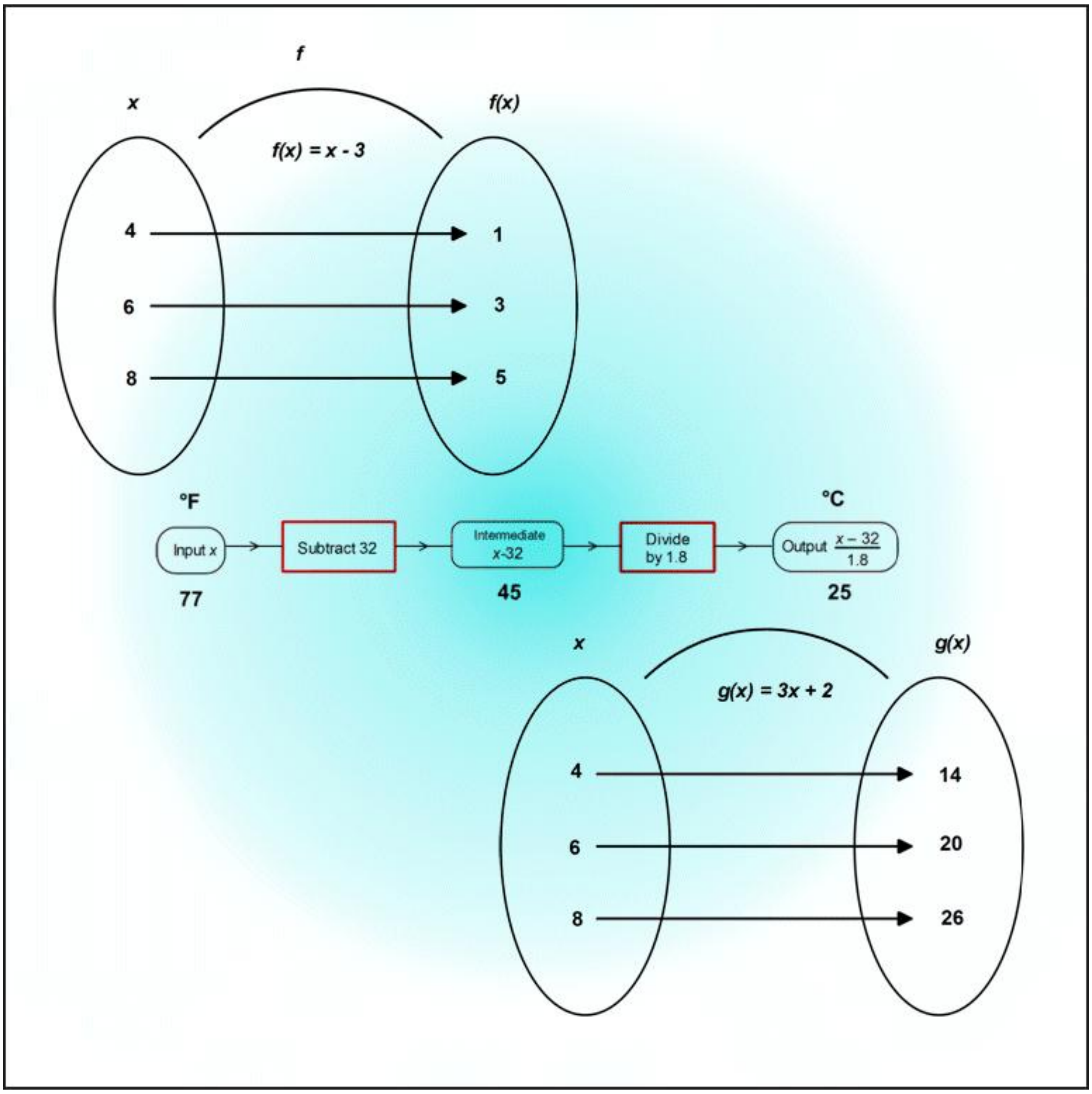
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Foundation Tier

FUNCTIONS



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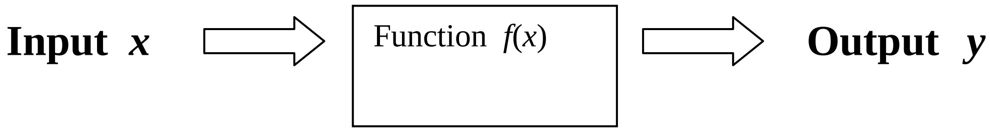
FUNCTIONS

A function can be thought of as being a rule where we start with an **input**, apply the rule, and end with an **output**.

In the diagram below, we assign x to the input and y to the output, while the function is called $f(x)$.



Functions are usually denoted by the letters f, g or h.

A function f which squares a number x can therefore be expressed in the following ways :

$y = x^2$ (if the result is assigned the variable y)

$f(x) = x^2$

Here, f produces an output of 16 when the input is 4, therefore we say that $f(4)= 16$.

Example (1) : A function *f* with input of *x* and output of *y* uses the following rule; subtract 3 from *x*. Express this in function notation, and write out $f(4)$.

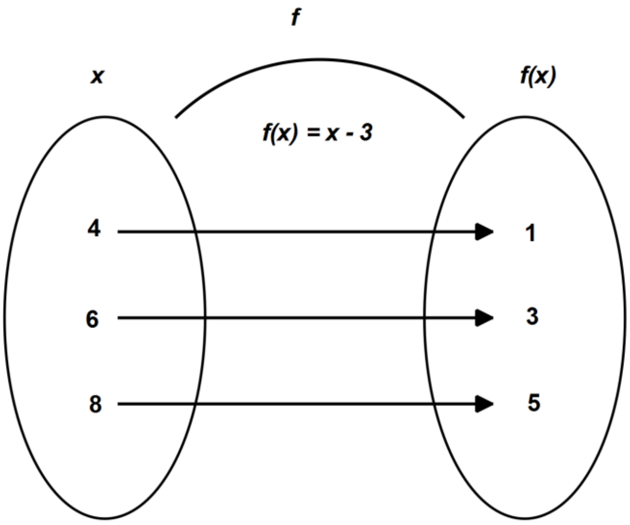
The function can be denoted by $f(x) = x – 3$, or $y = x – 3$.

Hence $f(4) = 1$.

Input $x = 4$ ⇨ $f(x) = x - 3$ ⇨ **Output** $y = 1$

Functions are also often illustrated by **mapping diagrams**.

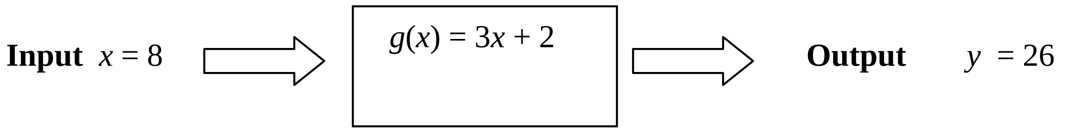
In the diagram below, the function *f* "fires" the values of 4, 6 and 8 from the "input" region to 1, 3 and 5 in the "output" region.



Mapping diagrams are a useful tool for visualising functions, although they can only display a limited set of mappings.

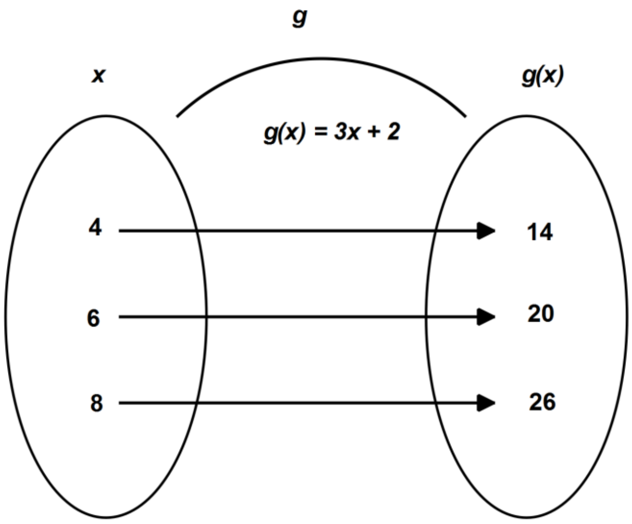
Example (2) : A function *g* maps *x* to *y* by the following rule; multiply *x* by 3, and then add 2. Express this in function notation, write out $g(8)$, and draw a mapping diagram featuring three values of *x* and their images in *g*.

The function can be denoted by $g(x) = 3x + 2$ or $y = 3x + 2$.



Hence $g(8) = 26$.

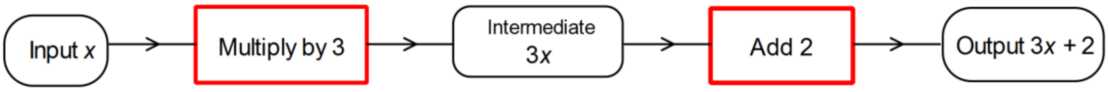
Mapping diagram below :



Multi-step functions.

The function in Example (2) was shown as a single-step function, whereby the rule "multiply x by 3, and then add 2" was treated as a single step.

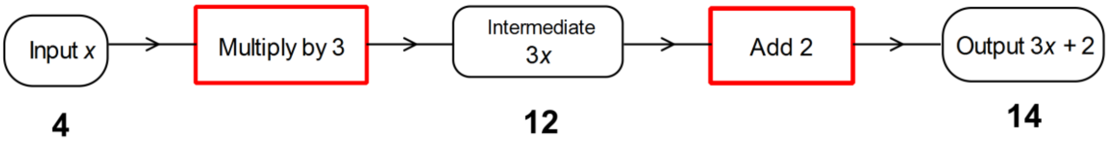
There is nothing to stop us from breaking it down into two functions, as shown below:



(We have described the functions in words rather than in algebraic notation.)

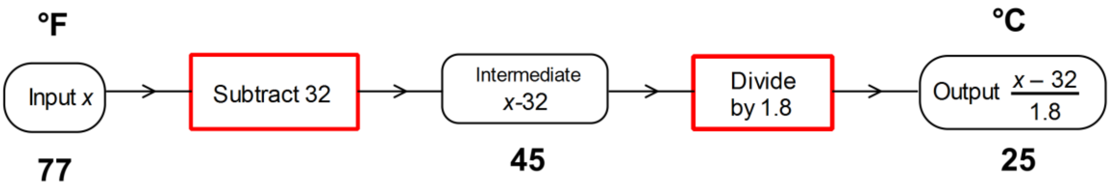
Such diagrams are often called **function machines.**

Example (3): Use the function machine above to find the output to this function, for an input of 4.



Example (4): Weather forecasts in the USA use the Fahrenheit scale for temperature, and the rule for converting temperature readings from Fahrenheit to Celsius is "Subtract 32, then divide by 1.8".

Express this as a function machine diagram, and use it to convert 77°F to the Celsius scale.

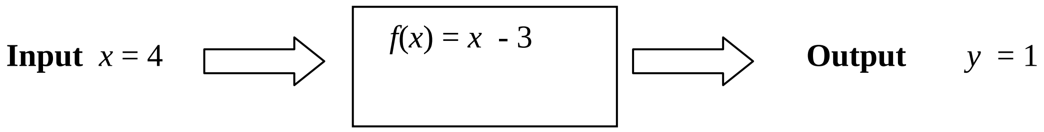


Hence 77°F = **25°C.**

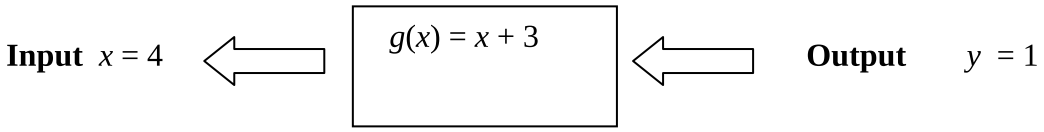
Although the "Intermediate" and "Output" boxes show expressions in x, they are often omitted from function machine diagrams.

Inverse functions.

The inverse of a function is another function that reverses whatever the original does.
We will take the simplest case .



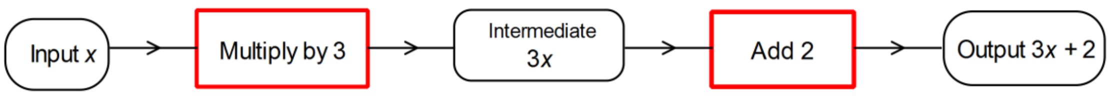
The function f takes an input value of x and subtracts 3 from it, so what happens if we were asked to find the input when given the output ?



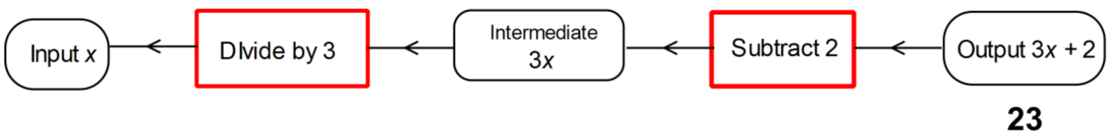
This time we are applying the function in reverse by starting off with the output and ending up with the input. We have called this function g, and it adds 3 to the output of 1, to give us the input of 4.

This method of "undoing" a function is familiarly similar to solving linear algebraic equations, as the next example shows.

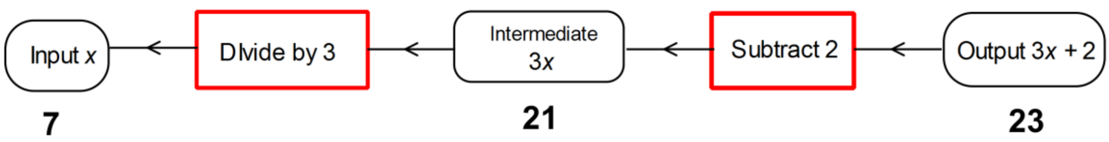
Example (5): Use the function machine below to find the **input** to this function, for an output of 23. The processes are the same as those in solving a linear equation. What is the equation ?



We reverse the processes in the function boxes, and also reverse the arrows as shown below :



Then, we start with the output of 23 and work left until we have the original input :

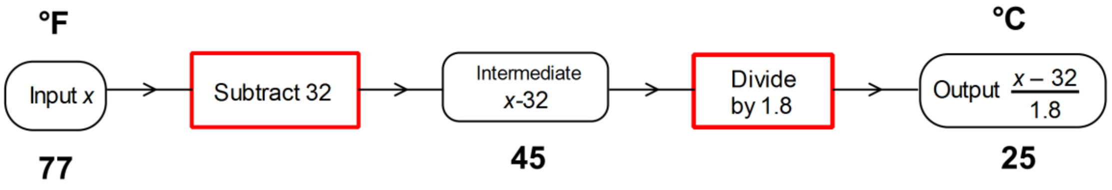


Hence the input is 7 when the output is 23.

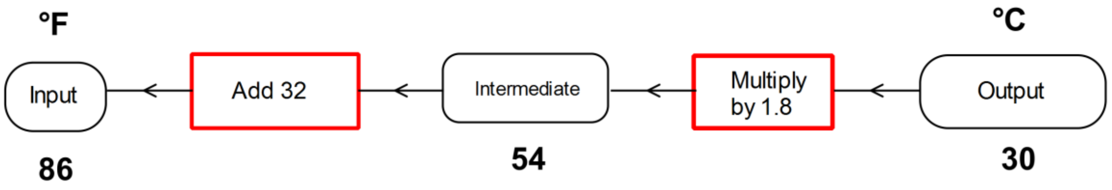
What we have done is solved the equation $3x + 2 = 23$ using function machine notation.

Example (6): In Example (4) we converted temperatures from Fahrenheit to Celsius.
The function diagram below shows how to convert 77°F to the Celsius scale, giving the result of 25°C .

Use the diagram to convert 30°C to the Fahrenheit scale.



The reversed diagram looks like this :



This time, we have a scenario of "rearranging the formula".