 as $-(6^2)$, or -36, rather than $(-6)^2$ or 36. As it turned out, this did not affect the final reasoning.

Example (19): Solve the equation $x^2 - 12x + 15 = 0$, giving your answers in the form $a \pm \sqrt{b}$, where a and b are integers.

$x^2 - 12x + 15 = 0$

$\rightarrow x = \dfrac{12 \pm \sqrt{144-60}}{2} = \dfrac{12 \pm \sqrt{84}}{2}$

$\rightarrow x = 6 \pm \sqrt{42}$ $\quad x = 6 + \sqrt{42},\ 6 - \sqrt{42}$

The pupil made an error in the surd arithmetic: $\dfrac{\sqrt{84}}{2} = \dfrac{\sqrt{84}}{\sqrt{4}} = \sqrt{\dfrac{84}{4}} = \sqrt{21}$, not $\sqrt{42}$.

The correct answer is :

$x^2 - 12x + 15 = 0$

$\rightarrow x = \dfrac{12 \pm \sqrt{144-60}}{2} = \dfrac{12 \pm \sqrt{84}}{2}$

$\rightarrow x = 6 \pm \sqrt{21}$ $\quad x = 6 + \sqrt{21},\ 6 - \sqrt{21}$

Example (20): Solve the equation $x^2 - 6x + 8 = -1$.

The pupil forgot to rearrange the equation into the correct form, i.e. $x^2 - 6x + 9 = 0$.

Always have zero on the RHS when solving a quadratic equation by factorising.

$$x^2 - 6x + 8 = -1$$
$$\rightarrow (x-2)(x-4) = -1$$
$$\rightarrow x - 2 = -1 \text{ or } x - 4 = -1$$
$$\therefore x = 1 \text{ or } x = 3$$

The correct working is :

$$x^2 - 6x + 8 = -1$$
$$\rightarrow x^2 - 6x + 9 = 0$$
$$\rightarrow (x-3)^2 = 0$$
$$\rightarrow x = 3.$$

Example (21):

Solve $\log(a + 15) = 2 \log(a - 15)$.

The interesting thing here is that the base of the logarithm is immaterial !

The pupil clearly spotted the disguised quadratic in the equation, but failed to spot that the 'solution' $a = 10$ was inadmissible, for reasons shown below.

$$\log(a+15) = 2\log(a-15)$$
$$\rightarrow \log(a+15) = \log((a-15)^2)$$
$$\rightarrow (a-15)^2 - (a+15) = 0$$
$$\rightarrow a^2 - 30a + 225 - a - 15 = 0$$
$$\rightarrow a^2 - 31a - 210 = 0$$
$$\rightarrow (a-21)(a-10) = 0 \quad \therefore a = 21, a = 10$$

Correct working:

$$\log(a+15) = 2\log(a-15)$$
$$\rightarrow \log(a+15) = \log((a-15)^2)$$
$$\rightarrow (a-15)^2 - (a+15) = 0$$
$$\rightarrow a^2 - 30a + 225 - a - 15 = 0$$
$$\rightarrow a^2 - 31a - 210 = 0$$
$$\rightarrow (a-21)(a-10) = 0 \quad \therefore a = 21$$

$\cancel{a = 10}$ inadmissible $10 - 15 = -5$
$\log(-5)$ undefined

Sometimes a simple question might have a subtler meaning:

Example (22):

i) Write down the value of $\sqrt{49}$.
ii) Hence solve the equation $x^2 - 49 = 0$.

(i) $\sqrt{49} = +7, -7$ (ii) solutions of $x^2 - 49 = 0$ are $x = 7,\ x = -7$.

Part ii) is correct here, but not part i). This is because the square root function is defined solely as the *positive* square root. The correct definition is shown below.

(i) $\sqrt{49} = 7$ (ii) solutions of $x^2 - 49 = 0$ are $x = \pm\sqrt{49} \rightarrow x = +7, -7$.

Notice how the $\pm$ sign is shown outside the square root.

Example (23):

i) Solve the equation $x^2 – 2x – 24 = 0$.
ii) Hence solve the inequality $x^2 – 2x – 24 > 0$.

(i) $x^2 - 2x - 24 = 0 \rightarrow (x-6)(x+4) = 0 \rightarrow x = 6,\ x = -4$.

(ii) $x^2 - 2x - 24 > 0 \rightarrow (x-6)(x+4) > 0 \rightarrow (x-6) > 0$ or $(x+4) > 0$

$\rightarrow x > 6$ or $x > -4$

$\rightarrow x > -4$.

The pupils had solved the quadratic equation correctly in part i), but used flawed reasoning when attempting to solve the inequality.

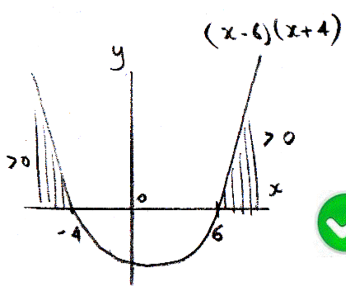
We could reason $(x - 6)(x + 4) > 0 \Rightarrow (x - 6) > 0$ and $(x + 4) > 0$, or $(x - 6) < 0$ and $(x + 4) < 0$ (two negatives give a positive product !) .

The easiest way, however, of solving a quadratic inequality is to include a sketch graph, as in the correct solution below. Note that the solution consists of two distinct inequalities combined.

(i) $x^2 - 2x - 24 = 0 \rightarrow (x-6)(x+4) = 0 \rightarrow x = 6,\ x = -4$.

(ii) $x^2 - 2x - 24 > 0 \rightarrow (x-6)(x+4) > 0$

(ii)



$(x-6)(x+4) > 0$
when $x > 6$ or
$x < -4$.

Infuriating Inequalities.

Example (24) : Solve the inequality $10 - x > 3 + 2x$.

$$\begin{aligned} 10 - x &> 3 + 2x \\ -x &> 2x - 7 \\ -3x &> -7 \\ x &> -7/3 \end{aligned} \qquad \begin{aligned} 10 - x &> 3 + 2x \\ -x &> 2x - 7 \\ -3x &> -7 \\ x &< -7/3 \end{aligned}$$

The pupil forgot to reverse the inequality sign when dividing by (-3) in the last step !
The correct answer is shown on the right.

That was a basic example; the next one is more sneaky.

Example (25) : A geometric series has a first term of 32 and a common ratio of 0.75.
i) Find the sum of the first 8 terms.
ii) Find the least number of terms needed for the sum to exceed 120.

(i) $a = 32$; $r = 0.75$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{32(1-0.75^n)}{1-0.75} = \frac{32(1-0.75^n)}{0.25}$$

$$S_8 = \frac{32(1-0.75^8)}{0.25} = 128(1-0.75^8) = 115.19$$

(ii) $S_n > 120 \rightarrow 128(1-0.75^n) > 120$

$$\rightarrow 1 - 0.75^n > \frac{15}{16}$$

$$\rightarrow -0.75^n > \frac{-1}{16}$$

$$\rightarrow 0.75^n < \frac{1}{16}$$

$$\rightarrow n \log_{10} 0.75 < \log_{10}\left(\frac{1}{16}\right)$$

So far, so good, but ...

$$\rightarrow n < \frac{\log_{10}(1/16)}{\log_{10}(0.75)}$$

$\rightarrow n < 9.63$ $\therefore$ 9 terms required for sum to exceed 120.

Positive numbers less than 1 have a negative logarithm. The pupil should have reversed the inequality sign after dividing both sides by lg(0.75).

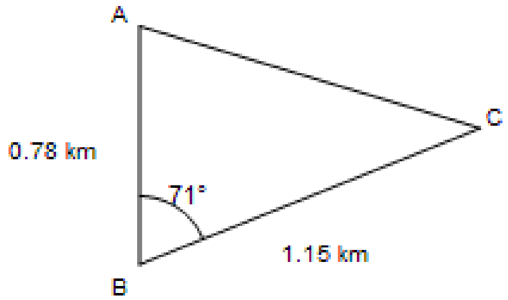
The correct final step is:

$$\rightarrow n > \frac{\log_{10}(1/16)}{\log_{10}(0.75)}$$

$\rightarrow n > 9.63$ $\therefore$ 10 terms required for sum to exceed 120.

Trigonometric Trauma.

Example (26): *A*, *B* and *C* are the corners of a triangular field.
Find i) the area of the field;   ii) the perimeter of the field.



Area = $\frac{1}{2}$ AB.BC sin 71° = $\frac{1}{2} \times 0.78 \times 1.15 \times \sin 71°$ 0.369 km^2.

To find AC, use cosine rule. $(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos 71°$

$\rightarrow AC^2 = 0.78^2 + 1.15^2 - (2 \times 0.78 \times 1.15 \cos 71°) = 1.347$

∴ Perimeter = 0.78 + 1.15 + 1.347 = 3.28 km

One step had been carelessly omitted from the working. The pupil had forgotten to take the square root when calculating the length *AC.*

Correct version:

Area = $\frac{1}{2}$ AB.BC sin 71° $\frac{1}{2} \times 0.78 \times 1.15 \times \sin 71°$ = 0.369 km^2.

To find AC, use cosine rule. $(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos 71°$

$\rightarrow AC^2 = 0.78^2 + 1.15^2 - (2 \times 0.78 \times 1.15 \cos 71°) = 1.347$

$\rightarrow AC = \sqrt{1347} = 1.1605$

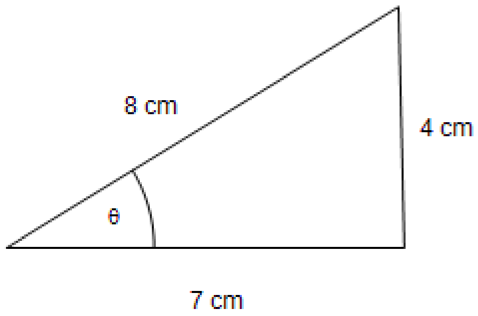
∴ Perimeter = 0.78 + 1.15 + 1.16 = 3.09 km.

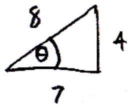
When a question is illustrated by a diagram, then the written annotations should be assumed to be correct, but that the diagram itself is *not* drawn accurately.

Example (27):

i) Find the value of cos θ.in the triangle shown on the right.

ii) Hence show that the area of the triangle = $\dfrac{\sqrt{3135}}{4}$ cm^2.





$\dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{7}{8} \quad \therefore \cos\theta = \dfrac{7}{8}$

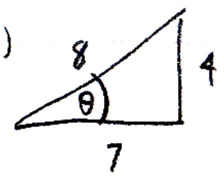
Area of triangle = $\frac{1}{2}$ base × height = $\frac{1}{2} \times 4 \times 7 = 14\text{ cm}^2$

Now, $14 = \dfrac{56}{4}$, and $56^2 = 3136 \rightarrow$ area $= \dfrac{\sqrt{3136}}{4}$ cm^2.

This 'solution' has one lethal flaw – the pupil assumed that, because the triangle *looked* right-angled, it actually *was* right-angled. (The sides do not satisfy Pythagoras' Theorem, as $4^2 + 7^2 = 65$, but $8^2 = 64$.) The pupil also tried to 'show' the result in ii) by using the faulty area and working backwards.

Correct solution:

(i)



$\cos\theta = \dfrac{7^2 + 8^2 - 4^2}{2 \times 7 \times 8} = \dfrac{97}{112}$

(ii) $\sin^2\theta = 1 - \dfrac{97^2}{112^2} = \dfrac{12544 - 9409}{12544} = \dfrac{3135}{12544}$

$\rightarrow \sin\theta = \dfrac{\sqrt{3135}}{112}$

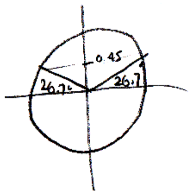
$\therefore \text{Area} = \frac{1}{2}(7)(8) \times \dfrac{\sqrt{3135}}{112} = \dfrac{28\sqrt{3135}}{112} = \dfrac{\sqrt{3135}}{4}\text{ cm}^2$

Because we are not dealing with a right-angled triangle, we need to use the cosine formula, the identity $\cos^2\theta + \sin^2\theta = 1$, and the general triangle area formula to answer the question.

Example (28): Solve the equation sin 2*x* = 0.45 giving all solutions in the range 0° ≤ *x* < 360°.

$\sin 2x = 0.45$

$2x = 26.7°$



$2x = 26.7°$

$2x = 153.3°$

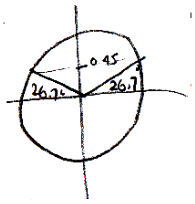
$\therefore x = 13.4°$

$x = 76.6°$

The pupil had 'lost' two solutions by forgetting to transform the range in which the solutions were to be found. The range 0° ≤ *x* < 360° should have been transformed to the range 0° ≤ 2*x* < 720° to prevent the losses. The correct solution is :

$\sin 2x = 0.45$

$2x = 26.7°, 153.3°$



Range $0° \leq x \leq 360°$

$\rightarrow 0° \leq 2x \leq 720°$

$\therefore 2x = 26.7°, 153.3°$

$386.7°, 513.3°$

$\therefore x = 13.4°, 76.6°$

$193.4°, 256.6°$

Example (29): Solve sin x + 2 cos x = 0 giving all solutions in radians where $0 \le x < 2\pi$.

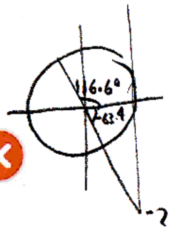
$\sin x + 2\cos x = 0$

$\rightarrow \sin x = -2\cos x$

$\rightarrow \dfrac{\sin x}{\cos x} = -2 \rightarrow \tan x = -2$

$\rightarrow x = -63.4°,$

$x = 116.6°$



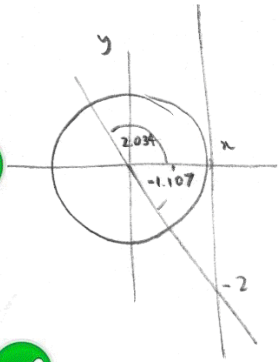
The pupil had given the solutions in degrees, but that is not the main issue. The value of -63.4° given on the calculator is outside the range, and another solution, which is in the range, had been omitted.

Correct working :

$\sin x + 2\cos x = 0$

$\rightarrow \sin x = -2\cos x$

$\rightarrow \dfrac{\sin x}{\cos x} = -2 \rightarrow \tan x = -2$



$x = \cancel{-1.107^c}$ not in range

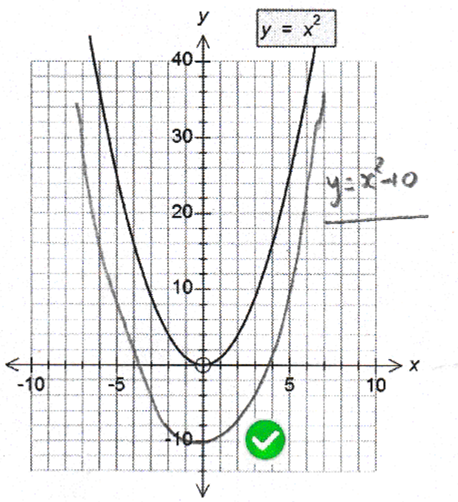
$x = 2.034^c \quad (-1.107 + \pi)$

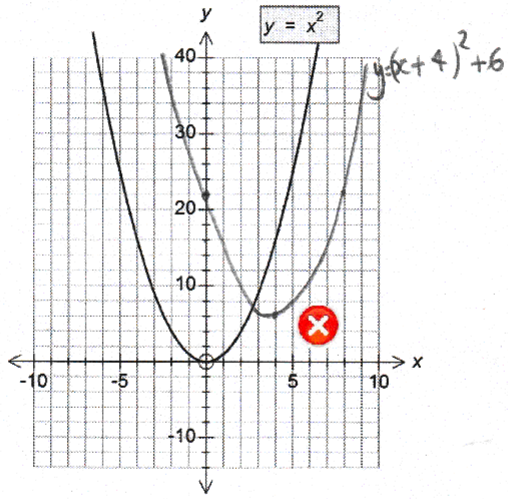
$x = 5.176^c \quad (-1.107 + 2\pi)$

(2 solutions)

Treacherous Transformations.

Example (30): Two graphs of $y = x^2$ are shown below. Sketch the following separately on each:
i) $y = x^2 - 10$; ii) $y = (x + 4)^2 + 6$.

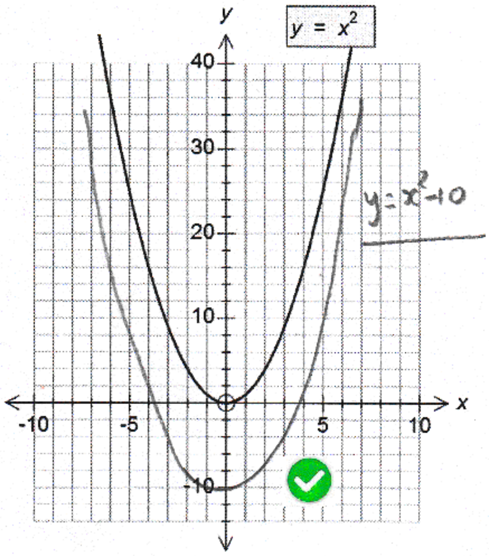


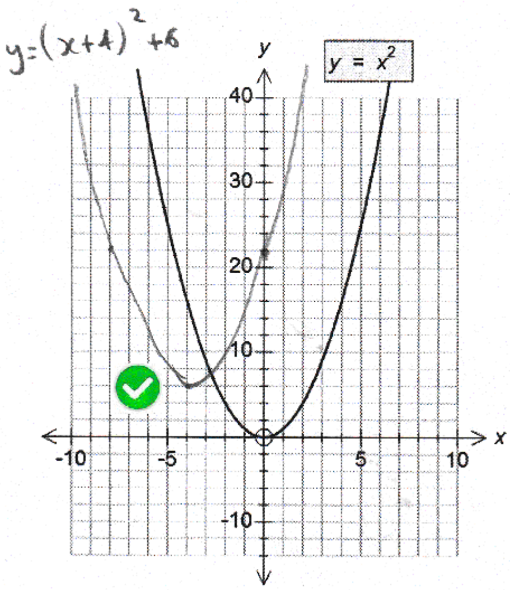


The pupil had sketched the graph of $y = x^2 – 10$ correctly by translating by -10 units vertically. The graph of $y = (x + 4)^2 + 6$ was wrong; the initial translation should have been by 4 units in the *negative* x-direction, although the y-translation was correct.

The x-translation seems to work the 'wrong way' at first.

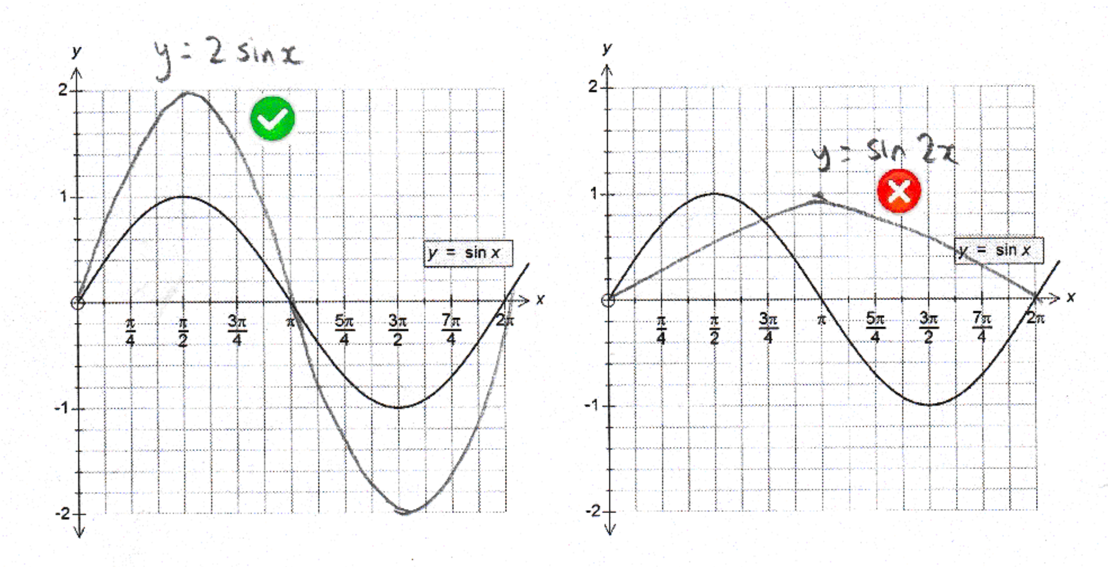
Correct answer:





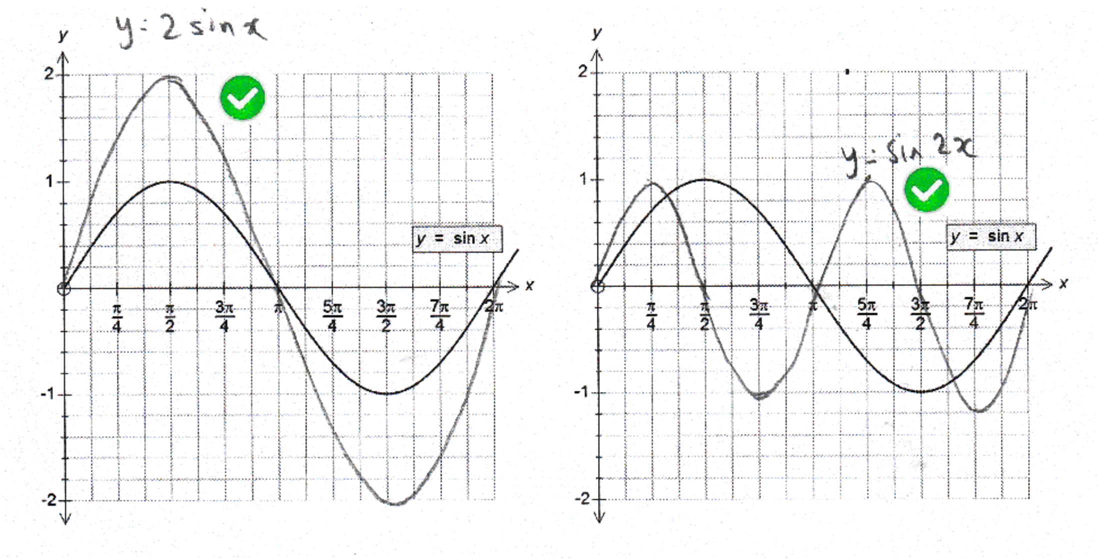
The graph of $y = (x + 4)^2 + 6$ is now correct, with its minimum point at (-4, 0) and y-intercept at (0, 22). (The question did not ask for those points).

Example (31): Two graphs of $y = \sin x$ are shown below. Sketch the following separately on each:
i) $y = 2 \sin x$; ii) $y = \sin 2x$.



The pupil transformed the first graph correctly with a y-stretch of scale factor 2. The second graph was stretched in the x-direction, but by the wrong scale factor, 2, instead of ½. .

Correct:



This time, the second graph was stretched in the x-direction by a scale factor of ½.. The periodicity of $\sin x$ is 2π, and that of $\sin 2x$ is π.

Example (32): Using the graph of $y = x^2$, use transformations to sketch the graph of $y = (2x - 3)^2$. Show any points where the graph of $y = (2x - 3)^2$ meets the coordinate axes.

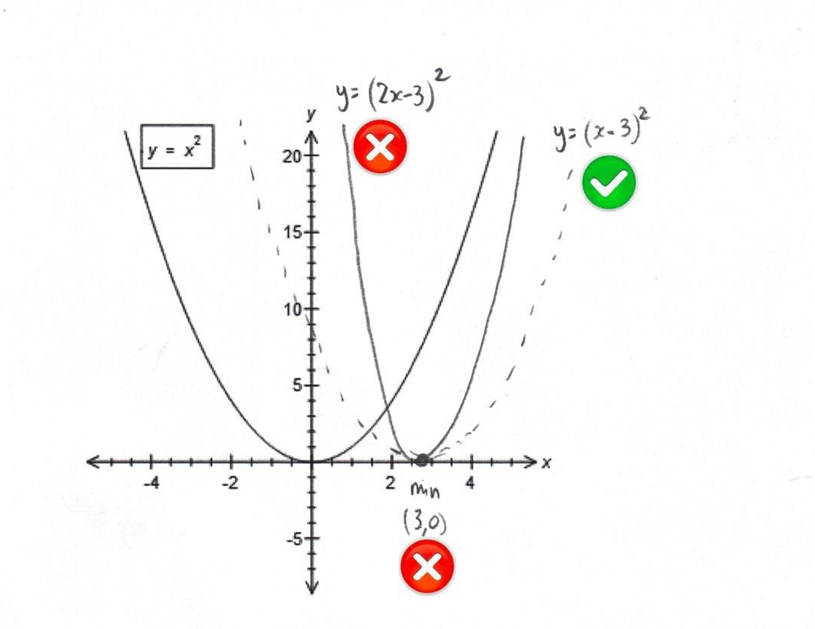
This is the most troublesome of all the composite transformations – the *x*-translation combined with the *x*-stretch.

The pupil had correctly transformed $y = x^2$ to $y = (x-3)^2$, using an *x*-translation of +3 units.

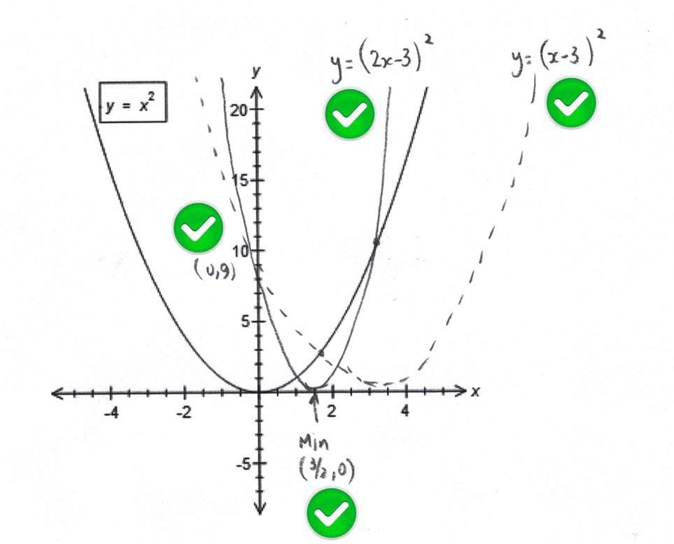
The next step – an *x*-stretch of scale factor ½, compressed the graph into the right shape, but using the wrong invariant line.

If $y = (2x - 3)^2$,
then if $x = 3$, $y = 9$.

The graph of $y = (2x - 3)^2$ is wrong, since it passes through (3, 0) and not (3, 9).



In the correct answer, $y = (x-3)^2$,:has a minimum of (3,0), The *x*-stretch is performed next, again with scale factor ½, but this time with the *y*-axis invariant. In other words, *x*-values are halved whilst *y*-values remain unchanged. The minimum point of the graph $y = (2x - 3)^2$ is now at (1½, 0)
Check: 2(1½)-3 = 0, and its square = 0.



We finish with some miscellaneous examples of algebraic abuse.

Example (33): Write down the values of i) 5^0 ; ii) 2^{-3}.

$$\text{(i) } 5^0 = 0, \quad \text{(ii) } 2^{-3} = -2^3 = -8$$

Those are two big blunders; the zero power of any positive number is always 1, not 0; also the negative power of a number is the *reciprocal* of the corresponding positive power and not the *negative* of it !

$$\text{(i) } 5^0 = 1 \quad \text{(ii) } 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Example (34):

Find the area bounded by the curve $y = x(x-2)(x-3)$, the two coordinate axes and the line $x = 3$.
Hint: sketch the curve.

This could be classed as 'misreading the question', since the pupil had simply integrated $x(x-2)(x-3)$ and arrived at what was apparently the correct result.

However, a sketch of the curve would have shown that part of the area was above the x-axis, and that part of the area was below it.

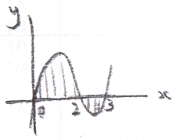
The correct solution is shown below, including the sketch graph and the two separate integrands. The *sign* of the area below the x-axis is ignored, i.e. the area is treated as positive.

$$x(x-2)(x-3) = x(x^2-5x+6)$$
$$= x^3 - 5x^2 + 6x$$

$$\text{Area} = \int_0^3 x^3 - 5x^2 + 6x \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_0^3$$

$$= \left[\frac{81}{4} - 45 + 27\right] - 0 = 2\tfrac{1}{4}$$

$$x(x-2)(x-3) = x(x^2-5x+6)$$
$$= x^3 - 5x^2 + 6x$$



From $x = 0$ to $x = 2$

$$\int_0^2 x^3 - 5x^2 + 6x \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_0^2 = \left[4 - \frac{40}{3} + 12\right] - [0] = \frac{8}{3}$$

From $x = 2$ to $x = 3$

$$\int_2^3 x^3 - 5x^2 + 6x \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_2^3 = \left[\frac{81}{4} - 45 + 27\right] - \left[4 - \frac{40}{3} + 12\right] = \frac{-5}{12}$$

below x-axis

$$\text{Total area} = \frac{8}{3} + \frac{5}{12} = \frac{37}{12} = 3\tfrac{1}{12}$$

Example (35) : Solve the equation $\frac{x+1}{x+3}+\frac{x-2}{2x-1}=1.$

$$\frac{x+1}{x+3}+\frac{x-2}{2x-1}=1 \rightarrow (x+1)(2x-1)+(x-2)(x+3)=1$$

$$\rightarrow 2x^2+x-1+x^2+x-6=1$$

$$\rightarrow 3x^2+2x-7=1 \rightarrow 3x^2+2x-8=0$$

$$\rightarrow (3x-4)(x+2)=0 \rightarrow x=4/3,\ x=-2$$

The incorrect working above is a result of the pupil attempting to cross-multiply the expression. There might be a follow-through credit for attempting the solve the resulting quadratic.

If an equation can be written in the form $\frac{a}{b}=\frac{c}{d}$ where *a*, *b*, *c* and *d* are expressions and *b* and *d* are not zero, we *can* cross-multiply and write $ad = bc$.

The equation needs to be rewritten into a suitable form for cross-multiplication to work – here is the correct working

$$\frac{x+1}{x+3}+\frac{x-2}{2x-1}=1 \rightarrow \frac{x+1}{x+3}=1-\frac{x-2}{2x-1}$$

$$\rightarrow \frac{x+1}{x+3}=\frac{2x-1}{2x-1}-\frac{x-2}{2x-1} \rightarrow \frac{x+1}{x+3}=\frac{x+1}{2x-1}$$

$$\rightarrow (x+1)(2x-1)=(x+3)(x+1)$$

$$\rightarrow 2x^2+x-1=x^2+4x+3$$

$$\rightarrow 2x^2+x-1-x^2-4x-3=0$$

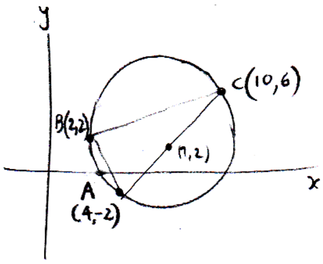
$$\rightarrow x^2-3x-4=0 \rightarrow (x-4)(x+1)=0$$

$$\rightarrow x=4,\ x=-1.$$

We finish with some examples of 'kickself' carelessness.

Example (36): Points *A*, *B* and *C* on a circle have coordinates (4, -2), (2, 2) and (10, 6) respectively.

i) Show that angle *ABC* is a right angle.
ii) Hence find the equation of the circle.



Gradient AB = $\frac{2-(-2)}{2-4} = -2$

Gradient BC = $\frac{6-2}{10-2} = \frac{1}{2}$

Gradient product = $-2 \times \frac{1}{2} = -1$

$\therefore$ AB, BC are perpendicular and $\angle ABC = 90°$

Angle in a semicircle = 90° $\therefore$ AC is a diameter.

AC has length $\sqrt{(10-4)^2 + (6-(-2))^2} = \sqrt{36+64} = \sqrt{100} = 10.$

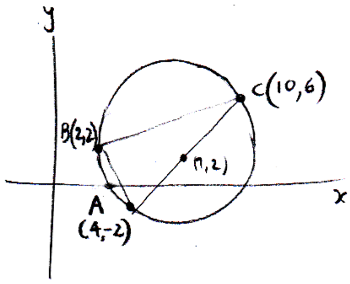
centre of circle = midpoint of AC = $\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$

$= (7, 2).$

$\therefore$ Equation of circle is $(x-7)^2 + (y-2)^2 = 100$

The pupil's working is correct, but for the error of forgetting to halve the diameter of the circle to get the radius.

The corrected result is here.



Gradient AB = $\frac{2-(-2)}{2-4} = -2$

Gradient BC = $\frac{6-2}{10-2} = \frac{1}{2}$

Gradient product = $-2 \times \frac{1}{2} = -1$

$\therefore$ AB, BC are perpendicular and $\angle ABC = 90°$

Angle in a semicircle = 90° $\therefore$ AC is a diameter.

AC has length $\sqrt{(10-4)^2 + (6-(-2))^2} = \sqrt{36+64} = \sqrt{100} = 10.$

centre of circle = midpoint of AC = $\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$

$= (7, 2).$ Radius of circle = $\frac{1}{2}$AC = 5

$\therefore$ Equation of circle is $(x-7)^2 + (y-2)^2 = 25$

Example (37): Find the solution(s) of the simultaneous equations

$y = x^2 - 5x + 9$
$y = 2x - 3$

$\rightarrow$ simultaneous solution when $x^2 - 5x + 9 = 2x - 3$

$\rightarrow x^2 - 5x + 9 - 2x - 3 = 0$

$\rightarrow x^2 - 7x + 6 = 0 \rightarrow (x-1)(x-6) = 0$

x-coords of intersection pts are 1, 6

$x = 1, y = -1$ (sub into $2x - 3$)
$x = 6, y = 9$

The pupil had made a careless sign error here when subtracting $2x - 3$ from both sides.

$x^2 - 5x + 9 = (2x - 3)$

is equivalent to

$x^2 - 5x + 9 - (2x - 3) = 0$ or
$x^2 - 5x + 9 - 2x + 3 = 0$.

A minus sign outside brackets
means reversing everything inside the brackets.

Correct version:

$\rightarrow$ simultaneous solution when $x^2 - 5x + 9 = 2x - 3$

$\rightarrow x^2 - 5x + 9 - 2x + 3 = 0$

$\rightarrow x^2 - 7x + 12 = 0 \rightarrow (x-3)(x-4) = 0$

x-coords of intersection pts are 3, 4

$x = 3, y = 3$ (sub into $2x - 3$)
$x = 4, y = 5$

Example (38): Find the term in x^2 in the expansion of $(4 + 3x)^5$.

$$(4+3x)^5 \rightarrow x^2 \text{ term is } \binom{5}{2} \times 4^3 \times 3x^2$$
$$= 10 \times 64 \times 3x^2 = 1920x^2$$

The pupil had forgotten to square the 3 as well as the x in $3x$!

Correct answer:

$$(4+3x)^5 \rightarrow x^2 \text{ term is } \binom{5}{2} \times 4^3 \times (3x)^2$$
$$= 10 \times 64 \times 3x^2 = 5760x^2$$

Example (39): Find the value of $\displaystyle\sum_{i=2}^{6} \frac{420}{i}$.

$$\sum_{i=2}^{6} \frac{420}{i} = \frac{420}{2} + \frac{420}{3} + \frac{420}{4} + \frac{420}{5} + \frac{420}{6} + \frac{420}{7}$$
$$= 210 + 140 + 105 + 84 + 70 + 60$$
$$= 669$$

The pupil had summed the series to one more term than required by thinking the 6 above the sigma symbol meant summing the series to six terms.

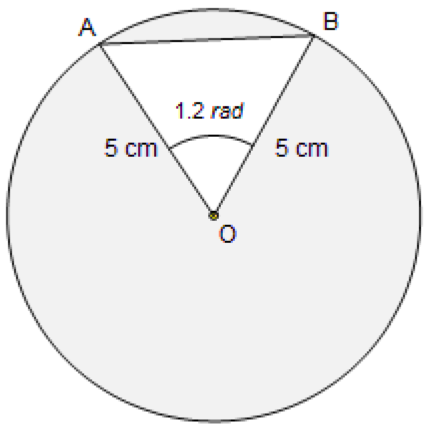
The 6 is the end value for the count variable i which takes a starting value of 2. Hence there are five terms in this series, not six.

Correct answer:

$$\sum_{i=2}^{6} \frac{420}{i} = \frac{420}{2} + \frac{420}{3} + \frac{420}{4} + \frac{420}{5} + \frac{420}{6}$$
$$= 210 + 140 + 105 + 84 + 70$$
$$= 609$$

Example (40):

Find i) the area of the shaded major sector *AOB*,
ii) the perimeter of the shaded segment *AB*,
where the acute angle *AOB* = 1.2 radians
and the radius of the circle = 5cm .



Area of major sector AOB = $\frac{1}{2}r^2\theta$
where θ = $\pi - 1.2^c$ = $12.5(\pi - 1.2^c)$ = 24.3 cm^2

Arc AB = $r\theta$ = 5×1.2 cm = 6 cm
Chord AB = 5 sin 1.2^c = 4.66 cm ∴ perimeter of segment = 10.7 cm

There are two blunders here: the angle of the major segment was calculated wrongly, by assuming π, rather than 2π, radians in a circle.

Secondly, the length of the chord was calculated wrongly, as being the product of the radius and the sine of the sector angle.

It should have been the product of the radius and *twice* the sine of *half* the sector angle.

Correct :

Area of major sector AOB = $\frac{1}{2}r^2\theta$
where θ = $2\pi - 1.2^c$ = $12.5(2\pi - 1.2^c)$ = 63.5 cm^2

Arc AB = $r\theta$ = 5×1.2 cm = 6 cm
Chord AB = 5 × 2sin 0.6^c = 5.64 cm ∴ perimeter of segment = 11.6 cm