M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

INDICES

$$273^1 = 273$$

$$2^{3+2} = 2^3 \times 2^2 = 2^5 = 8 \times 4 = 32$$

$$8^0 = 1$$

Powers of 10:

Power of 10	1	2	3	4	5	6
Number	10	100	1000	10000	100000	1000000

$$1^{24} = 1$$

$$3^{2\times3} = \left(3^2\right)^3 = 3^6 = 729$$

Powers of 2:

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

$$5^{3-1} = 5^3 \div 5^1 = 5^2 = 125 \div 5 = 25$$

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INDICES.

When we multiply a number by itself, we are said to square it, or raise it to the power of 2.
Thus we write $6 \times 6 = 6^2 = 36$, i.e "six squared"

When we multiply a number by itself twice, we are cubing it, or raising it to the power of 3.
Therefore we write $5 \times 5 \times 5 = 125$, or $5^3 = 125$, i.e "5 cubed".

Higher powers also exist, thus $2^4 = 2 \times 2 \times 2 \times 2 = 16$ ("2 to the fourth").

In the expression 5^3, the 5 is the **base** and the 3 is the **index** (plural: **indices**).

Here are some power tables:

Powers of 10:

Power of 10	1	2	3	4	5	6
Number	10	100	1000	10000	100000	1000000

Powers of 2:

Power of 2	1	2	3	4	5	6	7	8	9	10	11	12
Number	2	4	8	16	32	64	128	256	512	1024	2048	4096

Using powers of 2 is more convenient for illustration, because the numbers in question do not become unmanageably large.

Laws of Indices.

The basic laws of indices are as follows, applicable to all positive numbers *a*.

The Multiplication Law : $a^{m+n} = a^m \times a^n$

Addition of indices corresponds to multiplication of actual numbers.

Examples (1): $2^{3+2} = 2^3 \times 2^2 = 2^5$, or $8 \times 4 = 32$.

This law also holds for fractions, as do the division law and the "powers of powers" law :

$$\left(\frac{2}{3}\right)^{1+2} = \left(\frac{2}{3}\right)^{1} \times \left(\frac{2}{3}\right)^{2} = \left(\frac{2}{3}\right)^{3}, \text{ or } \frac{2}{3} \times \frac{4}{9} = \frac{8}{27}.$$

The Division Law : $a^{m-n} = a^m \div a^n$

Subtraction of indices corresponds to division of actual numbers.

Example(2): $5^{3-1} = 5^3 \div 5^1 = 5^2$, or $125 \div 5 = 25$.

Brackets ("Powers of powers"): $a^{m \times n} = \left(a^m\right)^n$

When we multiply indices, we take a "power of a power".

Example(3): $3^{2\times3} = \left(3^2\right)^3 = 3^6$, or $9^3 = 729$.

Zero power:

$$a^0 = 1$$

Any positive number raised to the zero power is equal to 1.

This can be demonstrated by the multiplication law:

$a^{m+0} = a^m \times a^0$, but as $m + 0$ is simply m, $a^m \times a^0 = a^m$.

Adding zero to a number leaves it unchanged; so does multiplying by 1.

Example(4): $8^0 = 1$

Powers of 1:

The number 1 raised to any power is just 1.
Any positive number raised to the power 1 is equal to the number itself.

Examples(5): $1^{24} = 1$; $273^1 = 273$

Examples (6): Show how you would use the 'Powers of 2' table to work out:

i) 32×64 ; ii) $2048 \div 128$; iii) 32^2

32 × 64 **add these powers** **get result**

Power of 2	1	2	3	4	**5**	**6**	7	8	9	10	**11**	12
Number	2	4	8	16	**32**	**64**	128	256	512	1024	**2048**	4096

Reading upwards, we see that $32 = 2^5$ and $64 = 2^6$.
Adding the indices gives 2^{11}, or $64 \times 32 = 2048$.

2048 ÷ 128 **subtract this** **from this**

Power of 2	1	2	3	**4**	5	6	**7**	**8**	9	10	**11**	12
Number	2	4	8	**16**	32	64	**128**	**256**	512	1024	**2048**	4096

get result

Reading upwards, we see that $2048 = 2^{11}$ and $128 = 2^7$.
Subtracting indices gives 2^4, or $2048 \div 128 = 16$.

32² **double this** **get result**

Power of 2	1	2	3	4	**5**	6	7	8	9	**10**	11	12
Number	2	4	8	16	**32**	64	128	256	512	**1024**	2048	4096

Reading upwards, we see that $32 = 2^5$.
Doubling the index gives 2^{10}, or $32^2 = 1024$.

Fractional and Negative Powers.

Looking back at the "Powers of 2" table, we can see how the entries in the table are **doubled** every time the power of 2, namely the index, is **increased** by 1. Conversely, when the index is **decreased** by 1, the corresponding entry in the table is **halved**.

Power of 2	**-4**	**-3**	**-2**	**-1**	**0**	1	2	3	4	5	6	7
Number	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	**1**	2	4	8	16	32	64	128

Negative Indices - Reciprocals:

$$a^{-m} = \frac{1}{a^m}$$

Any number raised to a negative power is the reciprocal of the same number raised to the corresponding positive power.

Examples (7) : $2^{-3} = \frac{1}{2^3}$, or $\frac{1}{8}$.

Avoid the common error: 2^{-3} is not -8.

$\left(\frac{1}{4}\right)^{-2} = 4^2$ or 16

Here we convert the expression with a negative power to an expression with a positive power by reversing the sides of the original fraction.

$\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^{2}$ or $\frac{25}{4}$

Fractional Indices - Roots:

This leaves us with having to find a meaning to expressions like $64^{½}$.

We will show the process of squaring 8 to obtain 64.

8^2 **double this** **get result**

Power of 2	1	2	**3**	4	5	**6**	7	8	9	10	11	12
Number	2	4	**8**	16	32	**64**	128	256	512	1024	2048	4096

To **square** a number, we **double** the index.

The inverse of squaring is to take square roots, and to do so, we **halve** the index.

$64^{½}$ **halve this**

Power of 2	1	2	**3**	4	5	**6**	7	8	9	10	11	12
Number	2	4	**8**	16	32	**64**	128	256	512	1024	2048	4096

get result

Hence $64^{\frac{1}{2}} = \sqrt{64} = 8.$

This last result can be generalised:

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

Any positive number raised to the reciprocal of an index *m* is equivalent to the *m*th root of that number.

Examples(8): $16^{\frac{1}{2}} = \sqrt{16} = 4.$

Recall the 'powers of powers' rule: $(16^{\frac{1}{2}})^2 = 16^{\frac{1}{2}\times 2} = 16^1 = 16.$

$$125^{\frac{1}{3}} = \sqrt[3]{125} = 5.$$

When we cube a number, we triple the index; when we take cube roots, we divide the index by 3.