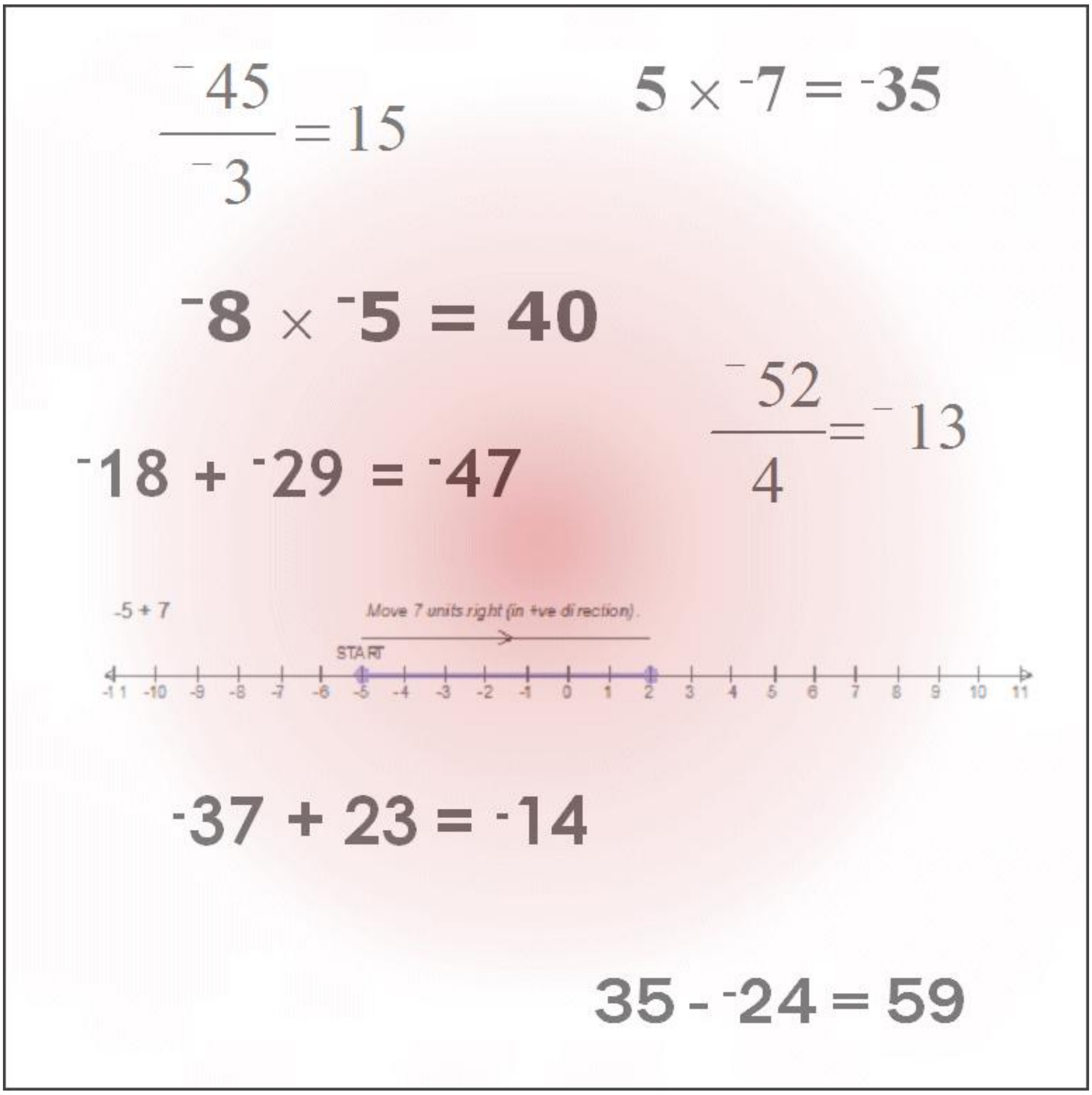
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

DIRECTED (SIGNED) NUMBERS



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DIRECTED (SIGNED) NUMBERS.

Directed or signed numbers are numbers prefixed by a positive (+) or negative (-) sign.
They are used to denote quantities on either side of zero.

Examples of everyday use are seen on weather maps in winter when temperatures are 'below zero', or on bank statements when accounts are 'in the red'.

The number line is a convenient way of showing directed numbers and performing basic arithmetic on them (addition and subtraction).

Positive numbers are not usually prefixed with a + sign, in other words, unsigned numbers are positive by default.

Using a number line for addition and subtraction.

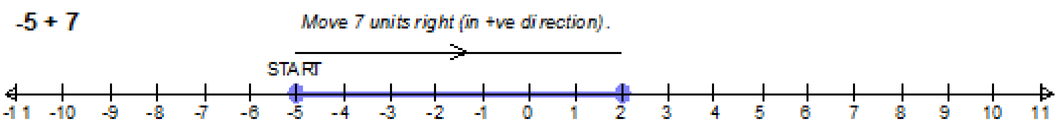
<u>To add a positive number, move right (positive direction) along the number line.</u>

Example (1) : The minimum temperature in Manchester one night was -5° C. It rose by 7° to a maximum the following afternoon. What was that maximum temperature ?

The question is asking us to find the value of $^{-}5 + 7$.

We begin by plotting the starting value , $^{-}5$, on the number line and then moving 7 units in the positive direction until we reach the answer, 2.

$\therefore$ $\mathbf{^{-}5 + 7 = 2.}$ (The maximum temperature that afternoon was 2°C).

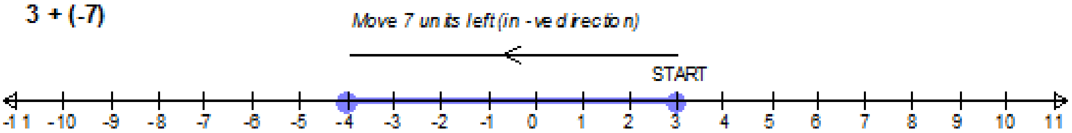


<u>To add a negative number, move left (negative direction) along the number line.</u>

Example (2) : Find the value of $3 + (^{-}7)$.

We plot 3 on the number line and then move 7 units in the negative direction until we reach the answer, namely -4.

$\therefore$ $\mathbf{3 + {}^{-}7 = {}^{-}4.}$



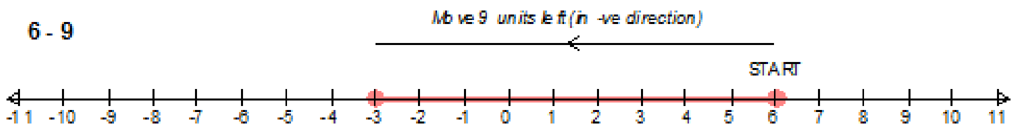
Note that this result is the same as $\mathbf{3 - 7 = {}^{-}4.}$
The adjacent + and – signs have been replaced by a single – sign.

<u>To subtract a positive number, move left (negative direction) along the number line.</u>

Example (3) : The maximum temperature in Manchester was 6° C one afternoon. It fell by 9° to an overnight minimum. What was that minimum?

The question asks us to find the value of 6 - 9.

We plot 6 on the number line and then move 9 units in the negative direction until we reach the answer, namely $^{-}3$.

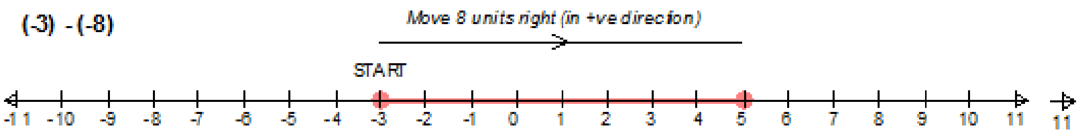


∴ **6 - 9 = $^{-}$3.** (The minimum overnight temperature was -3°C).

<u>To subtract a negative number, move right (positive direction) along the number line.</u>

Example (4) : Find the value of $^{-}3 – (^{-}8)$.

We plot $^{-}3$ on the number line and then move 8 units in the positive direction until we reach the answer, 5.

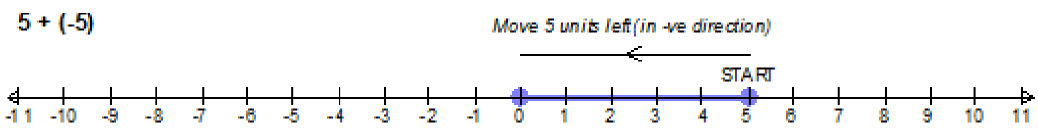


∴ **$^{-}$3 - $^{-}$8 = 5.**

Note that this result is the same as **$^{-}$3 + 8 = 5.**
The adjacent − and − signs have been replaced by a single + sign.

Example (5) : Find the value of $5 + (^{-}5)$.

We plot 5 on the number line and then move 5 units in the negative direction until we reach zero on the number line.



This is a general rule: adding a positive number to the negative one **of the same size** always gives an answer of zero.

Directed number arithmetic without a number line.

Without the use of a number line, the following methods are perhaps the easiest way of adding and subtracting signed numbers.

Addition.

Positive + positive always gives a **positive** - this is standard, i.e. $\mathbf{52 + 19 = 71}$.

Negative + negative always gives a **negative**.
This addition is most easily done by adding the **sizes** of the two numbers and then prefixing with a minus sign.

Thus, we work out $^{-}18 + {}^{-}29$ by adding 18 and 29 to give 47, and finally putting a minus sign in front. .
$\mathbf{^{-}18 + {}^{-}29 = {}^{-}47.}$

Note also that $\mathbf{^{-}18 + {}^{-}29 = {}^{-}47}$ is identical to $\mathbf{^{-}18 - 29 = {}^{-}47.}$
(The adjacent '+' and '–' signs have been reduced to a single '-' sign).

Positive + negative (or negative + positive) can give either a positive or a negative result.

These sums are best worked out by finding the unsigned **difference** between their **sizes**.
The sign of the result is the same as that of the number of larger size.

This is best illustrated by the following examples:

$\mathbf{52 + {}^{-}36}$: Ignoring signs, the difference between 52 and 36 is 16.
The positive number (52) in the original sum was larger in size, so the result is positive,

i.e. $\mathbf{52 + {}^{-}36 = 16}$.

Again, $\mathbf{52 + {}^{-}36}$ is identical to $\mathbf{52 - 36}$. **(Adjacent '+' and '–' signs reduced to a single '-' sign).**

$\mathbf{^{-}37 + 23}$: Ignoring signs, the difference between the sizes of 37 and 23 is 14.
The negative number ($^{-}37$) in the original sum was larger in size, so the result is negative,
i.e. $\mathbf{^{-}37 + 23 = {}^{-}14.}$

<u>Subtraction.</u>
This is a little trickier than addition – it may often be helpful to turn the sums into addition sums.

Positive - positive gives a **positive** if the right-hand number is smaller than the left-hand one - this is standard, i.e. $\mathbf{38 - 16 = 22}$.

If the number on the right is larger, we still take the difference but the sign is reversed.
Thus $\mathbf{16 - 38 = {}^{-}22}$.

Note: For any numbers a and b, b-a = - (a-b) : thus for example 8-5 = 3; 5-8 = -3.

Negative – positive always gives a negative.
Here we add the sizes of the two numbers and put a – sign in front.

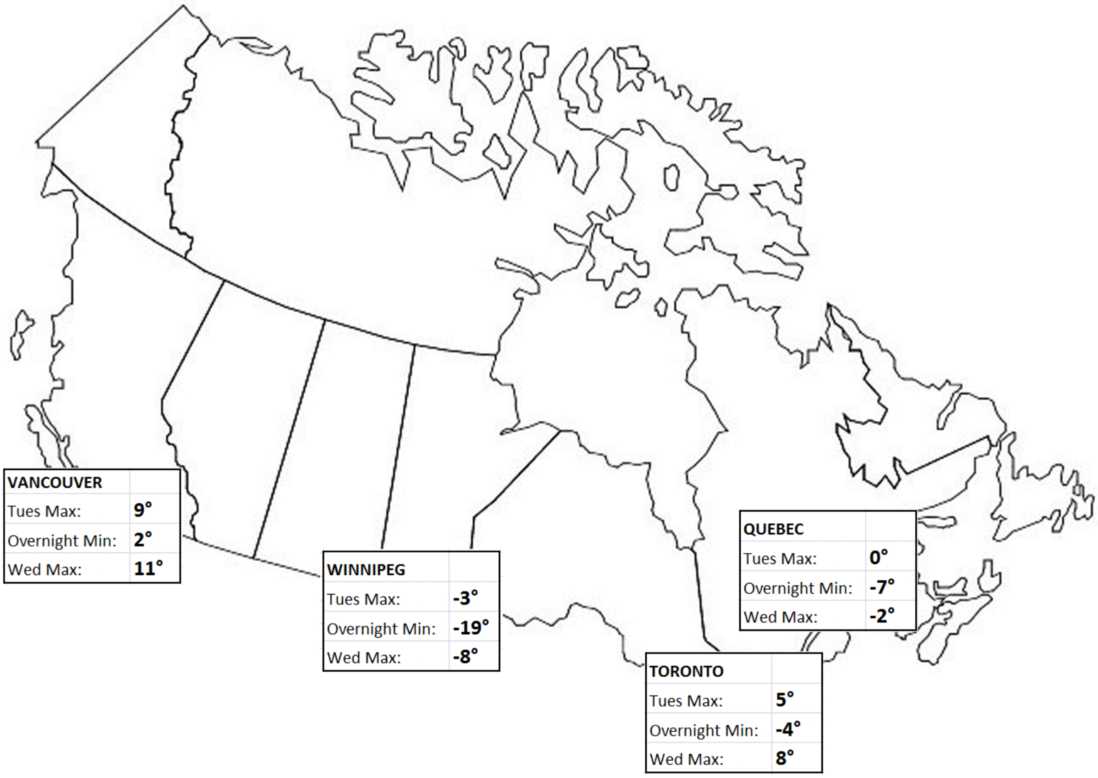
Thus $\mathbf{{}^{-}31 - 21}$ can be found by adding 31and 21 to give 52, and finally prefixing with a – sign to give $\mathbf{{}^{-}31 - 21 = {}^{-}52}$.

Subtracting a negative number is identical to **adding the positive number of the same size**.
Thus $\mathbf{35 - {}^{-}24 = 35 + 24 = 59}$.

(The two adjacent ‘–‘ signs have been replaced with a single ‘+‘ sign).

Also $\mathbf{-34 - {}^{-}19 = {}^{-}34 + 19 = {}^{-}15}$.

Example (6): This map of Canada shows the temperatures for four cities over two winter days.



i) In which city did the temperature never fall below 0°C ?
ii) Which city had the largest overnight temperature decrease ?
iii) Which city had the largest temperature increase from the overnight minimum ?
iv) Which two cities had the same overnight temperature decrease ?

v) The details for Montreal are missing from the map.
The maximum temperature on Tuesday was 2°C, and the overnight minimum represented an 8°C decrease. The maximum temperature on Wednesday was a 10°C increase over that minimum.
Work out the overnight minimum and the Wednesday maximum temperatures for Montreal.

i) The only city where the temperature never fell below 0°C was Vancouver.

ii) The overnight temperature decrease for Vancouver was from 9°C to 2°C, i.e. a 7°C decrease.
For Winnipeg, the maximum was -3°C and the minimum -19°C – a decrease of 16°C.
Toronto's temperature decreased from 5°C to -4°C, or by 9°C, and finally the temperature at Quebec decreased from 0°C to -7°C, i.e. by 7°C.

Hence Winnipeg had the largest overnight temperature decrease.

iii) The temperature at Vancouver increased from the overnight low of 2°C to the Wednesday maximum of 11°C – an increase of 9°C. At Winnipeg the temperature rose from -19°C to -8°C, or an increase of 11°C. The temperature rose at Toronto from -4°C to 8°C, - a 12°C increase. Finally, the temperature in Quebec rose from -7°C to -2°C, or a 5° rise.
Toronto had the largest temperature increase from the overnight minimum.

iv) Using the results from ii), both Vancouver and Quebec experienced the same overnight temperature decrease of 7°C.

v) Montreal's temperature decreased by 8°C from the Tuesday maximum of 2°C, giving an overnight low of (2 – 8)°C or -6°C.

The Wednesday maximum was a 10° increase from that overnight low, i.e.(-6 + 10)°C or 4°C.

Multiplication and division of directed numbers.

These are more straightforward than addition and subtraction.

Positive $\times$ positive = positive.
Positive $\times$ negative = negative.
Negative $\times$ positive = negative.
Negative $\times$ negative = positive.

Positive $\div$ positive = positive.
Positive $\div$ negative = negative.
Negative $\div$ positive = negative.
Negative $\div$ negative = positive.

When multiplying or dividing:

If the signs are **alike**, we have a **positive** result, but if they are **different**, we have a **negative** result.

Thus: $\mathbf{9 \times 6 = 54,\ 5 \times {}^{-}7 = {}^{-}35}$ **and** $\mathbf{{}^{-}8 \times {}^{-}5 = 40.}$

Similarly $\frac{60}{5} = 12$, $\frac{{}^{-}52}{4} = {}^{-}13$ and $\frac{{}^{-}45}{{}^{-}3} = 15$.

All of the examples shown so far involved integer arithmetic, but the same rules apply to fractional and decimal numbers as well.

For example,

$\mathbf{5.7 + {}^{-}2.1 = 3.6\ ;\quad 2.5 \times {}^{-}1.5 = {}^{-}3.75\ ;\quad 7 - {}^{-}2\frac{1}{2} = 9\frac{1}{2}}$