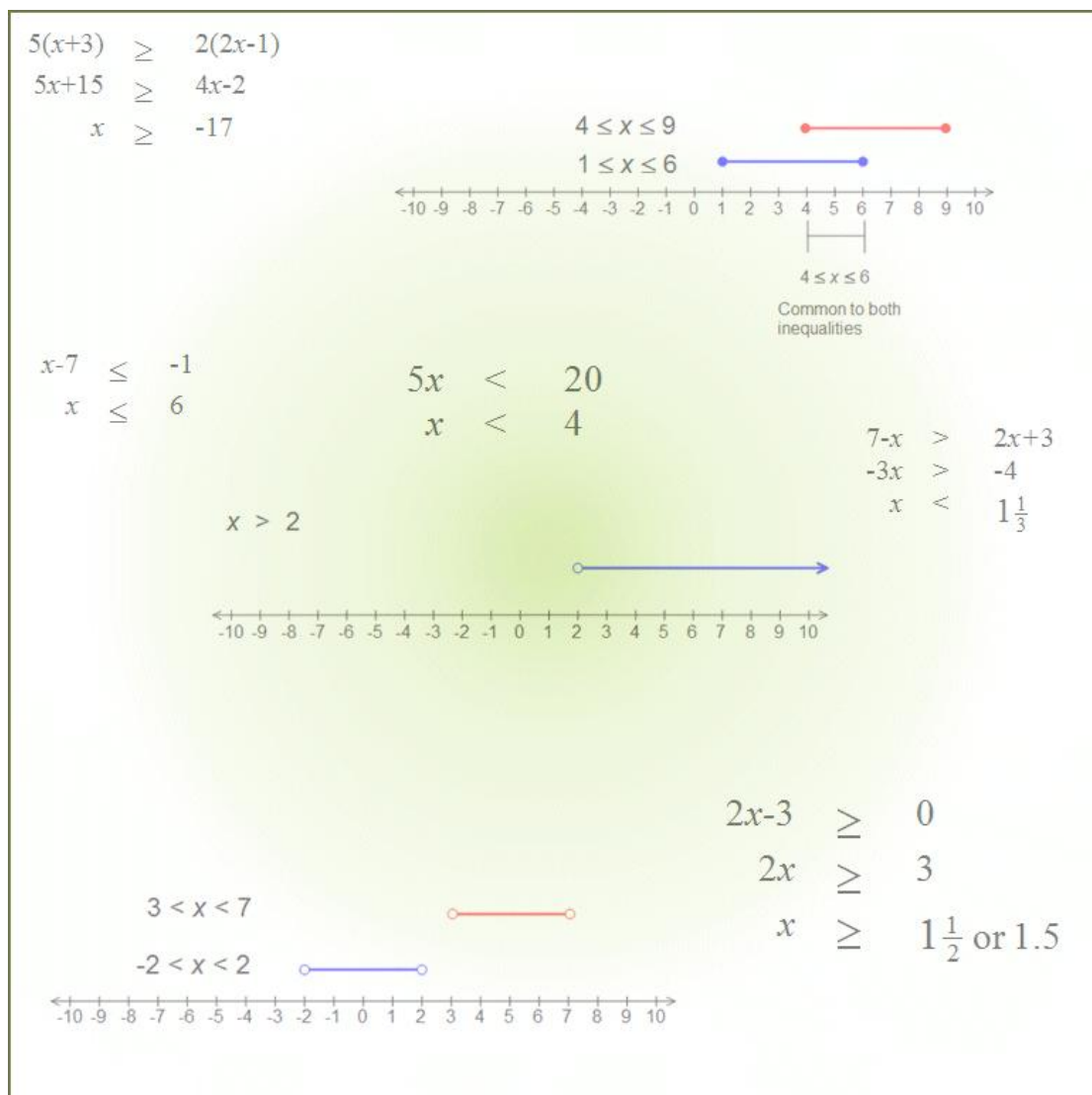


M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

LINEAR INEQUALITIES



LINEAR INEQUALITIES

Inequalities, like equations, connect two expressions involving definite unknown values.

An inequality must always include one of the following signs:

> (greater than)

< (less than)

≥ (greater than or equal to)

≤ (less than or equal to).

The symbols always open out towards the larger number: thus $7 > 5$.

Inequalities involving < and > signs are known as **strict** inequalities.

Inequalities are solved in the same way as equations, but with an important difference in detail. The **sign must be reversed** when multiplying or dividing by a negative number, or when turning the inequality around.

Examples (1): Solve the inequalities i) $5x < 20$; ii) $x-7 \leq -1$; iii) $2x - 3 \geq 0$

$5x < 20$

$$\begin{array}{rcl} 5x & < & 20 \\ x & < & 4 \end{array}$$

Divide both sides by 5

$x-7 \leq -1$

$$\begin{array}{rcl} x-7 & \leq & -1 \\ x & \leq & 6 \end{array}$$

Add 7 to both sides

$2x - 3 \geq 0$

$$\begin{array}{rcl} 2x-3 & \geq & 0 \\ 2x & \geq & 3 \\ x & \geq & 1\frac{1}{2} \text{ or } 1.5 \end{array}$$

Add 3 to both sides

Divide both sides by 2

Examples (2):

Solve the inequalities i) $5(x+3) \geq 2(2x-1)$; ii) $7-x > 2x+3$

$5(x+3) \geq 2(2x-1)$

$$\begin{array}{rcl} 5(x+3) & \geq & 2(2x-1) \\ 5x+15 & \geq & 4x-2 \\ x & \geq & -17 \end{array}$$

Expand both sides

Subtract $15 + 4x$ from each side

$7-x > 2x+3$

The final step here involves dividing both sides of the inequality by a negative number.

$$\begin{array}{rcl} 7-x & > & 2x+3 \\ -3x & > & -4 \\ x & < & 1\frac{1}{3} \end{array}$$

Subtract $(2x + 7)$ from each side

Divide both sides by -3 and **reverse the inequality sign**

Inequalities on the number line.

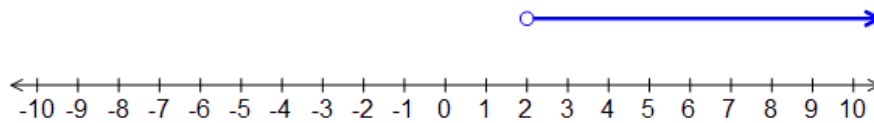
Inequalities can be illustrated on the number line as follows:

Examples (3): Illustrate the following inequalities on the number line: i) $x > 2$; ii) $x \leq 3$;
iii) $-5 < x < 6$.

i) The inequality $x > 2$ is shown below.

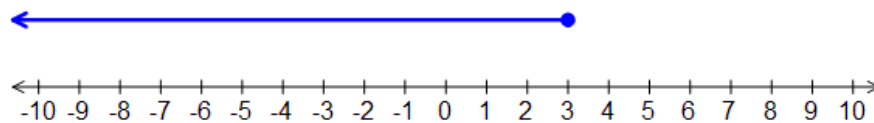
The arrow at the end is to signify that the range of x extends to (positive) infinity; the **outline** circle at $x = 2$ is to show that the inequality is a strict one, and that $x = 2$ does not satisfy it.

$$x > 2$$



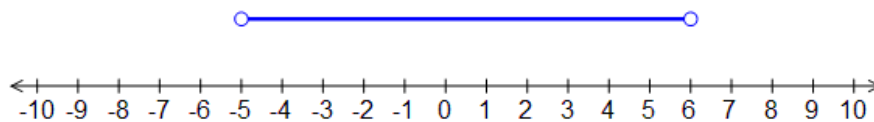
ii) For $x \leq 3$, there are two differences from the last case. The arrow at the end goes towards negative infinity; also the circle at $x = 3$ is shown as a **solid dot**, to show that the inequality is not a strict one, so that $x = 3$ does satisfy it.

$$x \leq 3$$



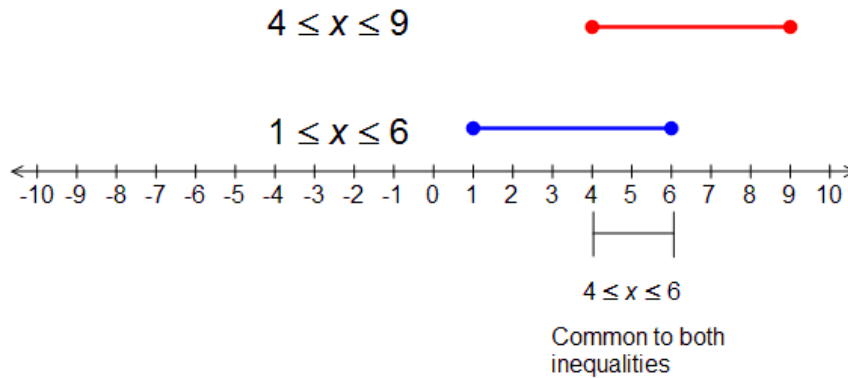
iii) The last two inequalities were open-ended, extending to infinity in one direction. For $-5 < x < 6$, we now have two ends to the inequality, at -5 and 6 . (This is a strict inequality, so the circles are shown outlined).

$$-5 < x < 6$$



Example (4): Show the inequalities $1 \leq x \leq 6$ and $4 \leq x \leq 9$ on the same number line. Do any values of x satisfy both inequalities ?.

There is a region of overlap; when x is between 4 and 6 inclusive, both inequalities are satisfied. Note the solid dots; the inequality is not strict.



The set of values satisfying both inequalities is therefore $4 \leq x \leq 6$.

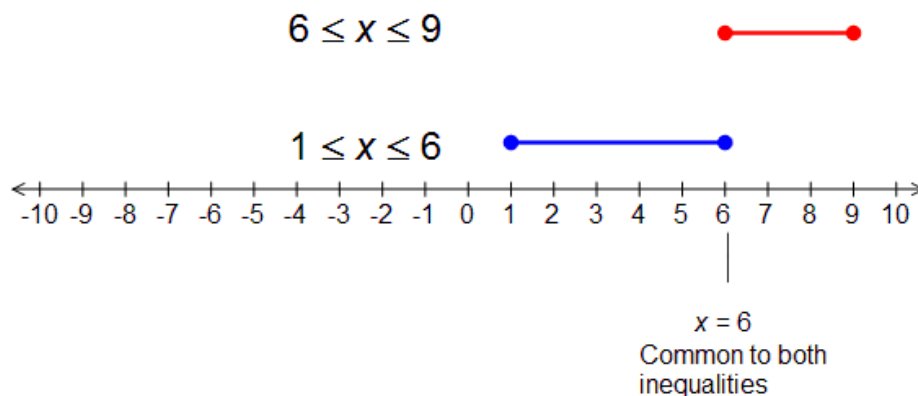
Example (5):

i) Show the inequalities $1 \leq x \leq 6$ and $6 \leq x \leq 9$ on the same number line. Do any values of x satisfy both inequalities ? . What if either inequality had been strict ?

ii) State all the integer values of x satisfying the inequality $6 \leq x \leq 9$.

i) The inequalities overlap at one point here, namely at $x = 6$, which is the only value of x satisfying both .

Had either inequality been strict, there would have been **no** value satisfying both, as at least one of them would not have included 6 in the allowable values for x .

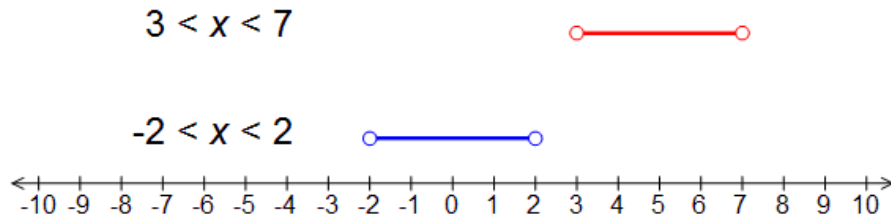


ii) The integer values of x satisfying the inequality $6 \leq x \leq 9$ are 6, 7, 8 and 9.

Example (6):

i) Show the inequalities $-2 < x < 2$ and $3 < x < 7$ on the same number line. Do any values of x satisfy both inequalities ?

ii) State all the integer values of x satisfying **either** inequality.



The inequalities do not overlap at all , so no value of x satisfies both .

The integers satisfying $-2 < x < 2$ are -1, 0 and 1; those satisfying $3 < x < 7$ are 4, 5 and 6.