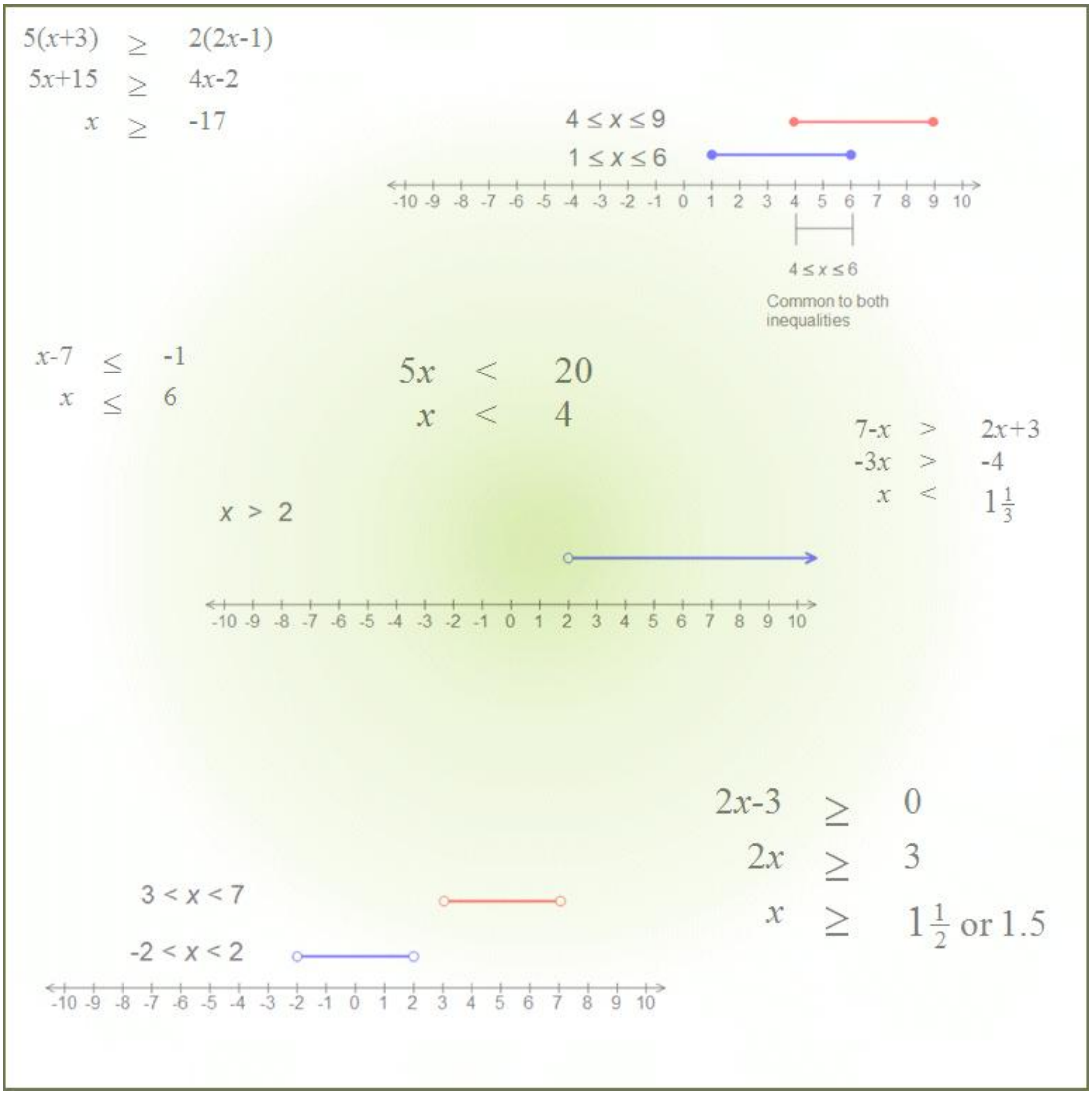
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

LINEAR INEQUALITIES



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LINEAR INEQUALITIES

Inequalities, like equations, connect two expressions involving definite unknown values.

An inequality must always include one of the following signs:
$>$ (greater than)
$<$ (less than)
$\geq$ (greater than or equal to)
$\leq$ (less than or equal to).

The symbols always open out towards the larger number: thus $7 > 5$.

Inequalities involving $<$ and $>$ signs are known as **strict** inequalities.

Inequalities are solved in the same way as equations, but with an important difference in detail. The **sign must be reversed** when multiplying or dividing by a negative number, or when turning the inequality around.

Examples (1): Solve the inequalities i) $5x < 20$; ii) $x-7 \leq -1$; iii) $2x - 3 \geq 0$

5*x* < 20

$5x$	$<$	20	
x	$<$	4	Divide both sides by 5

***x*-7 ≤ -1**

$x-7$	$\leq$	-1	
x	$\leq$	6	Add 7 to both sides

2*x* – 3 ≥ 0

$2x-3$	$\geq$	0	
$2x$	$\geq$	3	Add 3 to both sides
x	$\geq$	$1\frac{1}{2}$ or 1.5	Divide both sides by 2

Examples (2):

Solve the inequalities i) $5(x+3) \geq 2(2x-1)$; ii) $7-x > 2x+3$

5(*x*+3) ≥ 2(2*x*- 1)

$5(x+3)$	$\geq$	$2(2x-1)$	
$5x+15$	$\geq$	$4x-2$	Expand both sides
x	$\geq$	-17	Subtract $15 + 4x$ from each side

7-*x* > 2*x*+ 3

The final step here involves dividing both sides of the inequality by a negative number.

$7-x$	$>$	$2x+3$	
$-3x$	$>$	-4	Subtract $(2x + 7)$ from each side
x	$<$	$1\frac{1}{3}$	Divide both sides by -3 and **reverse the inequality sign**

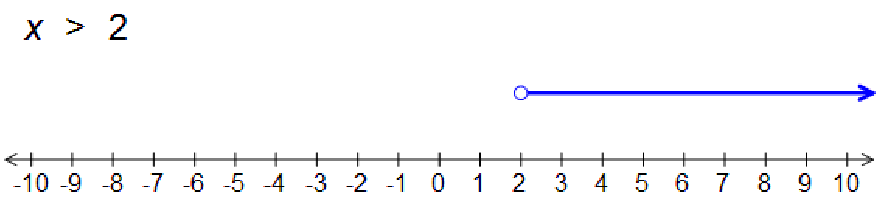
Inequalities on the number line.

Inequalities can be illustrated on the number line as follows:

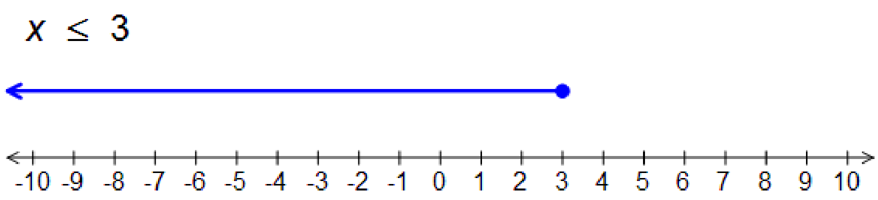
Examples (3): Illustrate the following inequalities on the number line: i) $x > 2$; ii) $x \leq 3$; iii) $-5 < x < 6$.

i) The inequality $x > 2$ is shown below.

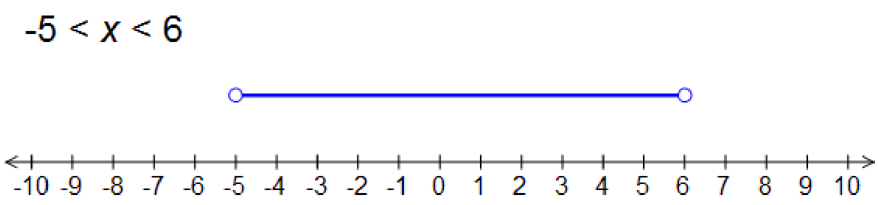
The arrow at the end is to signify that the range of x extends to (positive) infinity; the **outline** circle at $x = 2$ is to show that the inequality is a strict one, and that $x = 2$ does not satisfy it..



ii) For $x \leq 3$, there are two differences from the last case. The arrow at the end goes towards negative infinity; also the circle at $x = 3$ is shown as a **solid dot,** to show that the inequality is not a strict one, so that $x = 3$ does satisfy it.

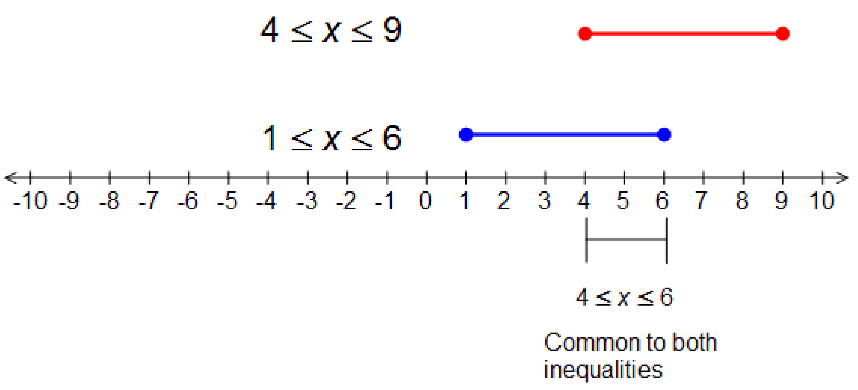


iii) The last two inequalities were open-ended, extending to infinity in one direction.
For $-5 < x < 6$, we now have two ends to the inequality, at –5 and 6. (This is a strict inequality, so the circles are shown outlined).



Example (4): Show the inequalities $1 \le x \le 6$ and $4 \le x \le 9$ on the same number line. Do any values of x satisfy both inequalities ?.

There is a region of overlap; when x is between 4 and 6 inclusive, both inequalities are satisfied. Note the solid dots; the inequality is not strict.



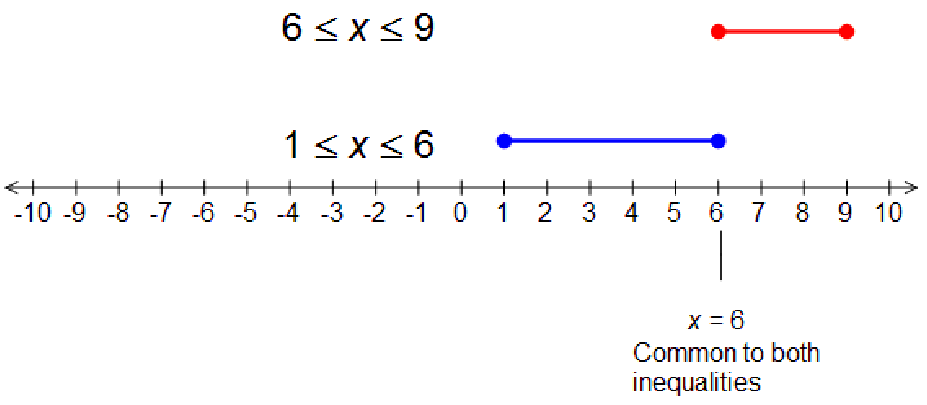
The set of values satisfying both inequalities is therefore $4 \le x \le 6$.

Example (5):

i) Show the inequalities $1 \le x \le 6$ and $6 \le x \le 9$ on the same number line. Do any values of x satisfy both inequalities ?. What if either inequality had been strict ?

ii) State all the integer values of x satisfying the inequality $6 \le x \le 9$.

i) The inequalities overlap at one point here, namely at $x = 6$, which is the only value of x satisfying both .
Had either inequality been strict, there would have been **no** value satisfying both, as at least one of them would not have included 6 in the allowable values for x.

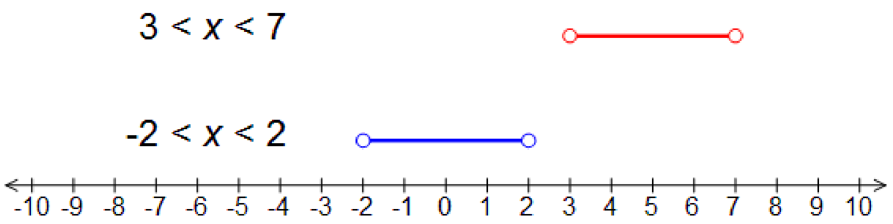


ii) The integer values of x satisfying the inequality $6 \le x \le 9$ are 6, 7, 8 and 9.

Example (6):

i) Show the inequalities -2 < x < 2 and 3 < x < 7 on the same number line. Do any values of x satisfy both inequalities ?

ii) State all the integer values of x satisfying **either** inequality.



The inequalities do not overlap at all , so no value of x satisfies both .

The integers satisfying -2 < x < 2 are -1, 0 and 1; those satisfying 3 < x < 7 are 4, 5 and 6.