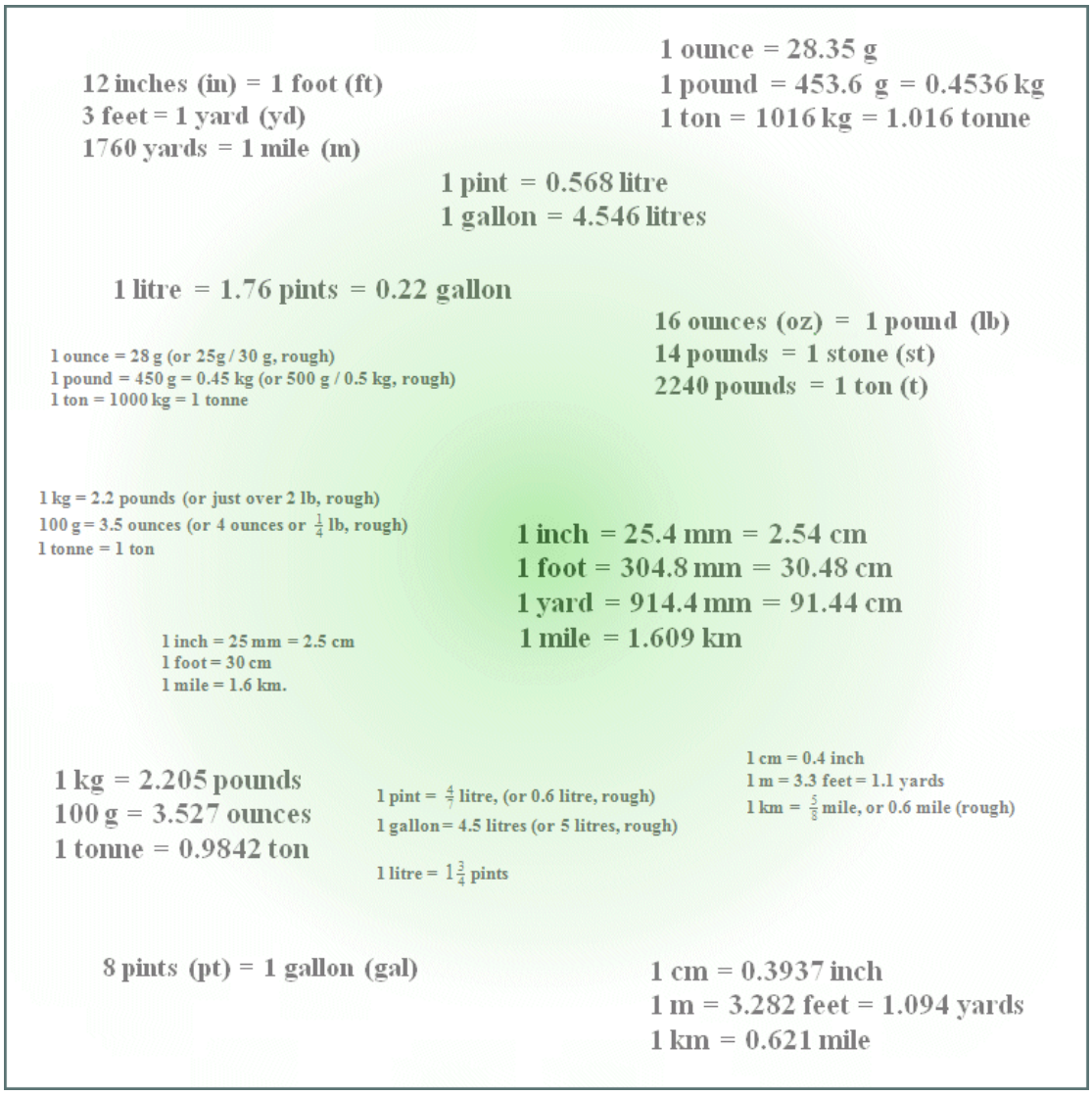
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

MEASURING SYSTEMS



MEASURING SYSTEMS

There are two systems of weights and measures currently in use in the United Kingdom – the metric system and the Imperial system.

The metric system is now used exclusively in science and almost so in trade, as it is so much more straightforward than the older Imperial system. Some remnants of the Imperial system still survive in everyday use, however.

The Metric System.

The metric system is based on three units – the **metre** for length, the **gram(me)** for weight and the **litre** for capacity.

Multiples of 10**,** 100 **and 1000 of the unit are prefixed with** deka- **,** hecto- **and kilo- respectively.**
Fractions of $\frac{1}{10}$ **,** $\frac{1}{100}$ **and** $\frac{1}{1000}$ **of the unit are prefixed with** deci- **, centi- , and milli- respectively.**

(The deci-, deka- and hecto- units are not in common use)

(Not in maths GCSE) Other prefixes are used in science, especially physics:

1,000,000 of a unit	- prefix mega- e.g. 1,000,000 joules = 1 megajoule (MJ). Note that 1,000,000 grams = 1000 kg is called a tonne, not a megagram.
1,000,000,000 of a unit	- prefix giga- e.g. 1,000,000,000 watts = 1 gigawatt (GW).
1/1,000,000 of a unit	- prefix micro (μ) – e.g. 1 microampere (1μA) Note that a millionth part of a metre is called a micron (μ) rather than a micrometre.
1/1,000,000,000 of a unit	- prefix nano – e.g. 400 nanometres (nm), also describing nanoparticles.

The more important metric units are:

LENGTH	**1000 millimetres (mm) = 100 centimetres (cm) = 1 metre (m).** **1000 metres = 1 kilometre (km).**
WEIGHT	**1000 milligrams (mg) = 1 gram (g).** **1000 grams = 1 kilogram (kg).** **1000 kilograms = 1 tonne (T).**
CAPACITY	**1000 millilitres (ml) = 100 centilitres (cl) = 1 litre (l)**

The following statements give a feel for the everyday use of the units.

"This room measures 4.2 metres by 3 metres."
"The blinds have a drop of 150 cm and an exact width of 655 mm."
"Jack is 172 cm tall and weighs 64 kilograms".
"We need 30 litres of petrol to fill up the tank."
"This hot-pot recipe needs 1 kg of potatoes and 250g of steak."
"I need a 250 milligram (250 mg) tablet and a 5 millilitre (5 ml) spoon of medicine for my cold."
"This 50-tonne lorry is doing 100 km per hour on the M62."
"Bury is 16 km from Manchester".

Converting between metric units.

Metric units can be converted between each other by multiplying or dividing by the appropriate power of 10. Thus:

172cm = 1.72m (divide by 100)
0.52kg = 520g (multiply by 1000)
0.835m = 835mm (multiply by 1000)
500 ml = 0.5 litre (divide by 1000)
5.8cm = 58mm (multiply by 10)
3000m = 3 km (divide by 1000)

The "base unit" in question can be the metre, the gram, or the litre.

		FROM		
		milli -	**centi -**	**(base unit)**
TO	**milli -**	**----------**	**× 10**	**× 1000**
	centi -	**÷ 10**	**----------**	**× 100**
	(base unit)	**÷ 1000**	**÷ 100**	**----------**

		FROM	
		(base unit)	**kilo-**
TO	**(base unit)**	**----------**	**× 1000**
	kilo -	**÷ 1000**	**----------**

Note also the relation between kilograms and tonnes:

2.7 T = 2700kg (multiply by 1000)
907kg = 0.907T (divide by 1000)

Examples (1): Convert the following to metres; a) 1.8 km; b) 59 cm; c) 1075 mm

a) 1.8 km = 1.8 × 1000 m = 1800m; b) 59 cm = $\frac{59}{100}$ m = 0.59 m; c) 1075 mm = $\frac{1075}{1000}$ m = 1.075 m.

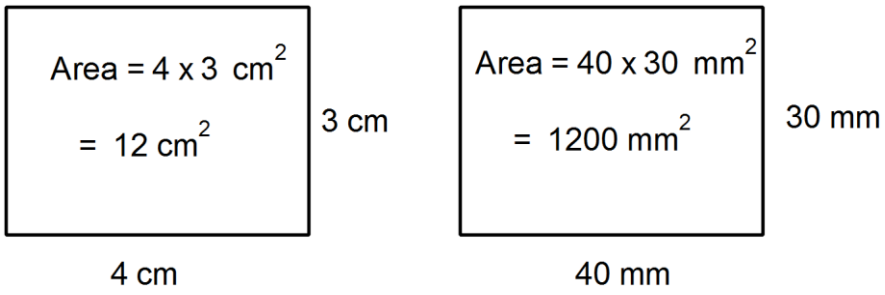
Examples (2): Convert the following to kilograms; a) 95 g; b) 9.65 T

a) 95 g = $\frac{95}{1000}$ kg = 0.095 kg; 9.65 T = 9.65 × 1000 kg = 9650 kg.

Converting between metric units for area and volume.

Care is needed when converting between metric units when dealing with areas and volumes.

Take the two rectangles below. Their dimensions are the same, except that one rectangle has them quoted in centimetres and the other in millimetres.



There are 10 millimetres in a centimetre, so 4 cm is obviously 40 mm.
When we examine the areas, 12 cm^2 is not 120 mm^2, but 1200 mm^2, implying that there are 100, and not 10, square millimetres in a square centimetre. Note that 100 is the square of 10.

AREA **100 square millimetres (mm^2) = 1 square centimetre (cm^2)**
10,000 square centimetres = 1 square metre (m^2).
10,000 square metres = 1 hectare (Ha)
1,000,000 square metres = 100 hectares = 1 square kilometre (km^2).

There are 100 centimetres in a metre, so the number of square centimetres in a square metre is the *square* of 100, or 10,000. Similarly, as there are 1000 metres in a kilometre, we need to square that to obtain the number of square metres in a square kilometre – i.e. one million.

Note the special case of the hectare – it is the area of a 100m × 100m square, or a square hectometre.

A similar situation occurs with volumes, except that we take cubes instead of squares.

VOLUME **1000 cubic millimetres (mm^3) = 1 cubic centimetre (cm^3)**
1,000,000 cubic centimetres = 1 cubic metre (m^3).

There are 10 millimetres in a centimetre, so the number of cubic millimetres in a cubic centimetre is the *cube* of 10, or 1000. For the same reason, the number of cubic centimetres in a cubic metre is the cube of 100, or 1,000,000.

Note also that a cubic centimetre is equivalent to a millilitre, i.e. 1 cm^3 = 1 ml, and that a cubic metre contains 1000 litres.

Examples (3): Convert the following areas and volumes:
a) 24 cm^2 to mm^2 ; b) 7400 cm^2 to m^2; c) 0.165 m^3 to cm^3 ; d) 3650 mm^3 to cm^3; e) 320 Ha to km^2

a) 24 cm^2 = 24 × 100 mm^2 = 2400 mm^2 (multiply by the square of 10, i.e.100)
b) 7400 cm^2 = $\frac{7400}{10000}$ m^2 = 0.74 m^2 (divide by the square of 100, i.e.10,000)
c) 0.165 m^3 = 0.165 × 1,000,000 cm^3 = 165,000 cm^3 (multiply by 1,000,000, i.e. the cube of 100)
d) 3650 mm^3 = $\frac{3650}{1000}$ cm^3 = 3.65 cm^3 (divide by 1000, i.e. the cube of 10)
e) 320 Ha = $\frac{320}{100}$ km^2 = 3.2 km^2 (divide by 100)

The Imperial System.

The Imperial system of weights and measures is falling into disuse, but some terms still survive in everyday use, especially in travel and shopping.

"This room measures 15 feet 6 inches by 12 feet."

"This hot-pot recipe needs 2 pounds of potatoes and 10 ounces of steak."

"Jack is 5 feet 7 inches tall and weighs 10 stones 8 pounds".

"We need 6 gallons of petrol to fill up the tank. While we're there, we must get a pint of milk."

"The Post Office is 400 yards down the road".

"This 50-ton lorry is doing 60 miles per hour on the M62."

"Bury is 9 miles from Manchester".

The surviving Imperial units in everyday use are:

LENGTH	**12 inches (in) = 1 foot (ft)** **3 feet = 1 yard (yd)** **1760 yards = 1 mile (m)**
WEIGHT	**16 ounces (oz) = 1 pound (lb)** **14 pounds = 1 stone (st)** **2240 pounds = 1 ton (t)** (The stone is only used for personal weights and is dying out).
CAPACITY	**8 pints (pt) = 1 gallon (gal)**

Accurate Equivalents (no need to memorise, you'll be pleased to know !) .

LENGTH
1 inch = 25.4 mm = 2.54 cm
1 foot = 304.8 mm = 30.48 cm
1 yard = 914.4 mm = 91.44 cm
1 mile = 1.609 km.

1 cm = 0.3937 inch
1 m = 3.282 feet = 1.094 yards
1 km = 0.621 mile

WEIGHT
1 ounce = 28.35 g
1 pound = 453.6 g = 0.4536 kg
1 ton = 1016 kg = 1.016 tonne

1 kg = 2.205 pounds
100 g = 3.527 ounces
1 tonne = 0.9842 ton

CAPACITY
1 pint = 0.568 litre
1 gallon = 4.546 litres

1 litre = 1.76 pints = 0.22 gallon

Example (4): A Marathon course is 26.22 miles in length. Convert it into kilometres, giving the result to 4 significant figures.

To convert miles to kilometres, we multiply by 1.609, so 26.22 miles = 26.22×1.609 km = 42.19 km.

Approximate Equivalents.

(Again, there is no need to memorise those, as the equivalents will be given in questions)

LENGTH **1 inch = 25 mm = 2.5 cm**
1 foot = 30 cm
1 mile = 1.6 km.

1 cm = 0.4 inch
1 m = 3.3 feet = 1.1 yards
1 km = $\frac{5}{8}$ mile, or 0.6 mile (rough)

WEIGHT **1 ounce = 28 g (or 25g / 30 g, rough)**
1 pound = 450 g = 0.45 kg (or 500 g / 0.5 kg, rough)
1 ton = 1000 kg = 1 tonne

1 kg = 2.2 pounds (or just over 2 lb, rough)
100 g = 3.5 ounces (or 4 ounces or $\frac{1}{4}$ lb, rough)
1 tonne = 1 ton

CAPACITY **1 pint = $\frac{4}{7}$ litre, (or 0.6 litre, rough)**
1 gallon = 4.5 litres (or 5 litres, rough)

1 litre = $1\frac{3}{4}$ pints = 0.22 gallon (or 0.2 gallon, rough)

Example (5): The onboard luggage limit per passenger on a flight is 25 kg. A tourist has weighed his luggage on an Imperial scale and the reading is 57 lb. Does this weight exceed the baggage limit ? Use the estimate 5 kg = 11 lb.

To convert kilograms to pounds, we multiply the limiting weight of 25 kg by $\frac{11}{5}$ **,**

so 25 kg = $25 \times \frac{11}{5} = 55$ lb. Therefore the tourist's luggage of 57 lb exceeds the limit by two pounds.

Example (6): A cake recipe in an old cookery book recommends the use of
2 lb flour, 6 oz of sugar, and ½ lb of dried fruit. Convert these measurements into metric.

Since 1 lb is about 450g, (500g), 2 lb is about 1000 g or 1 kg.
6 oz is roughly 6 × 30 g or 180 g , and ½ lb is about half of 500g, or 250g.

A metric equivalent of the recipe is about 1 kg flour, 180g sugar and 250 g dried fruit.

Example (7): A milk dispensing machine serves milk in half-pint cups. Given that the capacity of the machine is 40 litres, how many cups can be served out of a full machine?

Since 1 litre is about $1\frac{3}{4}$ pints, 40 litres is about $1\frac{3}{4}$ × 40 pints, or 70 pints.
Also, as two half-pint cups can be filled from one pint, it follows that 140 cups can be served out of a full machine.

Example (8): A motorist travelling down to Nice arrives at Calais on a full tank of petrol, with a range of 420 miles under normal motorway conditions. He plans to refill the tank at Lyon, 760 km from Calais. Will he run out of fuel before he gets to Lyon ?

One kilometre is about $\frac{5}{8}$ of a mile, so the distance from Calais to Lyon is $760 \times \frac{5}{8} = 475$ miles.
He has fuel in his tank for 420 miles, so he would have to fill up quite a while before he reaches Lyon.

Foreign Currency Exchange.

Another popular topic in exams is foreign exchange, where money is changed from one currency to another.

The headline exchange rate is usually quoted in terms of how many foreign currency units can be obtained for £1 sterling, e.g. £1 = $1.53 (US), £1 = €1.24.

The following examples are simpler than the usual commercial practice of using different rates for buying and selling foreign currency.

Example (9): Steve goes on holiday to France and exchanges £500 into euros. At the end of his holiday, he has €75 left over to convert back into sterling. The exchange rate is £1 = €1.22 throughout.

i) How many euros does Steve buy in total ?
ii) How much in sterling does he receive back ?

i) Since £1 = €1.22, £500 = 500 × 1.22 euros, or €610.
We **multiply** by the exchange rate to convert from sterling into euros.

ii) This time were are changing 75 euros back into sterling, so we **divide** 75 by the exchange rate:

$$€\ 75 = £\ \frac{75}{1.22} = £\ 61.48.$$

Example (10): Lisa has bought a camera in the USA for $109.99, where the same model costs £89.99 in the UK. Given that the exchange rate was £1 = $1.52 at the time of purchase, how much cheaper was it for her to buy the camera in the USA? Give the answer in sterling.

The camera's US price of $109.99 must be **divided** by the exchange rate of 1.52 to give the result of

$$\$\ 109.99 = £\ \frac{109.99}{1.52} = £\ 72.36.$$

Lisa therefore has saved £(89.99 – 72.36) or £17.63 by buying the camera in the USA.

Example (11): Bill is on a motoring tour of North America, near the U.S. / Canadian border.

Petrol in the U.S.A. costs $3.30 (U.S.) per gallon; in Canada, $1.22 (Canadian) per litre.
Additionally, $1 U.S. = $1.25 Canadian and 1 U.S. gallon = 3.79 litres.

He wants to fill up with 10 gallons of petrol, but is not sure whether to do so in the U.S. or in Canada. Which country offers the better deal ? Justify your answer fully, showing all calculations.

With petrol at $3.30 per gallon in the U.S. the cost of the fill-up would be $3.30 × 10 = **$33 U.S.**
To find the equivalent price in Canada, we convert 10 U.S. gallons to litres, i.e. 10 × 3.79 = 37.9 litres.

Petrol in Canada costs $1.22 (Canadian) per litre, hence the fill-up would cost Bill
37.9 × 1.22 = **$46.24 Canadian.**

Finally we need to convert from Canadian to U.S. dollars, so we divide by the exchange rate of 1.25.

The U.S. dollar equivalent of $46.24 (Canadian) is $\$\ \frac{46.24}{1.25}$ U.S. = **$36.99 U.S.**

It is therefore cheaper by $4 U.S. for Bill to fill up in the U.S.A. rather than in Canada.