

M.K. HOME TUITION

Mathematics Revision Guides
 Level: GCSE Foundation Tier

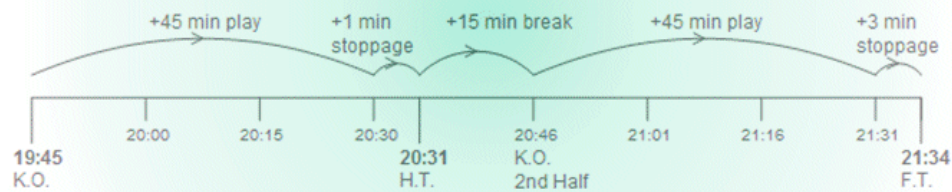
TIME CALCULATIONS

| | | | | | |
|------------------------------------|------|------|------|------|------|
| Manchester Piccadilly (dep) | 0700 | 0735 | 0815 | 0835 | 0915 |
| Stockport (dep) | | 0743 | 0823 | 0842 | 0923 |
| Macclesfield (dep) | | 0756 | | 0855 | |
| Stoke-on-Trent (dep) | | 0812 | 0850 | 0912 | 0950 |
| London Euston (arr) | 0858 | 0952 | 1023 | 1042 | 1123 |

Stockport to London = 178 miles
 09:23 to 11:23 = 2 hours
 average speed = $\frac{178}{2}$ mph = 89 mph

From 07:35 to 09:35 is 2 hours; another 17 minutes brings us to 09:52.
 \therefore travel time between Manchester and London on the 07:35 is 2 hr 17 min.

From 07:56 to 09:56 is 2 hours; subtract 4 minutes to bring us back to 09:52.
 \therefore travel time between Macclesfield and London on the 07:56 is 1 hr 56 min.



| | | | | | | | | | | | |
|------------------|-----|-----|---------------|-----|-----|---------------|-----|-----|---------------|-----|-----|
| Minutes | 6 | 12 | 15 | 18 | 24 | 30 | 36 | 42 | 45 | 48 | 54 |
| Decimal of hour | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 |
| Fraction of hour | | | $\frac{1}{4}$ | | | $\frac{1}{2}$ | | | $\frac{3}{4}$ | | |

TIME CALCULATIONS

Many everyday situations involve calculations based around **time**.

The 12 and 24 hour clocks.

The standard clock face has 12 hours on it, but most timetables and businesses now use the 24-hour clock.

The 12-hour clock uses the suffixes “a.m.” (before noon) and “p.m.” (after noon) to distinguish seemingly identical times. Thus 7 a.m. is in the morning, but 7 p.m. is in the evening. Note that most people are asleep at 12.05 a.m. and having their lunch at 12.05 p.m. !

In the 24-hour clock, the hours run from 00 to 11 for times before noon, and 12 to 23 for times after noon. The minutes may be separated by a dot or a colon. (This document favours the colon.)

Thus, 9.30 a.m. = 0930 or 09:30; 12.45 a.m. = 0045 or 00:45; 8.25 p.m. = 2025 or 20:25.

Time arithmetic.

Recall: 60 seconds = 1 minute; 60 minutes = 3600 seconds = 1 hour; 24 hours = 1 day.

Most time calculations are best carried out mentally using various methods; a few are illustrated here.

Example (1): A football match kicks off at 19:45, but the referee adds 1 minute of stoppage time at the end of the 45 minutes of the first half. At what time does the referee blow the half-time whistle ?

Method: “Go forward 15 minutes to 20:00, but as we’re adding 45 in total, we still need another 30, which brings the time to 20:30. Add the one minute of injury time, and the ref blows at 20:31.”

∴ The referee blows the half-time whistle at 20:31.

Example (2): The second half of the football match in Example (1) continues 15 minutes after the half-time whistle has been blown. The second half lasts 45 minutes plus three minutes of stoppage time.

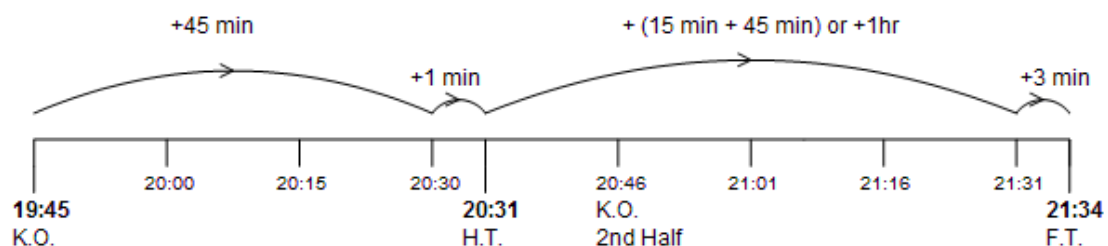
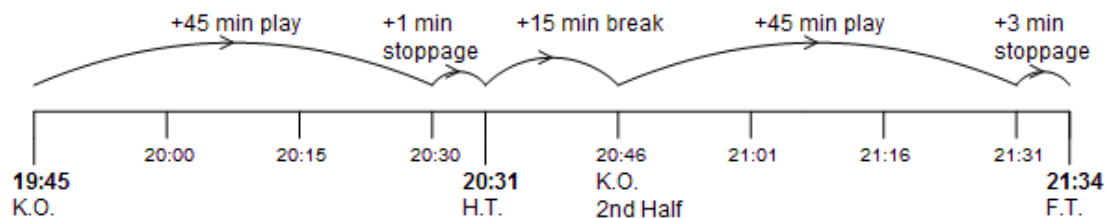
When does the referee blow the final whistle ?

Method: “We’re talking a 15 minute break plus 45 minutes of play, giving us 60 minutes or an hour forward from 20:31, namely to 21:31. The ref adds 3 minutes, so the final whistle will go at 21:34.”

∴ The referee blows the full-time whistle at 21:34.

The last two questions could also have been worked out using a timeline diagram. The first one keeps the half-time break and the second half’s play separate; the second one combines the two to give an exact hour.

With practice, some of the intermediate times on the timelines, such as 20:00, can be omitted.



Example (3) (Harder): An office worker clocks in to work at 08:27 and out for lunch at 12:06. He then clocks back in to work at 12:53 and clocks off for home at 16:44. How many hours and minutes has he worked all day according to the clock ?

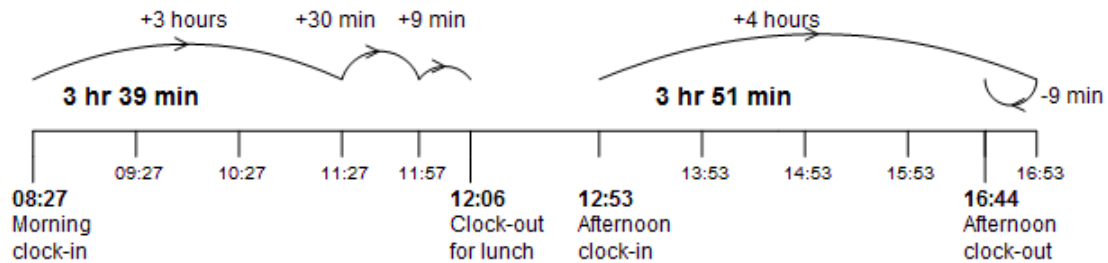
Method (1): Calculate morning and afternoon hours separately and add.

We reckon three hours forward from 08:27 to 11:27, plus another 30 minutes to 11:57, and another 9 minutes to 12:06. He has therefore worked 3 hours 39 minutes in the morning.

We then reckon on from 12:53, and four hours on from that is 16:53. He clocks off at 16:44, nine minutes short of four hours, or 3 hours 51 minutes, so he has worked 3 hours and 51 minutes in the afternoon.

Adding 3 hours 39 minutes to 3 hours 51 minutes gives 6 hours plus 90 minutes, or 7 hours and 30 minutes.

Again, the intermediate times of 09:27, 10:27 ... are not really necessary and can be omitted.



Total time worked: 3 hr 39 min + 3 hr 51 min = 7 hr 30 min

Mechanically, the arithmetic is shown below. Notice carefully how we have to ‘borrow’ 60 minutes or 1 hour to carry out the subtractions. For instance, 27 cannot be subtracted from 6, we had to borrow 60 minutes to subtract 27 from 66. A similar problem occurs when subtracting 53 from 44.

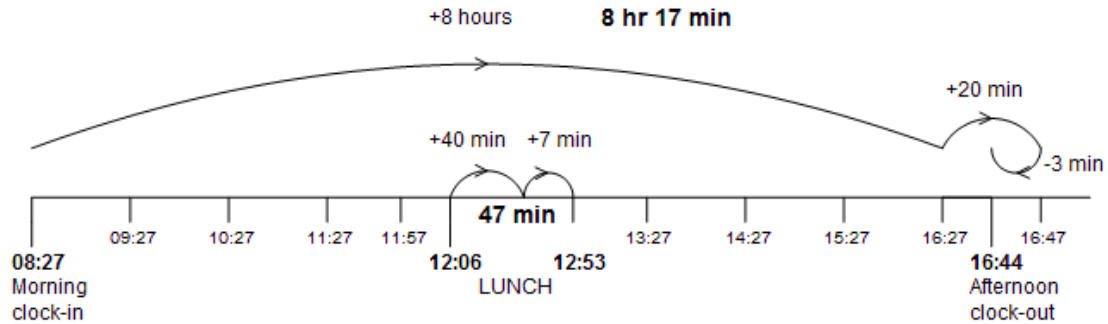
Also, in the final addition sum, adding 39 minutes to 51 gives 90 minutes, which is over 1 hour, so we ‘carry’ 60 minutes or 1 hour in adding up the hours.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--|---------------|-----|-----|--|----|----|---|---------------|---|--|---|----|--|--|--|--|---|----|---|--|----|-----|--|----|-----|---|---------------|----|--|----|----|--|--|--|--|---|----|--|--|----|-----|--|---|----|---|---|----|--|--|--|--|---|----|
| <table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: right;">hr</td><td style="text-align: right;">min</td></tr> <tr><td></td><td style="text-align: right;">11</td><td style="text-align: right;">66</td></tr> <tr><td style="text-align: right;">-</td><td style="text-align: right;">12</td><td style="text-align: right;">6</td></tr> <tr><td></td><td style="text-align: right;">8</td><td style="text-align: right;">27</td></tr> <tr><td colspan="3" style="border-top: 1px solid black; padding-top: 5px;"></td></tr> <tr><td></td><td style="text-align: right;">3</td><td style="text-align: right;">39</td></tr> </table> | | hr | min | | 11 | 66 | - | 12 | 6 | | 8 | 27 | | | | | 3 | 39 | <table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: right;">hr</td><td style="text-align: right;">min</td></tr> <tr><td></td><td style="text-align: right;">15</td><td style="text-align: right;">104</td></tr> <tr><td style="text-align: right;">-</td><td style="text-align: right;">16</td><td style="text-align: right;">44</td></tr> <tr><td></td><td style="text-align: right;">12</td><td style="text-align: right;">53</td></tr> <tr><td colspan="3" style="border-top: 1px solid black; padding-top: 5px;"></td></tr> <tr><td></td><td style="text-align: right;">3</td><td style="text-align: right;">51</td></tr> </table> | | hr | min | | 15 | 104 | - | 16 | 44 | | 12 | 53 | | | | | 3 | 51 | <table style="border-collapse: collapse; margin-left: auto; margin-right: auto;"> <tr><td></td><td style="text-align: right;">hr</td><td style="text-align: right;">min</td></tr> <tr><td></td><td style="text-align: right;">1</td><td style="text-align: right;">39</td></tr> <tr><td style="text-align: right;">+</td><td style="text-align: right;">3</td><td style="text-align: right;">51</td></tr> <tr><td colspan="3" style="border-top: 1px solid black; padding-top: 5px;"></td></tr> <tr><td></td><td style="text-align: right;">7</td><td style="text-align: right;">30</td></tr> </table> | | hr | min | | 1 | 39 | + | 3 | 51 | | | | | 7 | 30 |
| | hr | min | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 11 | 66 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| - | 12 | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | 27 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | 39 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | hr | min | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 15 | 104 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| - | 16 | 44 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 12 | 53 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | 51 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | hr | min | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | 39 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| + | 3 | 51 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | 30 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Method (2): Calculate hours worked for whole day and deduct for lunch.

We reckon eight hours forward from 08:27 to 16:27, and another 20 minutes to 16:47. We then go back three minutes to 16:44. He has therefore worked 8 hours 17 minutes, but we must still deduct lunch.

He took lunch from 12:06 to 12:53, which is 47 minutes, so we must subtract 47 minutes from 8 hours and 17 minutes. Taking 17 minutes off makes it 8 hours, but we've still got 30 to subtract, giving us 7 hours and 30 minutes.



Total time worked: 8 hr 17 min - 47 min = 7 hr 30 min

The corresponding mechanical method is shown below. (Notice the 'borrowing' in the final subtraction.)

| | | | | | | | | |
|---|----|-----|--|----|-----|--|----|---------------|
| | hr | min | | hr | min | | hr | min |
| - | 16 | 44 | | 12 | 53 | | 7 | 77 |
| | 8 | 27 | | 12 | 06 | | 8 | 17 |
| | 8 | 17 | | 0 | 47 | | 0 | 47 |

| | |
|---|----|
| 7 | 30 |
| 7 | 30 |

Example (4): A plumber charges £28 per hour for labour, with partial hours reckoned to the nearest half hour **above** at £14 per half hour.

In other words, a job lasting 1 hour 30 minutes is reckoned as exactly that, but a job lasting 1 hour 35 minutes is treated as 2 hours.

The plumber started his work at 11:07 and finished at 14:12.
 What were his labour costs for the work done ?

From 11:07 to 14:07 is 3 hours; from 14:07 to 14:12 is another 5 minutes.
 His work therefore took 3 hours and 5 minutes.

This is more than 3 hours, so his labour costs were for the next half hour above, or 3½ hours.

This works out at $3 \times £28$, or £84, plus $1 \times £14$ for the half hour, giving a total labour cost of £98.

Decimalisation of time.

Time units are not wholly decimal since they involve numbers like 60 and 24.
 In the plumber example above, we worked out a labour rate for 3½ hours, or 3 hours 30 minutes.
 Now, 30 minutes is half an hour or 0.5 of an hour, and certainly not 0.30 of an hour. Similarly 45 minutes is 0.75 of an hour, and definitely not 0.45.

Conversion rules:

Minutes to a decimal of an hour: divide by 60.
 Decimal of an hour into minutes: multiply by 60.

Seconds to a decimal of a minute: divide by 60.
 Decimal of a minute into seconds: multiply by 60.

Hours to a decimal of a day: divide by 24.
 Decimal of a day into hours: multiply by 24.

Example (5): Express the following into the required units:

- i) 42 minutes as a decimal of an hour; ii) 0.65 of an hour in minutes;
- iii) 2 minutes 12 seconds as decimal minutes; iv) 4.35 minutes as minutes and seconds;
- v) 7 hours 12 minutes as a decimal of a day; vi) 0.4 days in hours and minutes

i) $42 \text{ minutes} = \frac{42}{60} = \frac{7}{10} = 0.7 \text{ hour}$; ii) $0.65 \text{ hour} = 0.65 \times 60 = 39 \text{ minutes}$

iii) $2 \text{ minutes } 12 \text{ seconds} = 2 + \frac{12}{60} = 2 \frac{2}{10} = 2.2 \text{ minutes}$

iv) $4.35 \text{ minutes} = 4 \text{ minutes} + (0.35 \times 60) \text{ seconds} = 4 \text{ minutes } 21 \text{ seconds}$.

v) $7 \text{ hours } 12 \text{ minutes} = 7.2 \text{ hours}$ (since 12 minutes = 0.2 of an hour) = $\frac{7.2}{24}$ of a day = 0.3 day.

vi) $0.4 \text{ days} = 0.4 \times 24 \text{ hours} = 9.6 \text{ hours} = 9 \text{ hours } 36 \text{ minutes}$ (0.6 hour = $0.6 \times 60 = 36 \text{ minutes}$).

The following results are useful in non-calculator questions and should be memorised;

| | | | | | | | | | | | |
|------------------|-----|-----|---------------|-----|-----|---------------|-----|-----|---------------|-----|-----|
| Minutes | 6 | 12 | 15 | 18 | 24 | 30 | 36 | 42 | 45 | 48 | 54 |
| Decimal of hour | 0.1 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.75 | 0.8 | 0.9 |
| Fraction of hour | | | $\frac{1}{4}$ | | | $\frac{1}{2}$ | | | $\frac{3}{4}$ | | |

These values also apply when expressing seconds as decimals of a minute, as both involve 60.

Example (6): Kip is about to run a 10,000 metres race on a standard track.
(A lap on this track is 400 metres.)

He plans to run his first 6 laps at 70 seconds per lap, his last lap at 55 seconds, and all the laps in between at 65 seconds per lap. Calculate his planned total running time in minutes and seconds.

Firstly we calculate the number of 400-metre laps in 10,000 metres – it is $\frac{10000}{400}$, i.e. 25 laps.

We could either work throughout in seconds alone and divide by 60 in the final part of the sum, or we could convert the times for each stage into minutes and seconds and add the subtotals in minute / second format. The working shows both methods.

He plans to run his first 6 laps at 70 seconds each, so the time taken to run them is $6 \times 70 = 420$ seconds, or 7 minutes.

Kip has to run 25 laps, so we deduct the first six laps and the last lap to obtain the number of laps in the middle section, i.e. $25 - (6 + 1)$ laps, or 18 laps. He plans to run these 18 laps at 65 seconds each, and so his time for that part of the race is $18 \times 65 = 1170$ seconds, or 19 minutes and 30 seconds.

He plans to finish the race with a final lap of 55 seconds, so his total running time for the entire race is :

| | |
|-------------------------------------|--------------------------------------|
| First 6 laps @ 70 seconds per lap : | 420 seconds = 7 minutes |
| Next 18 laps @ 65 seconds per lap : | 1170 seconds = 19 minutes 30 seconds |
| Final lap @ 55 seconds : | 55 seconds |

Add for the total time for 10,000 metres (25 laps) : 1645 seconds = **27 minutes 25 seconds.**

Timetables.

Many time calculations rely on understanding timetables.

Here is an extract from a train timetable between Manchester (Piccadilly) and London (Euston).

| | | | | | | |
|------------------------------------|------|------|------|------|------|------|
| Manchester Piccadilly (dep) | 0700 | 0715 | 0735 | 0815 | 0835 | 0915 |
| Stockport (dep) | | 0723 | 0743 | 0823 | 0842 | 0923 |
| Macclesfield (dep) | | | 0756 | | 0855 | |
| Stoke-on-Trent (dep) | | 0750 | 0812 | 0850 | 0912 | 0950 |
| London Euston (arr) | 0858 | 0923 | 0952 | 1023 | 1042 | 1123 |

Example (7):

i) Work out the following travel times:

- a) Manchester-London on the 07:35; b) Macclesfield – London on the 07:56.

ii) Which is the fastest train from Manchester to London?

iii) A businessman from Stockport has to attend a meeting at 11:00 in central London, and he allows 40 minutes for lateness and taxi transfers between Euston and his London office. Which trains from the above list can he use to arrive on time? (Give the departure times from Stockport.)

iv) The 08:15 train from Manchester has arrived slightly late at Stoke-on-Trent at 09:01. Assuming no further change in lateness, when is it expected to arrive at London?

v) The distance from Stockport to London is 178 miles. What is the start-to-stop average speed between Stockport and London on the 09:23 from Stockport?

vi) The 09:12 service from Stoke-on-Trent to London is booked to run at an average speed of 97 mph between the two cities. How far is Stoke-on-Trent from London Euston?

i) a) From 07:35 to 09:35 is 2 hours; another 17 minutes brings us to 09:52.
∴ travel time between Manchester and London on the 07:35 is 2 hr 17 min.

i) b) From 07:56 to 09:56 is 2 hours; subtract 4 minutes to bring us back to 09:52.
∴ travel time between Macclesfield and London on the 07:56 is 1 hr 56 min.

ii) Inspection of the timetable shows that the 07:00 from Manchester arrives in London in 1 hr 58 min, the only booked time under 2 hours. The other clue is that it appears to be a non-stop service.

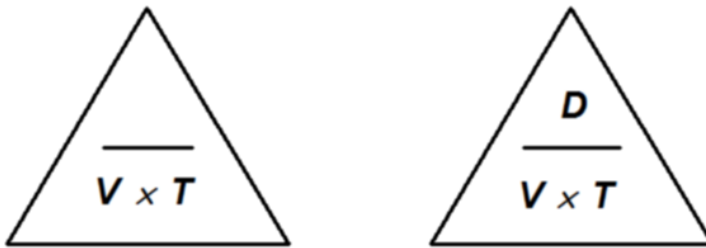
iii) The businessman's meeting is at 11:00, but as he must allow 40 minutes for transport from the station to the office, his train must arrive 40 minutes earlier, at 10:20.

The 07:43 from Stockport would get him into London at 09:52 with 28 minutes to spare, and the 07:23 would leave him with 57 minutes, arriving at 09:23.

The 08:23 would arrive at 10:23, perhaps cutting it too fine, allowing only 37 minutes for lateness and taxis !

iv) The actual arrival time at Stoke-on-Trent is 09:01, which is 11 minutes later than the expected time of 08:50. The corresponding expected arrival time at London is thus 11 minutes after 10:23, or 10:34.

(Recall the distance / time / speed formula triangle for the next two sections.)



D = distance; V = velocity (speed); T = time

v) The time difference between 09:23 and 11:23 is exactly 2 hours, so the average start-to-stop speed from Stockport to London is $\frac{178}{2}$ mph, or 89 mph.

vi) The 09:12 is booked to arrive at London Euston at 10:42, or 1 hr 30 min from Stoke-on-Trent. We must not multiply 97 by 1.30 to get a distance of 126 miles, but we must recognise that 30 minutes = $\frac{1}{2}$ hour, so we multiply 97 by $1\frac{1}{2}$, to get the correct distance of $145\frac{1}{2}$ miles.

Example (18): Dave drove off from Newcastle at 10:15 to attend a conference in Liverpool at 14:00. He by-passed Leeds, 105 miles past Newcastle, at 11:55, but the second part of his journey to Liverpool was much slower, averaging only 45 mph, while still taking the same time as the Newcastle-Leeds section.

- i) How many minutes did Dave have to spare when he arrived at Liverpool ?
- ii) Calculate Dave's average speed in miles per hour between Newcastle and Leeds.
- iii) Calculate the distance between Leeds and Liverpool.
- iv) Hence calculate Dave's average speed between Newcastle and Liverpool.

i) The time interval from 10:15 to 11:55 is 1 hour and 40 minutes, or 100 minutes, for the Newcastle-Leeds section, and it took Dave the same time to continue on to Liverpool, so he arrived there at 13:35, leaving him with 25 minutes to spare before the start of the conference.

ii) Dave's average speed between Newcastle and Leeds is obtained by dividing the distance of 105 miles by the time of 100 minutes, but that would give the average speed in miles *per minute* rather than in miles *per hour*, so we have to multiply by 60.

Hence Dave's average speed between Newcastle and Leeds was $\frac{105}{100} \times 60 = 63$ m.p.h.

iii) Distance is speed multiplied by time, so we multiply the speed of 45 m.p.h. by the time, which is 100 minutes, or 1 hour and 40 minutes, or $1\frac{2}{3}$ hours. (Because the speed is stated in m.p.h. we use hours rather than minutes as our time units).

Hence the distance from Leeds to Liverpool is $45 \times 1\frac{2}{3} = 75$ miles.

iv) The total distance from Newcastle to Liverpool is 180 miles, and Dave's total driving time was 200 minutes.

Calculating as in ii), the average speed between Newcastle and Liverpool was $\frac{180}{200} \times 60 = 54$ m.p.h.