

M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

FORMULAE

$$v = u + at$$

$$u = 5, t = 2 \text{ and } a = 3$$

$$v = 5 + (3 \times 2) = 11$$



M = mass; V = volume; D = density

$$V = 15 \times 15 \times 15 = 3375 \text{ cm}^3$$

$$D = 7.8 \text{ g/cm}^3$$

$$M = 3375 \times 7.8 \text{ g} = 26,325 \text{ g} = 26.3 \text{ kg}$$

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F$$



A train takes 2 hours and 18 minutes to cover the 184 miles between Manchester and London. Find the average speed in miles per hour.

$$D = 184 \quad T = 2.3 \quad V = \frac{184}{2.3} = 80 \text{ m.p.h.}$$

D = distance; V = velocity (speed); T = time

FORMULAE

A formula is a group of mathematical symbols, usually letters, expressing a relationship between quantities. Typically, a formula consists of a letter on the left-hand side and an expression on the right-hand side, with an equality sign in between.

Take the relationship between mass, volume and density. Since we obtain the mass by multiplying the density by the volume, we can express this relationship as a formula

$M = DV$ where M represents mass, D represents density and V represents volume.

Another example is $A = \pi r^2$ for the area of a circle, where r is the radius and π is a constant number whose value is approximately 3.14 or $\frac{22}{7}$.

Substituting into a formula.

Example (1) : In the formula $v = u + at$, find v when $u = 5$, $t = 2$ and $a = 3$.

The solution is obtained by substituting numbers for the letters in the original formula, thus

$$v = 5 + (3 \times 2) = 11.$$

Written this way, the formula is ideal for finding v when u , t and a are known. The answer to the last question was simply obtained by substituting values. The subject of the formula is v , it being the only term on the left-hand side.

Changing the formula subject.

Example (2).

Supposing we wanted to find a , given that $v = 12$, $t = 2$ and $u = 2$. This would require rearranging the formula by changing the subject from v to a .

The rules for changing the subject of a formula are the same as those for equations - just perform the same arithmetic on both sides.

Importantly, the subject must end up entirely on one side of the formula - you cannot have it referred to on both !

$$\begin{array}{rcl} v & = & u + at \\ v - u & = & at & \text{Subtract } u \text{ from both sides} \\ \frac{v - u}{t} & = & a & \text{Divide both sides by } t \end{array}$$

The same formula, but rearranged to make a the subject, is

$$a = \frac{v - u}{t} .$$

The value of a can then be found by simple substitution with $v = 12$, $t = 2$ and $u = 2$.

$$a = \left(\frac{12 - 2}{2} \right) = 5.$$

Alternatively, we could have substituted numbers for letters in the original formula to turn it into an equation:

$$v = u + at \rightarrow 12 = 2 + 2a \quad (\text{substituting for } v, u \text{ and } t).$$

$$\text{From there, } 2 + 2a = 12 \rightarrow 2a = 10 \rightarrow a = 5.$$

Finding a formula from requirements.

Example (3): Weather forecasts in the USA use the Fahrenheit scale for temperature, and the rule for converting temperature readings from Fahrenheit to Celsius temperature is

“Subtract 32, then divide by 1.8”.

Rewrite this rule as a formula, and use it to convert 77°F to the Celsius scale.

Firstly, we choose letter symbols, say F to represent the Fahrenheit temperature and C to represent Celsius.

Starting with F , we subtract 32 to obtain $F - 32$. Then we divide by 1.8 to complete the formula

$$C = \frac{(F - 32)}{1.8}. \text{ Substituting } F = 77 \text{ gives } C = \frac{(77 - 32)}{1.8} = \frac{45}{1.8} = 25.$$

$$\therefore 77^\circ\text{F} = 25^\circ\text{C}.$$

Example (4): Deduce a formula which converts a temperature in Celsius to one in Fahrenheit, by changing the formula subject. Use the result to convert 15°C to the Fahrenheit scale.

$$\begin{aligned} C &= \frac{(F - 32)}{1.8} \\ 1.8C &= F - 32 && \text{Multiply both sides by 1.8} \\ 1.8C + 32 &= F && \text{Add 32 to both sides} \end{aligned}$$

The formula to convert Celsius to Fahrenheit temperature is therefore $F = 1.8C + 32$.

Substituting $C = 15$ gives $F = (1.8 \times 15) + 32 = 27 + 32 = 59$.

$$\therefore 15^\circ\text{C} = 59^\circ\text{F}.$$

Example (5): A bank offers to exchange pounds sterling for euros. If the market euro rate is currently £1 = €1.16, i) write a formula that connects the pounds spent with euros received, and hence find out how many euros can be obtained for £150, and ii) write a formula to convert euros back to sterling at the same rate, and hence convert €203 back to sterling.

Let P denote the number of pounds spent and E the number of euros received.

i) Since £1 = €1.16, the required formula is $E = 1.16P$.

Substituting $P = 150$, £150 will be converted into 1.16×150 euros, or €174.

ii) Since $E = 1.16P$, the formula can be rewritten with P as the subject by dividing both sides by 1.16.

Hence $1.16P = E$, and dividing by 1.16 gives $P = \frac{E}{1.16}$.

Thus, €203 = £ $\frac{203}{1.16} = \text{£}175$.

The Formula Triangle.

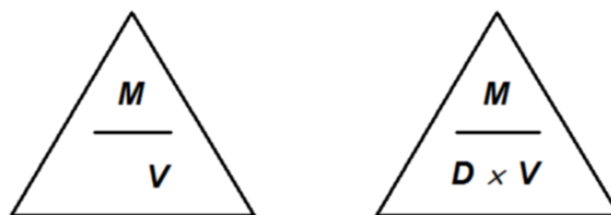
If a formula has three variables, where one is a product or a quotient of the other two, it is useful to illustrate the relationship by a **formula triangle**. The last example, on currency conversion, is a case in point. The next examples also feature strongly in science, especially physics.

Example (6): Density is measured by dividing the mass of a substance by its volume, i.e. $D = \frac{M}{V}$.

Draw the formula triangle to show the relationship.

Using the symbols **D** for density, **M** for mass and **V** for volume, we first put the quotient into the empty triangle below left, and then complete the triangle by putting **D** and the \times sign on the bottom row.

We can now use the completed formula triangle as follows:



M = mass; V = volume; D = density

If we need to find the mass, we cover up the **M** to leave the required formula $D \times V$.

To find the density, we cover up the **D** and the \times sign to leave M/V .

To find the volume, we cover up the **V** and the \times sign to leave M/D .

The next two examples make use of the following densities:

Iron: 7.8 g/cm^3 Lead: 11.4 g/cm^3 Gold: 19.3 g/cm^3

Example (7): Find the mass of an iron cube of side 15 cm. Give your answer in kilograms to the nearest 0.1 kg.

The volume **V** of the block is $15 \times 15 \times 15 = 3375 \text{ cm}^3$ and its density **D** is 7.8 g/cm^3 .
Covering up **M** on the formula triangle leaves us with $D \times V$ which works out at

$3375 \times 7.8 \text{ g}$, or $26,325 \text{ g}$. In kilograms, this is 26.3 kg to the nearest 0.1 kg.

Example (8): A gold ingot was stolen from a display at a museum. Its mass was 41 kg and its dimensions were $30 \text{ cm} \times 12 \text{ cm} \times 10 \text{ cm}$. Was this ingot actually made of gold ?

The volume **V** of the ingot was $30 \times 12 \times 10 = 3600 \text{ cm}^3$ and its mass **M** was 41000 g (remember to convert kg to g by multiplying by 1000).

Covering up **D** on the formula triangle leaves us with M/V which works out at

$\frac{41000}{3600}$ or 11.4 g/cm^3 .

The density of the ingot was 11.4 g/cm^3 - the 'gold' ingot was actually made of lead !

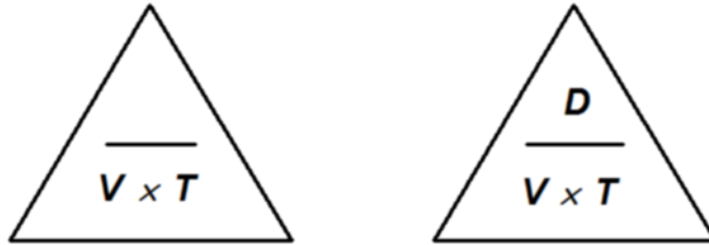
Example (9): Distance is measured by multiplying speed (velocity) by time, i.e $D = VT$.
Draw the formula triangle to show the relationship.

Using the symbols D for distance, V for speed and T for time, we first put the product into the empty triangle below left, and then complete the triangle by putting D and the division bar above it .

To find the distance, we cover up the D to leave the required formula $V \times T$.

To find the speed (velocity), we cover up the V and the \times sign to leave D/T .

To find the time, we cover up the T and the \times sign to leave D/V .



$D = \text{distance}; V = \text{velocity (speed)}; T = \text{time}$

Example (10): A train takes 2 hours and 18 minutes to cover the 184 miles between Manchester and London. Find the average speed in miles per hour.

Here we have $D = 184$ and $T = 2.3$ (remember that 18 minutes = 0.3 hours) and we require V .

We therefore cover up V to be left with D/T , which works out at $\frac{184}{2.3}$ or 80 m.p.h.

Example (11): Another train takes 2 hours and 6 minutes to cover the distance between London and York at an average speed of 90 miles per hour. What is the distance between London and York ?

The time $T = 2.1$ hours and the speed $V = 90$, but we require D .

Covering up D on the formula triangle leaves us with $T \times V$ which works out at 2.1×90 , or 189.

The distance between London and York is therefore 189 miles.