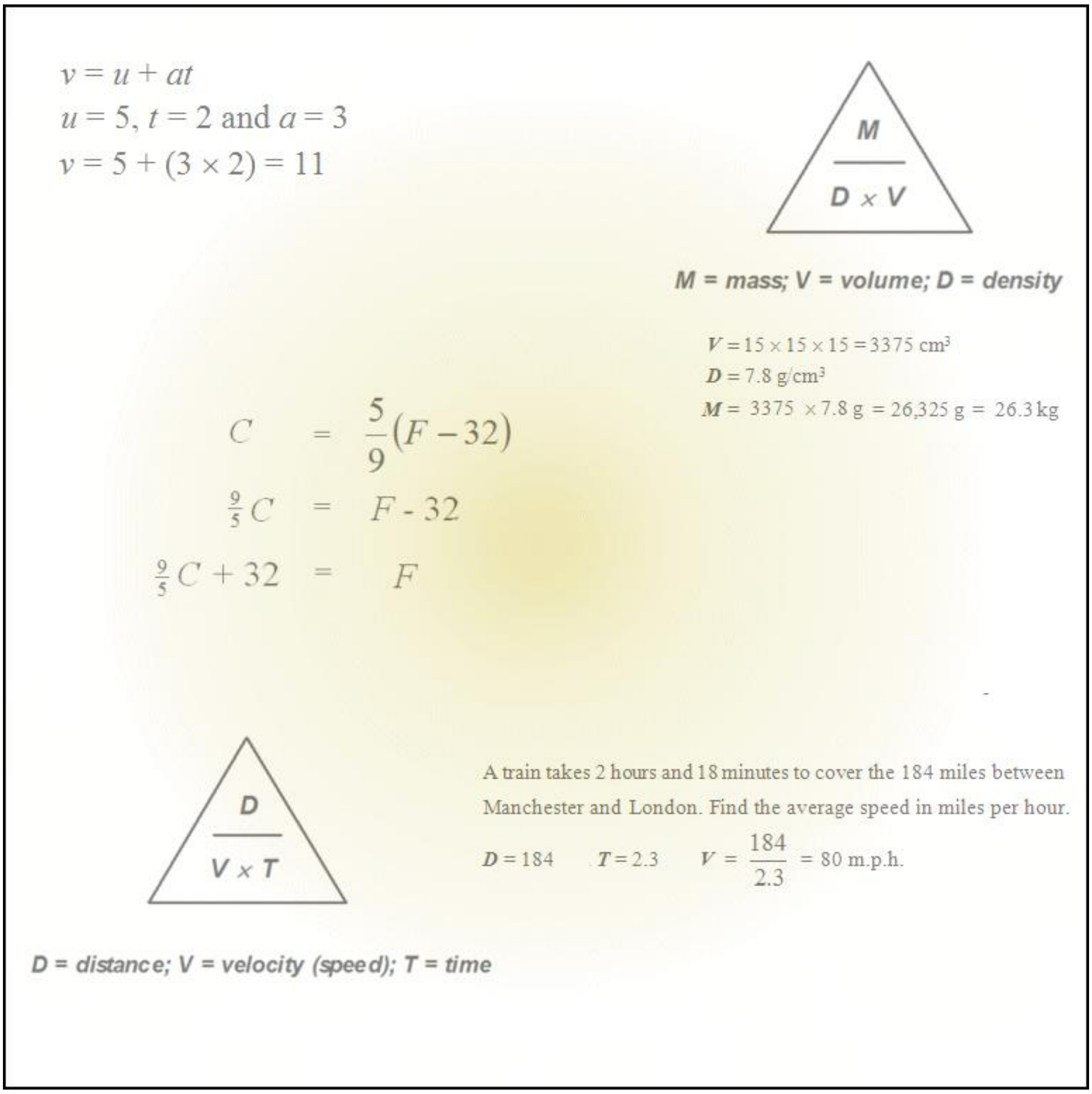
# M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

# FORMULAE

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# FORMULAE

A formula is a group of mathematical symbols, usually letters, expressing a relationship between quantities. Typically, a formula consists of a letter on the left-hand side and an expression on the right-hand side, with an equality sign in between.

Take the relationship between mass, volume and density. Since we obtain the mass by multiplying the density by the volume, we can express this relationship as a formula

$M = DV$ where $M$ represents mass, $D$ represents density and $V$ represents volume.

## Substituting into a formula.

**Example (1a)** : In the formula $v = u + at$, find $v$ when $u = 5$, $t = 2$ and $a = 3$.

The solution is obtained by substituting numbers for the letters in the original formula, thus

$v = 5 + (3 \times 2) = 11$.

Written this way, the formula is ideal for finding $v$ when $u$, $t$ and $a$ are known. The answer to the last question was simply obtained by substituting values. The subject of the formula is $v$, it being the only term on the left-hand side.

## Changing the formula subject.

**Example (1b).**

Supposing we wanted to find $a$, given that $v = 12$, $t = 2$ and $u = 2$. This would require rearranging the formula by changing the subject from $v$ to $a$.

We could substitute for $t$, $u$ and $v$ in the original formula $v = u + at$ to obtain the equation
$2 + 2a = 12$ , rearrange the terms and solve for $a$ : $2 + 2a = 12 \rightarrow 2a = 10 \rightarrow a = 5$.

However, this chapter is about formulae, so we shall take the approach of rearranging the formula by changing the subject from $v$ to $a$.

The rules for changing the subject of a formula are the same as those for equations - just perform the same arithmetic on both sides.

**Importantly, the subject must end up entirely on one side of the formula - you cannot have it referred to on both !**

| | | | |
|---|---|---|---|
| $v$ | = | $u + at$ | |
| $v - u$ | = | $at$ | Subtract $u$ from both sides |
| $\frac{v-u}{t}$ | = | $a$ | Divide both sides by $t$ |

The same formula, but rearranged to make $a$ the subject, is

$$a = \frac{v-u}{t}.$$

The value of $a$ can then be found by simple substitution with $v = 12$, $t = 2$ and $u = 2$.

$$a = \left(\frac{12-2}{2}\right) = 5.$$

**Example (2) :** The formula for the area *A* of a trapezium is $A = \frac{1}{2}(a+b)h$
where *a* and *b* are the parallel sides and *h* is the vertical height of the trapezium.

i) Find the area *A* of the trapezium where *a* = 7 cm, *b* = 13 cm and *h* = 6cm.
ii) Change the subject of the formula from *A* to *b*.
iii) Hence find the length of the other parallel side of a trapezium with an area of 54 $cm^2$, a vertical height of 9 cm, and one of its parallel sides 7 cm long.

i) Substituting for *a* , *b* and *h* we have the area of the trapezium; $A = \frac{1}{2}(7+13)\times 6 = 60$ $cm^2$.

ii) We rearrange the formula by changing the subject from *A* to *b*:

| | | | |
|---|---|---|---|
| $A$ | = | $\frac{1}{2}(a+b)h$ | |
| $2A$ | = | $(a+b)h$ | Double both sides to get rid of the ½ |
| $\frac{2A}{h}$ | = | $a+b$ | Divide both sides by *h* |
| $\frac{2A}{h} - a$ | = | $b$ | Subtract *a* from both sides |

The formula $b = \frac{2A}{h} - a$ now has *b* as its subject after exchanging left and right-hand sides.

iii) Substitute *A* = 54, *h* = 9 and *a* = 7 into the rearranged formula $b = \frac{2A}{h} - a$.

Hence the length of the other parallel side of the trapezium, *b*, is $\frac{2\times 54}{9} - 7 = 5$ cm.

**Finding a formula from requirements.**

**Example (3):** Weather forecasts in the USA use the Fahrenheit scale for temperature, and the rule for converting temperature readings from Fahrenheit to Celsius temperature is

"Subtract 32, then divide by 1.8".

Rewrite this rule as a formula, and use it to convert 77°F to the Celsius scale.

Firstly, we choose letter symbols, say $F$ to represent the Fahrenheit temperature and $C$ to represent Celsius.

Starting with $F$ , we subtract 32 to obtain $F$ – 32. Then we divide by 1.8 to complete the formula

$C = \frac{(F-32)}{1.8}$. Substituting $F$ = 77 gives $C = \frac{(77-32)}{1.8} = \frac{45}{1.8} = 25$.

$\therefore$ **77°F = 25°C.**

**Example (4):** Deduce a formula which converts a temperature in Celsius to one in Fahrenheit, by changing the formula subject. Use the result to convert 15°C to the Fahrenheit scale.

| | | | |
|---|---|---|---|
| $C$ | = | $C = \frac{(F-32)}{1.8}$ | |
| $1.8C$ | = | $F - 32$ | Multiply both sides by 1.8 |
| $1.8C + 32$ | = | $F$ | Add 32 to both sides |

The formula to convert Celsius to Fahrenheit temperature is therefore $F = 1.8C + 32$.

Substituting $C$ = 15 gives $F$ = (1.8 × 15) + 32 = 27 + 32 = 59.

$\therefore$ **15°C = 59°F.**

**Example (5):** A bank offers to exchange pounds sterling for euros. If the market euro rate is currently £1 = €1.16, i) write a formula that connects the pounds spent with euros received, and hence find out how many euros can be obtained for £150, and ii) write a formula to convert euros back to sterling at the same rate, and hence convert €203 back to sterling.

Let $P$ denote the number of pounds spent and $E$ the number of euros received.

i) Since £1 = €1.16, the required formula is $E = 1.16P$.
Substituting $P$ = 150, £150 will be converted into 1.16 × 150 euros, or €174.

ii) Since $E = 1.16P$, the formula can be rewritten with $P$ as the subject by dividing both sides by 1.16.

Hence $1.16P = E$ , and dividing by 1.16 gives $P = \frac{E}{1.16}$ .

Thus, €203 = £$\frac{203}{1.16}$ = £175.

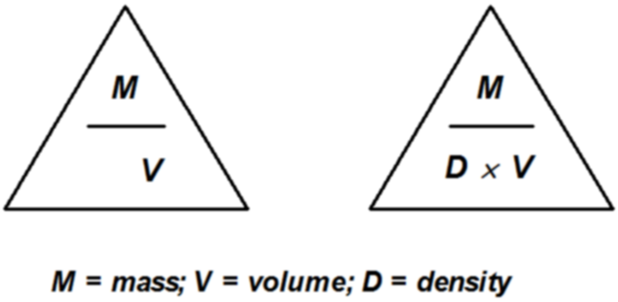
**The Formula Triangle.**

If a formula has three variables, where one is a product or a quotient of the other two, it is useful to illustrate the relationship by a **formula triangle**. The last example, on currency conversion, is a case in point. The next examples also feature strongly in science, especially physics.

**Example (6):** Density is measured by dividing the mass of a substance by its volume, i.e. $D = \frac{M}{V}$.

Draw the formula triangle to show the relationship.

Using the symbols ***D*** for density, ***M*** for mass and ***V*** for volume, we first put the quotient into the empty triangle below left, and then complete the triangle by putting ***D*** and the × sign on the bottom row.
We can now use the completed formula triangle as follows:




If we need to find the mass, we cover up the ***M*** to leave the required formula ***D*** × ***V***.

To find the density, we cover up the ***D*** and the × sign to leave ***M/V***.

To find the volume, we cover up the ***V*** and the × sign to leave ***M/D***.

The next two examples make use of the following densities:

Iron: 7.8 g/cm$^3$ Lead: 11.4 g/cm$^3$ Gold: 19.3 g/cm$^3$

**Example (7):** Find the mass of an iron cube of side 15 cm. Give your answer in kilograms to the nearest 0.1 kg.

The volume ***V*** of the block is 15 × 15 × 15 = 3375 cm$^3$ and its density ***D*** is 7.8 g/cm$^3$.
Covering up ***M*** on the formula triangle leaves us with ***D*** × ***V*** which works out at

3375 × 7.8 g, or 26,325 g. In kilograms, this is 26.3 kg to the nearest 0.1 kg.

**Example (8):** A gold ingot was stolen from a display at a museum. Its mass was 41 kg and its dimensions were 30 cm × 12 cm × 10 cm. Was this ingot actually made of gold ?

The volume ***V*** of the ingot was 30 × 12 × 10 = 3600 cm$^3$ and its mass ***M*** was 41000 g (remember to convert kg to g by multiplying by 1000).

Covering up ***D*** on the formula triangle leaves us with ***M/V*** which works out at
$\frac{41000}{3600}$ or 11.4 g/cm$^3$.

The density of the ingot was 11.4 g/cm$^3$ - the 'gold' ingot was actually made of lead !

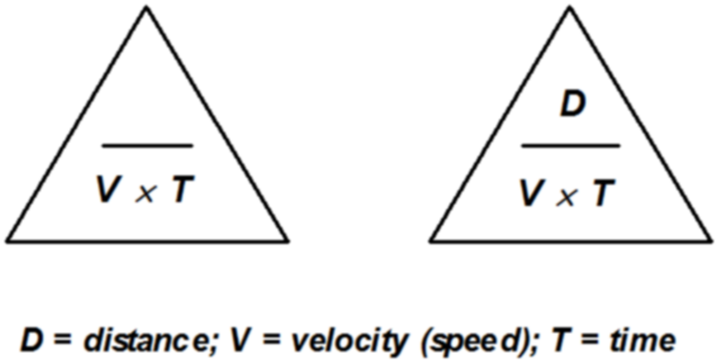
**Example (9):** Distance is measured by multiplying speed (velocity) by time, i.e ***D*** = ***VT***.
Draw the formula triangle to show the relationship.

Using the symbols ***D*** for distance, ***V*** for speed and ***T*** for time, we first put the product into the empty triangle below left, and then complete the triangle by putting ***D*** and the division bar above it .

To find the distance, we cover up the ***D*** to leave the required formula ***V*** × ***T***.

To find the speed (velocity), we cover up the ***V*** and the × sign to leave ***D/T***.

To find the time, we cover up the ***T*** and the × sign to leave ***D/V***.




**Example (10):** A train takes 2 hours and 18 minutes to cover the 184 miles between Manchester and London. Find the average speed in miles per hour.

Here we have ***D*** = 184 and ***T*** = 2.3 (remember that 18 minutes = 0.3 hours) and we require ***V***.

We therefore cover up ***V*** to be left with ***D/T*,** which works out at $\frac{184}{2.3}$ or 80 m.p.h.

**Example (11):** Another train takes 2 hours and 6 minutes to cover the distance between London and York at an average speed of 90 miles per hour. What is the distance between London and York ?

The time ***T* =** 2.1 hours and the speed ***V* =** 90, but we require ***D***.
Covering up ***D*** on the formula triangle leaves us with ***T*** × ***V*** which works out at 2.1 × 90, or 189.

The distance between London and York is therefore 189 miles.