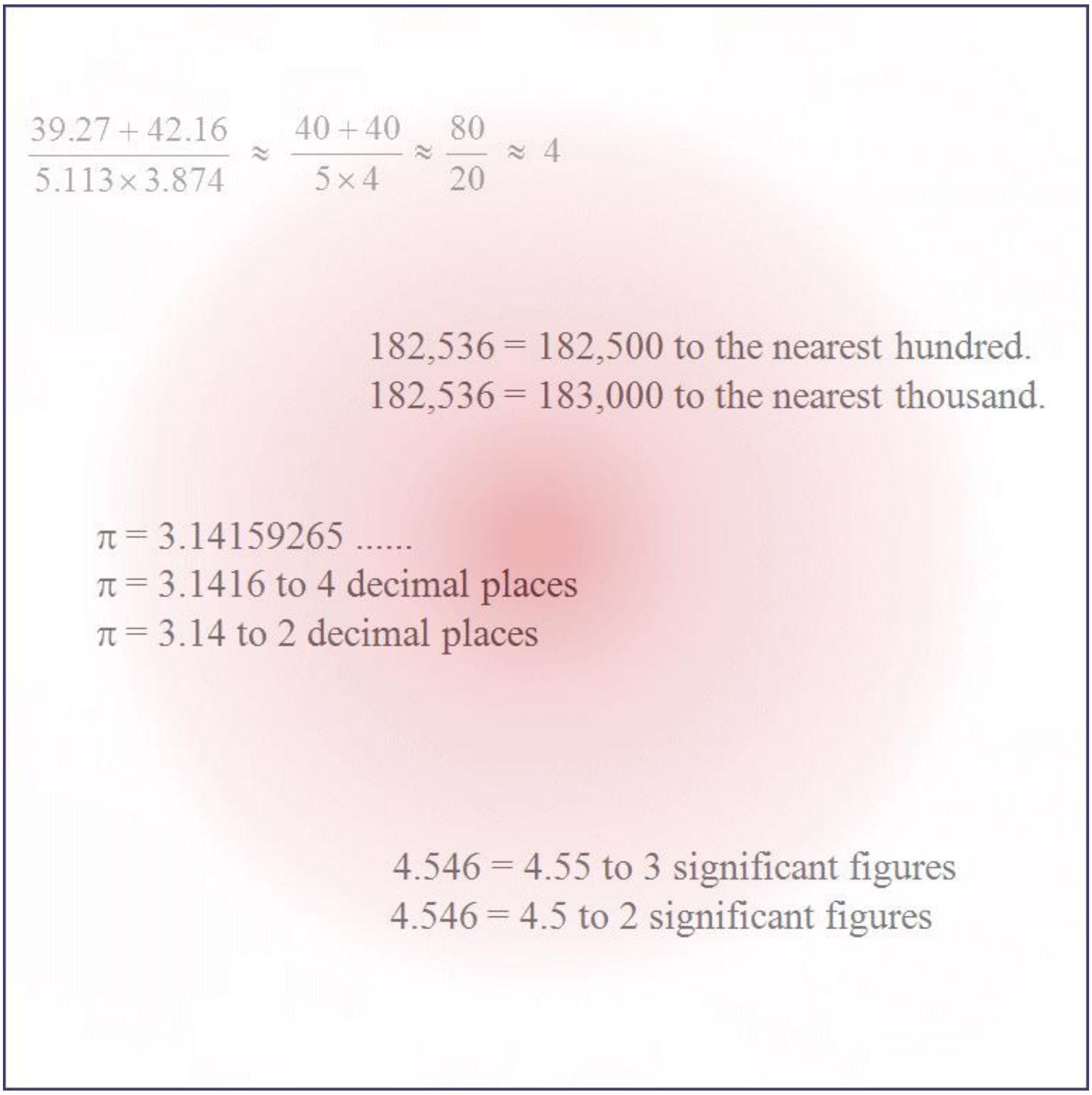
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

APPROXIMATION



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APPROXIMATION

Numbers are approximated (or rounded) for various reasons, mainly because they are measurements and cannot be exact, or because it is pointless to be 'over-accurate'.

Thus, if we drew a right-angled triangle whose short sides were 10 cm each, then it is only possible to measure the long side to the nearest millimetre (or 0.1 cm), to give a value of 14.1cm.

Calculations would give a theoretical value would be 14.14213562.... cm, but it is impossible to measure beyond the first decimal place, and so all the extra digits are pointless.

If we were asked to find the circumference of a wheel whose diameter was 60cm, we could multiply by π on the calculator to get 188.4955...cm, but it is again pointless to quote more than three or at most 4 figures, so we would give an answer of 188cm or 188.5 cm.

Rounding methods.

Numbers can be described as being rounded to the 'nearest 10', 'nearest 100', ' 1 decimal place', '3 significant figures' among other descriptions.

In all cases, the rounding rule is the same:

We check the final 'desired' digit and the first 'discarded' digit on either side of the 'cut-off' place.
If the first 'discarded' digit **is less than 5**, the last 'desired' digit is **left unchanged**.
If the first 'discarded' digit **is 5 or more**, the last 'desired' digit is **increased by 1**.

Example (1): The population of Bury Metropolitan Borough is 182,536. Round this figure to i) the nearest hundred, ii) the nearest thousand.

We check the place values of the digits of 182,536. Working from right to left, the 6 represents 6 units, but the 3 represents 3 tens, so its place value is 30. Similarly, the place value of the 5 is 500 and that of the 2 is 2000.

When we round to the nearest hundred, we are asked to ignore the tens and units digits.
The last 'desired' digit in 182,**5**36 is the 5 with its place value of 500, and the first 'discarded' one is 3.
Because 3 is less than 5, we leave the last 'desired' digit as 5 and say **182,500**.
(536 is nearer to 500 than it is to 600).

When rounding to the nearest thousand, we are asked to ignore the last three digits.
The last 'desired' digit in 18**2**,536 is the 2 (place value 2000), and the first 'discarded' one is 5.
Because 5 is greater than or equal to 5, we increase the last 'desired' digit by 1 to give 3,
and say **183,000** because 2536 is closer to 3000 than it is to 2000.

Example (2): The value of the number π is 3.1415927 to 7 decimal places. Give its value to i) 2 decimal places ii) 4 decimal places.

When we round to 2 decimal places, the cut-off point is after the 4, as in 3.1**4**15927.
The digit after the cut-off is 1 (less than 5) so we say 3.14. not 3.15.

When we round to 4 decimal places, the cut-off point is after the 5, as in 3.141**5**927.
The digit after the cut-off is 9 (greater than or equal to 5) so we say 3.1416. not 3.1415.

Example (2a): There are 1.7598 Imperial pints in a litre. Express this quantity to 3 decimal places.

The cut-off point is after the 9, as in 1.75**9**8.
The digit after the cut-off is 8 (5 or greater), so we round the 9 up to 10, but in so doing, we must carry 1 forward into the second decimal place, giving a value of 1.760.

Note that we must still include the final zero in the 1.760, as the question asks for 3 decimal places.
Disregarding the zero and stating 1.76 is wrong, as this implies only 2 decimal places.

Another way of describing rounding is by the use of **significant figures.**
The first significant figure in any number is either the first digit of a number greater than or equal to 1, or the first non-zero digit of a number less than 1.

Example (3): Give the significant figures in the following numbers:
i) 265.4; ii) 0.00534; iii) 716,000 as rounded to the nearest thousand; iv) 54,000 as rounded to the nearest hundred; v) 0.03075

i) The significant figures in 265.4 are 2, 6, 5 and 4. (4 in total)

ii) The leading zeros in 0.00534 are not significant; the significant figures are 5, 3 and 4.

iii) The fact that the number 716,000 is rounded to the nearest thousand implies that the zeros in the hundreds, tens and units places are not significant, and thus the significant figures are 7, 1 and 6.

iv) This is similar to the previous example, but here we say that the number 54,000 is rounded to the nearest hundred. The zeros in the tens and units places are therefore not significant, but the zero in the hundreds place is. The significant figures are 5, 4 and 0.

v) In the number 0.03075, the significant figures begin with the 3, and continue with 0, 7 and 5.
Note that embedded zeros within a string of non-zero digits are significant.

Example (4):
i) One mile is equal to 1.6093 kilometres. Give this number to 3 significant figures.
ii) Would the significant figures have been different if the value had been quoted in metres ?

iii) One Imperial gallon is equal to 4.546 litres. Give this number to 3 significant figures, and also to 2 significant figures.

i) The first 3 significant figures of 1.6093 are 1, 6 and (significant !) 0; the fourth one is 9.
Because the 4^{th} s.f. is a 9, (i.e more than or equal to 5), this value is 1.61 kilometres.

ii) If metres been been used, the estimate would have had the same significant figures in it to give 1610 metres, with the final zero not significant.

iii) 4.546 = 4.55 to 3 significant figures (the 4^{th} s.f, 6, is greater than or equal to 5)
4.546 = 4.5 to 2 significant figures (the 3^{rd} s.f, 4, is less than 5)

It is very dodgy to round in stages: 4.546 = 4.55 to 3 s.f. ; 4.55 = 4.6 to 2 s.f. which is incorrect.
Don't do it !

Use of rough estimates in calculations.
Another reason for working out approximate values for calculations is to act as a double-check on results obtained using a calculator.

Example (5): A pupil was given the sum $\frac{39.27+42.16}{5.113\times 3.874}$ to work out on his calculator.

He obtained an answer of 71.2136417. Was this correct or not ?

We can estimate what the answer should have been by simplifying the numbers so that the sum could be worked out mentally – this means reducing the numbers to one, or at most two, significant figures.

39.27 and 42.16 can each be rounded to 40, 5.113 to 5 and 3.874 to 4.

This simplifies the sum to $\frac{40+40}{5\times 4}$, or approximately $\frac{80}{20}$ or 4.
The pupil had forgotten to put brackets around the top and bottom lines, so the calculator treated the sum as

$39.27+\left(\frac{42.16}{5.113}\times 3.874\right)$ instead of $\frac{(39.27+42.16)}{(5.113\times 3.874)}$, due to BIDMAS rules.

By placing brackets round the top and bottom line expressions, we obtain the answer 4.1110.... which agrees well with the rough estimate of 4.

Modern calculators now have the feature of inputting expressions like the one above in fractional form, and, as a result, such BIDMAS errors are now less common in practice.

Example (6): Find the value of $\frac{(1.083+7.212)\times(0.326+1.176)}{0.78^2}$ using a calculator, and verify the result by using a rough estimate.

The calculator result comes out as 20.47845...

We can add 1.083 to 7.212 to obtain 8.295, simplifying to 8.
Likewise we can simplify 0.326 + 1.176 to 1.5 (1 or 2 are a little too far away from the result).
0.78 is just under 0.8, whose square is 0.64, so we can rough-guess 0.78^2 as 0.6.

The simplified sum becomes $\frac{8\times 1.5}{0.6}$ or $\frac{12}{0.6}$, or roughly 20.

The rough approximation of 20 agrees well enough with the actual result to suggest that the correct values were entered in the calculator.

There are other reasons in real-life situations where the approximation rules do not follow the usual pattern, usually as a result of division sums where a whole number result is required and there is a remainder.

Example (7): Electrical cable is produced in 500-metre long spools. How many 3-metre lengths of electrical flex can be cut from such a spool ?

Division gives $\frac{500}{3}$ = 166.66.... and since we are interested in a whole number result, the usual rules of rounding might suggest that it is possible to cut 167 lengths of flex, each 3 metres long.
However, $167 \times 3 = 501$, and we only have 500 metres of cable. The best we can do is to cut 166 lengths of flex from the 500m cable, with 2m cable left over as waste.

In this case we had to round down from 166.66... to 166.
Another term for this rounding down is **truncation**.

Example (8): Paint is sold in 1-litre cans, where a can covers 11 square metres. How many cans are needed to paint a wall 48 square metres in area ?

Division gives $\frac{48}{11}$ = 4.3636.... suggesting that 4 cans of paint are needed, by the usual rounding rules.

However, $11 \times 4 = 44$, and so 4 cans of paint can only cover 44 square metres. We need to break into another can of paint to cover those remaining 4 square metres of the 48, i.e. we need 5 cans.

In this case we had to round up from 4.3636... to 5.

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Error Bounds.

Many practical or experimental measurements are accurate only to a limited number of figures – i.e. only correct to within error bounds .

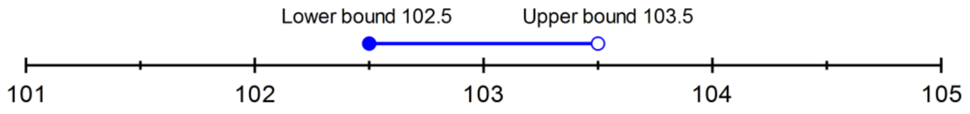
Example (9): A football pitch measures 103 metres by 68 metres to the nearest metre. What are the bounds for its length and its width ?

The quoted length and width are only correct to the nearest metre, and so the length of the pitch in metres can be anything between 102.5 m and 103.49...... m, or 0.5 metres, either side of 103 m.

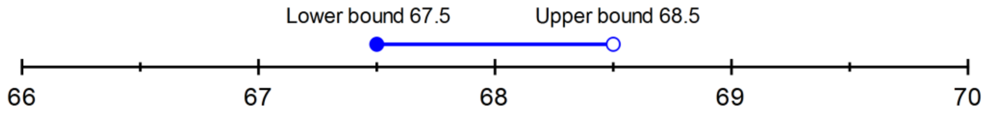
The degree of uncertainty is half a metre either side of 103 metres.

In practice the upper bound is taken as 103.5 m rather than 103.49... m, even though 103.5 m is 104 m to the nearest metre using the standard rounding rules.

We can also use inequality symbols: $102.5 \leq l < 103.5$, where l is the length of pitch in metres. Number line representation shown below:



Similarly the width of the pitch can take any value between 67.5 and 68.49.... m, with the upper bound rounded to 68.5 m for convenience. Using inequality symbols, we can say $67.5 \leq w < 68.5$ where w is the width of the pitch in metres. .



Example (10): A builder's lorry can carry a maximum safe load of 25 tonnes to the nearest tonne and delivers pallets of sand to a building site. A pallet of sand weighs 750 kg to the nearest 50 kg.

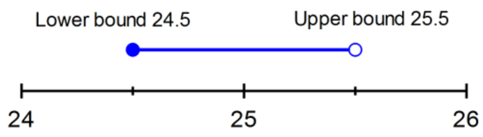
State the bounds for i) the safe load limit of the lorry; ii) the weight of a pallet of sand.

ii) The safe load limit of the lorry is quoted to the nearest tonne, so the margin of error either way is half a tonne.

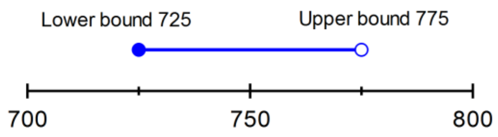
The lower weight limit is therefore 24.5 tonnes and the upper limit is 25.5 tonnes.
We can also state that $24.5 \leq l < 25.5$, where l is the load limit of the lorry in tonnes. .

ii) Since the weight of a pallet of sand is quoted to the nearest 50 kg, the margin of error either way is half of 50 kg, or 25 kg.

The lower weight limit is therefore 750 - 25, or 725 kg ; the upper limit is 750 + 25, or 775 kg.
We can also state that $725 \leq w < 775$, where w is the weight of the pallet of sand in kg.



Weight of lorry (tonnes)



Weight of pallet of sand (kg)