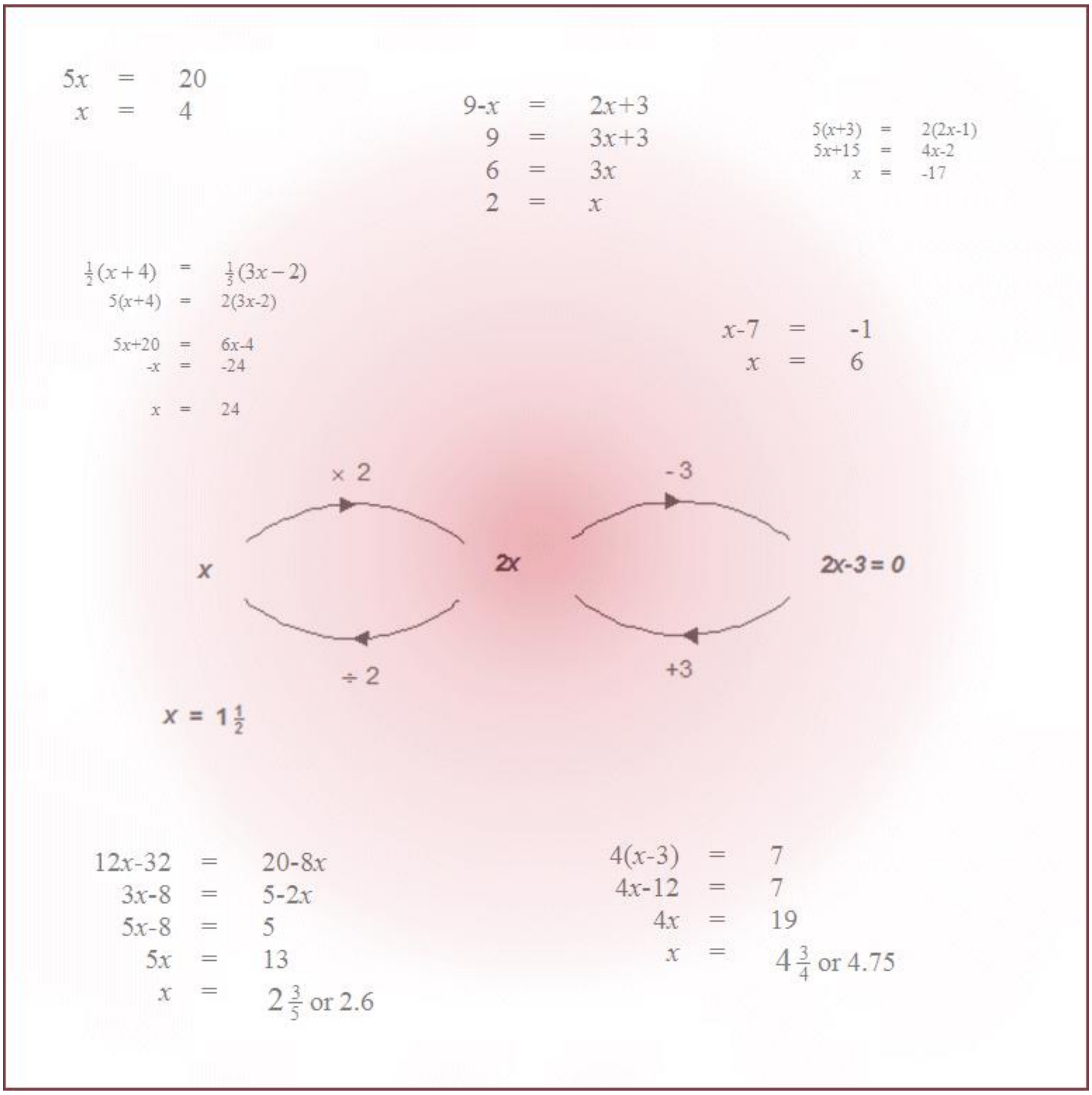
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

SIMPLE EQUATIONS



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LINEAR EQUATIONS

An **equation** connects two algebraic expressions, and always includes an equality sign.

A few examples of equations are:

$2A + 3B = 4A - B$

$7x = 42$

$2x^2 + 5x - 3 = 0$

Setting up an equation from a mathematical statement.

Example (1).
Write the equation corresponding to the instructions
"Take a number x, double it and then add 5. Store the result in y."

Doubling x gives us $2x$, and adding 5 to $2x$ gives $2x + 5$.

The equation is $y = 2x + 5$.

Example (2).

A man is currently three times as old as his son. Five years ago, he was four times as old. Form two equations, using M for the man's current age and S for the son's.

The first equation, stating that the man is three times as old as his son, is $M = 3S$.
Five years earlier, the man was four times as old as his son, the man's age was M-5 years and the son's was S-5 years. The second equation is therefore $M - 5 = 4(S-5)$.

Linear Equations.

If an equation contains no power of a variable greater than the first, it is termed a **linear** equation.
Thus the equations $2x + 5 = 11$ and $8-y = 5$ are linear, but $x^2 - 2x + 1 = 5$ is not, because of the presence of the x^2 term.

Solving linear equations in one dependent variable (here x) involves performing the same arithmetic operations on each side of the equality sign, such that the terms in x appear on one side of the equation and the numbers on the other.

The simplest equations have an expression containing the variable on the left, and a number alone on the right.

Examples (3): Solve the equations i) $5x = 20$; ii) $x-7 = -1$; iii) $2x - 3 = 0$; iv) $4(x - 3) = 7$.

5*x* = 20

$5x$	=	20	
x	=	4	Divide both sides by 5

***x*-7 = -1**

x-7	=	-1	
x	=	6	Add 7 to both sides

2*x* – 3 = 0

$2x$-3	=	0	
$2x$	=	3	Add 3 to both sides
x	=	$1\frac{1}{2}$ or 1.5	Divide both sides by 2

4(*x* – 3) = 7

$4(x$-3)	=	7	
$4x$-12	=	7	Expand brackets
$4x$	=	19	Add 12 to both sides
x	=	$4\frac{3}{4}$ or 4.75	Divide both sides by 4

Correct use of the equality sign.

The solution to equation i) in the last example could have been written in shorter form as follows:

$5x = 20, \therefore x = 4$ using the sign for ‘therefore’

or $5x = 20$, hence $x = 4$ in words

or $5x = 20 \rightarrow x = 4$ using arrow symbol to say ‘ if $5x = 20$, then $x = 4$’

Sometimes a linear equation might have the variable on both sides of the equality sign.
In such cases, we must bring the variable onto one side of the equation (separate the variable).

Examples (4): Solve the equations i) $9-x = 2x+3$; ii) $12x-32 = 20-8x$

$9-x = 2x+3$

The aim here is to bring the x to one side of the equation and any numbers on to the other.

$9-x$	=	$2x+3$	
$9-3x$	=	3	Subtract $2x$ from each side
$-3x$	=	-6	Subtract 9 from each side (this isolates x)
x	=	2	Divide both sides by –3

We could have avoided the division by a negative if we had brought x to the right-hand-side.

$9-x$	=	$2x+3$	
9	=	$3x+3$	Add x to each side
6	=	$3x$	Subtract 3 from each side (this isolates x)
2	=	x	Divide both sides by 3

The statements $2 = x$ and $x = 2$ are equivalent, of course.

With practice, the two subtractions can be carried out in one go:

$9-x$	=	$2x+3$	
6	=	$3x$	Add $x - 3$ to each side (separating the x-term from the number).
2	=	x	Divide both sides by 3

Adding $x - 3$ to each side of the equation separates the x-term and the number term in one step.

$12x-32 = 20-8x$

$12x-32$	=	$20-8x$	
$3x-8$	=	$5-2x$	Divide both sides by the common factor of 4
$5x-8$	=	5	Add $2x$ to each side
$5x$	=	13	Add 8 to each side
x	=	$2\frac{3}{5}$ or 2.6	Divide both sides by 5

Again, the two additions can be carried out in one go:

$12x-32$	=	$20-8x$	
$3x-8$	=	$5-2x$	Divide both sides by the common factor of 4
$5x$	=	13	Add $2x + 8$ to each side (separate x term from number)
x	=	$2\frac{3}{5}$ or 2.6	Divide both sides by 5

Examples (5): Solve the equations i) $5(x+3) = 2(2x-1)$; ii) $\frac{x+4}{2} = \frac{3x-2}{5}$

5(*x*+3) = 2(2*x*- 1)

This time, we need to expand brackets.

$5(x+3)$	=	$2(2x-1)$	
$5x+15$	=	$4x-2$	Expand both sides
x	=	-17	Subtract $(4x+15)$ from each side (separate terms)

ii) $\frac{x+4}{2} = \frac{3x-2}{5}$

When an equation includes fractional terms, it is best to get rid of the fractions by using a suitable multiplier – i.e. the lowest common multiple of the denominators.

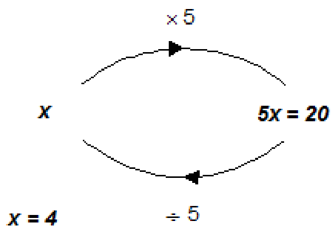
$\frac{x+4}{2}$	=	$\frac{3x-2}{5}$	
$\frac{10(x+4)}{2}$	=	$\frac{10(3x-2)}{5}$	Eliminate the fractions by multiplying both sides by 10 (the LCM of the denominators)
$5(x+4)$	=	$2(3x-2)$	With practice, the two last steps can be combined mentally.
$5x+20$	=	$6x-4$	Expand both sides
$-x$	=	-24	Subtract $(6x + 20)$ from each side to separate the terms
x	=	24	Multiply both sides by –1

Inverse Operations.

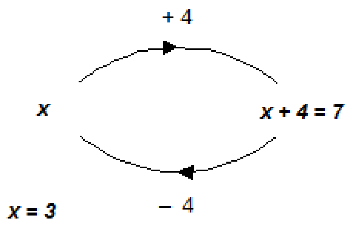
The last examples demonstrated the idea of **inverse** operations.

When solving $5x = 20$, we realised that we had to **multiply** x by 5 to give 20.

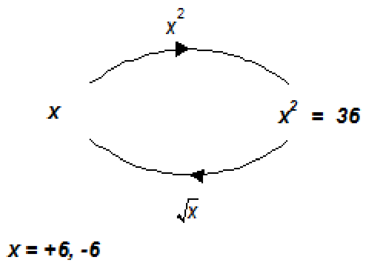
The opposite process of finding x from 20 involved **dividing** by 5.



Similarly to solve an equation like $x + 4 = 7$, we must have **added** 4 to x to give 7.
Therefore to find x we **subtract** 4 from 7 to get 3.



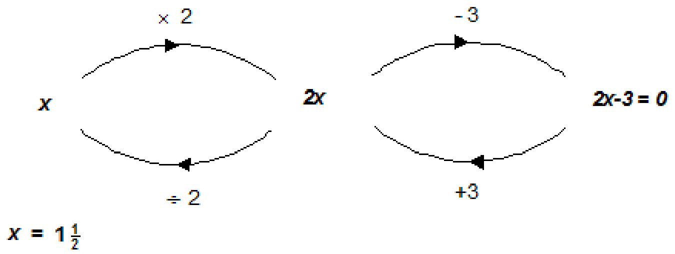
The equation $x^2 = 36$ is not linear, but we can see that we took the **square** of x to get 36. Therefore we take the **square root** of 36 to get the original x. This gives us the solution $x = 6$. However, because squaring a negative quantity gives a positive one, there is a second solution, i.e. $x = -6$.



When the solution of an equation takes more than one step, the order of inverse operations is also reversed.

When solving $2x-3 = 0$, we had to perform two operations; firstly to multiply x by 2 and then to subtract 3 from the result.

The inverse process is to add 3 and then divide by 2 to get the solution $x = 1½$.



From the examples above, it can be seen that addition and subtraction are inverse operations, as are multiplication and division.

Other examples of inverse pairs are taking squares and square roots, cubes and cube roots.

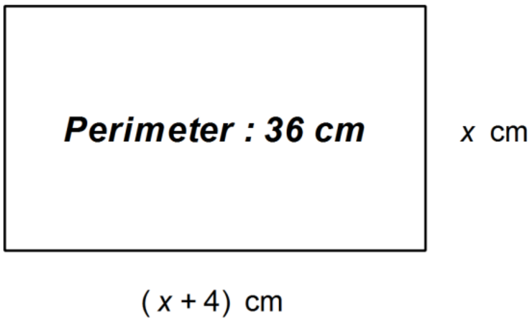
Some operations are self-inverse, such as taking the reciprocal of a number (dividing it into 1).
Thus $1 \div 2 = ½$ and $1 \div ½ = 2$.

Linear equations with a "shape and space" tie-in.

The following examples require us to form equations from geometrical facts, and then to solve it.

N.B. **None of the diagrams in those examples are drawn accurately.**

Example (6): The long sides of a rectangle are 4 cm longer than the short sides. Given that the perimeter of the rectangle is 36 cm, find the lengths of the sides of the rectangle.



The length of a short side is x cm, and that of a long side is (x + 4) cm.

The perimeter is 36 cm, so we can form this equation from the formula for the perimeter of a rectangle:

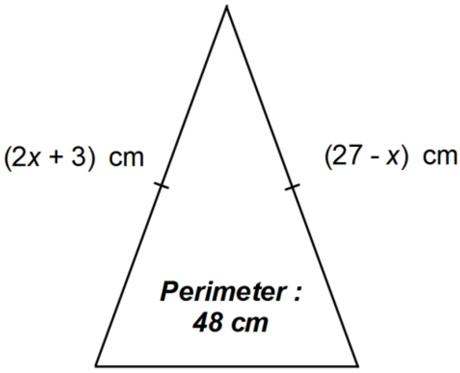
$x + (x + 4) + x + (x + 4) = 36$

Tidying up, we have the equation $4x + 8 = 36$, and then we solve it:

$4x + 8 = 36 \;\rightarrow\; 4x = 28 \;\rightarrow\; x = 7$. (Intermediate steps not shown in detail)

Since $x = 7$, the short sides are x cm, or **7 cm** long and the long sides are (x + 4) cm, or **11 cm** long.

Example (7): An isosceles triangle has a perimeter of 48 cm. The lengths, in centimetres, of the two equal sides are given as $2x + 3$ and $27 – x$. Find the length of the third side.



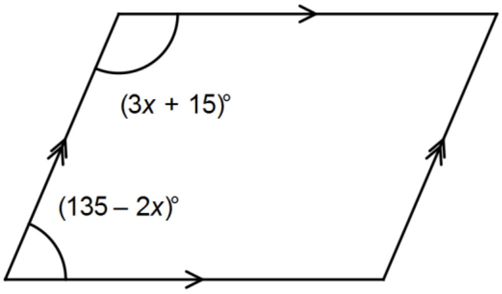
From the given data, we can form the linear equations:

$\mathbf{2x + 3 = 27 – x}$ (given the two equal sides)

and then use the resulting solution in x and the perimeter formula to find the third side.

$2x + 3 = 27 – x \;\rightarrow\; 3x + 3 = 27 \;\rightarrow\; 3x = 24 \;\rightarrow\; x = 8$.

By substituting $x = 8$ into either $2x + 3$ or $27 – x$, we find that the two equal sides are each 19 cm long. As the perimeter is 48 cm, we subtract twice 19 from 48, giving the length of the third side as **10 cm**.



Example (8): An obtuse angle of a parallelogram is $(3x + 15)°$ and an acute one is $(135 – 2x)°$. Find the sizes of both angles.

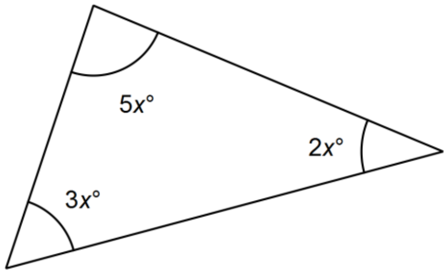
The two labelled angles are allied (co-interior) and therefore their sum is 180°.

We can therefore form the equation
$\mathbf{3x + 15 + 135 – 2x = 180}$ and solve it.

We simplify this as $\mathbf{x + 150 = 180}$, and thus $x = 30$.

Substituting $x = 30$ into $3x + 15$ gives us the obtuse angle of **105°.**
The acute angle is thus **75°** by the laws of allied angles.

(Check: when $x = 30$, $135 – 2x = 75$).



Example (9): The angles in a triangle are $3x°$, $2x°$ and $5x°$.

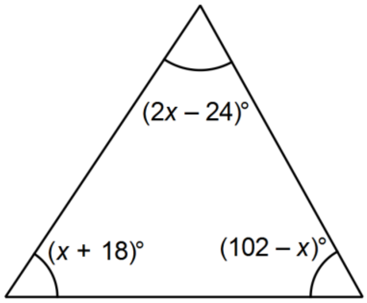
What type of triangle is it ? Justify the answer with full working.

The angle sum of a triangle is 180°, so we form the equation $\mathbf{3x + 2x + 5x = 180}$ and solve it.

This simplifies to $10x = 180 \rightarrow x = 18$.

The angles in the triangle are $\mathbf{3x° = 54°, 2x° = 36°}$ **and** $\mathbf{5x° = 90°}$.

This particular triangle is therefore **right-angled.**



Example (10): The angles in another triangle are $(x + 18)°$, $(102 - x)°$ and $(2x – 24)°$.

Find x, and hence all the angles, showing your working. What type of triangle is it ?

The angle sum of a triangle is 180°, giving us the equation

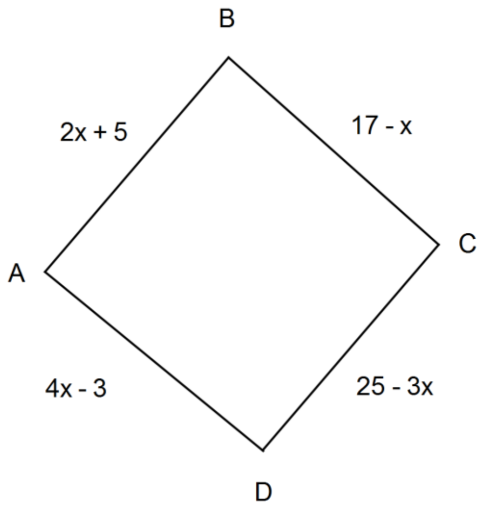
$\mathbf{(x + 18) + (102 - x) + (2x - 24) = 180}$ to solve.

This simplifies to

$2x + 96 = 180 \rightarrow 2x = 84 \rightarrow x = 42.$

The angles in the triangle are $\mathbf{(x + 18)° = 60°,\ (102\text{-}x°) = 60°}$ **and** $\mathbf{(2x\text{-}24)° = 60°}$**.**

This particular triangle is thus **equilateral**.



Example (11): A quadrilateral *ABCD* is shown below, and we are given that sides *AB* and *BC* are equal.

Lengths are in centimetres.

What type of quadrilateral is it ? Justify the answer with full working.

We begin by solving the equation $\mathbf{2x + 5 = 17 – x}$**.**

$2x + 5 = 17 – x \rightarrow 3x + 5 = 17 \rightarrow 3x = 12$

$\rightarrow$ $\mathbf{x = 4}$**.**

We then substitute $x = 4$ into all the side length expressions:

$2x + 5 = 13$, so $AB = 13$ cm, as is $17 – x$, so $BC = 13$ cm as well. (It should be, as we've just solved this equation !)

Now $AD = 4x – 3 = 13$ cm, and $DC = 25 – 3x = 13$ cm.

All four sides of the quadrilateral *ABCD* are equal, hence *ABCD* is a **rhombus.**

(We have no details of the angles, so we cannot assume it's a square.)