

M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

INDICES

$$273^1 = 273$$

$$2^{3+2} = 2^3 \times 2^2 = 2^5 = 8 \times 4 = 32$$

Powers of 10:

$$8^0 = 1$$

Power of 10	1	2	3	4	5	6
Number	10	100	1000	10000	100000	1000000

$$3^{2 \times 3} = (3^2)^3 = 3^6 = 729$$

$$1^{24} = 1$$

Powers of 2:

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

$$5^{3-1} = 5^3 \div 5^1 = 5^2 = 125 \div 5 = 25$$

INDICES.

When we multiply a number by itself, we are said to square it, or raise it to the power of 2. Thus we write $6 \times 6 = 6^2 = 36$, i.e “six squared”

When we multiply a number by itself twice, we are cubing it, or raising it to the power of 3. Therefore we write $5 \times 5 \times 5 = 125$, or $5^3 = 125$, i.e “5 cubed”.

Higher powers also exist, thus $2^4 = 2 \times 2 \times 2 \times 2 = 16$ (“2 to the fourth”)
Powers are also called **indices** (singular: **index**).

Here are some power tables:

Powers of 10:

Power of 10	1	2	3	4	5	6
Number	10	100	1000	10000	100000	1000000

Powers of 2:

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

Using powers of 2 is more convenient for illustration, because the numbers in question do not become unmanageably large.

Laws of Indices.

The basic laws of indices are as follows, applicable to all positive numbers a .

Multiplication and division:

The Multiplication Law : $a^{m+n} = a^m \times a^n$

Addition of indices corresponds to multiplication of actual numbers.

Examples (1): $2^{3+2} = 2^3 \times 2^2 = 2^5$, or $8 \times 4 = 32$.

This law also holds for fractions, as do the division law and the “powers of powers” law :

$$\left(\frac{2}{3}\right)^{1+2} = \left(\frac{2}{3}\right)^1 \times \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^3, \text{ or } \frac{2}{3} \times \frac{4}{9} = \frac{8}{27}.$$

The Division Law : $a^{m-n} = a^m \div a^n$

Subtraction of indices corresponds to division of actual numbers.

Example(2): $5^{3-1} = 5^3 \div 5^1 = 5^2$, or $125 \div 5 = 25$.

Brackets (“Powers of powers”): $a^{m \times n} = (a^m)^n$

When we multiply indices, we take a "power of a power".

Example(3): $3^{2 \times 3} = (3^2)^3 = 3^6$, or $9^3 = 729$.

Zero power:

$$a^0 = 1$$

Any positive number raised to the zero power is equal to 1.

This can be demonstrated by the multiplication law:

$$a^{m+0} = a^m \times a^0, \text{ but as } m + 0 \text{ is simply } m, a^m \times a^0 = a^m.$$

Adding zero to a number leaves it unchanged; so does multiplying by 1.

Example(4): $8^0 = 1$

Powers of 1:

The number 1 raised to any power is just 1.

Any positive number raised to the power 1 is equal to the number itself.

Examples(5): $1^{24} = 1$; $273^1 = 273$

Examples (6): Show how you would use the 'Powers of 2' table to work out:

- i) 32×64 ; ii) $2048 \div 128$; iii) 32^2

32 × 64 **add these powers** **get result**

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

Reading upwards, we see that $32 = 2^5$ and $64 = 2^6$.
 Adding the powers gives 2^{11} , or $64 \times 32 = 2048$.

2048 ÷ 128 **subtract this** **from this**

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

get result

Reading upwards, we see that $2048 = 2^{11}$ and $128 = 2^7$.
 Subtracting powers gives 2^4 , or $2048 \div 128 = 16$.

32² **double this** **get result**

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

Reading upwards, we see that $32 = 2^5$.
 Doubling the power gives 2^{10} , or $32^2 = 1024$.

Fractional and Negative Powers.

Looking back at the “Powers of 2” table, we can see how the entries in the table are **doubled** every time the power of 2 is **increased** by 1. Conversely, when the power of 2 is **decreased** by 1, the corresponding entry in the table is **halved**.

Power of 2	-4	-3	-2	-1	0	1	2	3	4	5	6	7
Number	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32	64	128

Negative Indices - Reciprocals:

$$a^{-m} = \frac{1}{a^m}$$

Any number raised to a negative power is the reciprocal of the same number raised to the corresponding positive power.

Examples (7) : $2^{-3} = \frac{1}{2^3}$, or $\frac{1}{8}$.

Avoid the common error: 2^{-3} is not -8.

$$\left(\frac{1}{4}\right)^{-2} = 4^2 \text{ or } 16$$

Here we convert the expression with a negative power to an expression with a positive power by reversing the sides of the original fraction.

$$\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 \text{ or } \frac{25}{4}$$

Fractional Indices - Roots:

This leaves us with having to find a meaning to expressions like $64^{\frac{1}{2}}$.

We will show the process of squaring 8 to obtain 64.

8^2	double this					get result						
Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

To **square** a number, we **double** the power.

The inverse of squaring is to take square roots, and to do so, we **halve** the power.

$64^{\frac{1}{2}}$	halve this											
Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

get result

Hence $64^{\frac{1}{2}} = \sqrt{64} = 8$.

This last result can be generalised:

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

Any positive number raised to the reciprocal of a power m is equivalent to the m^{th} root of that number.

Examples(8): $16^{\frac{1}{2}} = \sqrt{16} = 4$.

Recall the ‘powers of powers’ rule: $(16^{\frac{1}{2}})^2 = 16^{\frac{1}{2} \times 2} = 16^1 = 16$.

$$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$$

When we cube a number, we triple the power; when we take cube roots, we divide the power by 3.