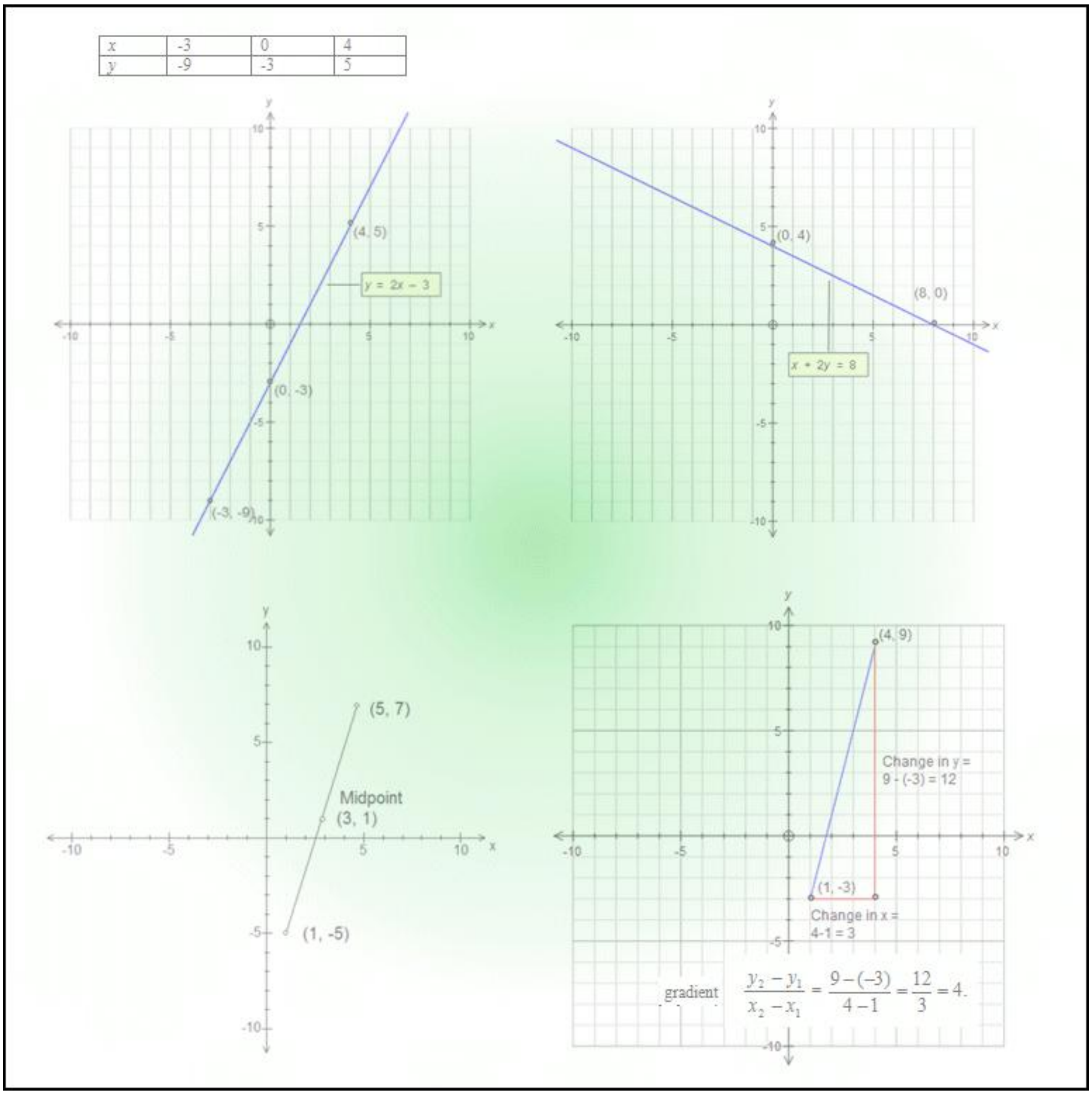
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

STRAIGHT LINE GRAPHS



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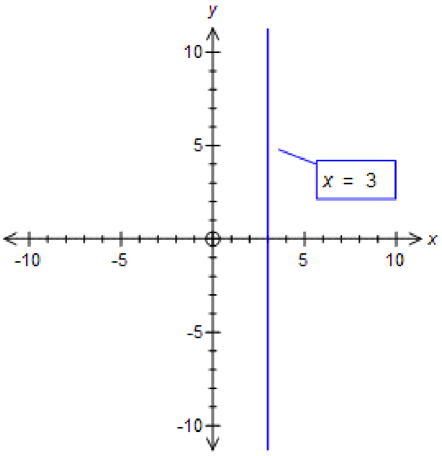
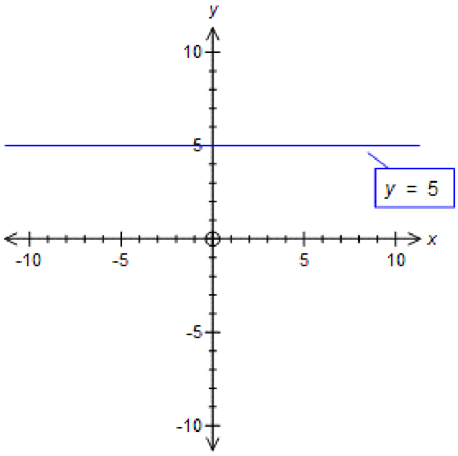
STRAIGHT LINE GRAPHS

A function's graph is a straight line if x and y are related linearly; in other words there are no terms in x^2, $\frac{1}{x}$, x^3 or any other powers or functions of x.

Therefore the graphs of $y = 2x - 3$, $y = 1 - 4x$, $x + 3y = 5$ and $3y = 5x$ are all linear, even though their equations appear different in form.

The simplest graphs are shown below.





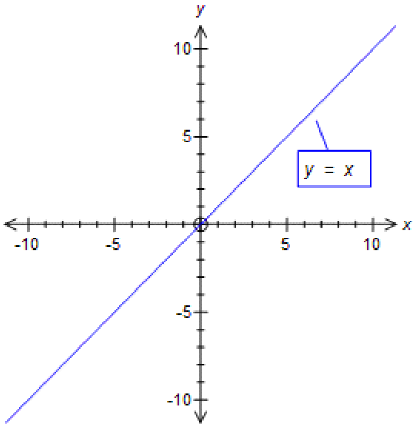
The graphs above are of *constant* functions; in the first one, x is always equal to 3 regardless of the value of y, but in the second one, y always equals 5, no matter what value x takes.

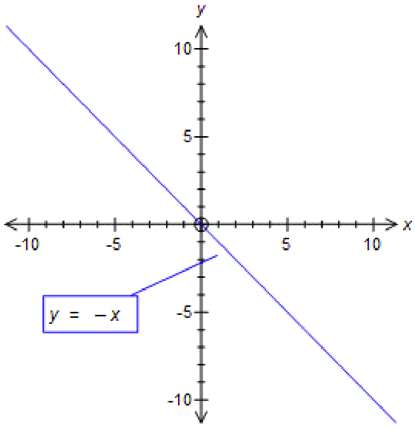
In general, any constant graph of $x = k$ is parallel to the y-axis and k units away from it.
Similarly, any constant graph of $y = k$ is parallel to the x-axis and k units away from it.

The graph of $x = 0$ coincides with the y-axis; that of $y = 0$ coincides with the x-axis.

The next pair of graphs are the main diagonals $y = x$ and $y = -x$.
Notice how each passes through the origin; also notice the slope or gradient of each.

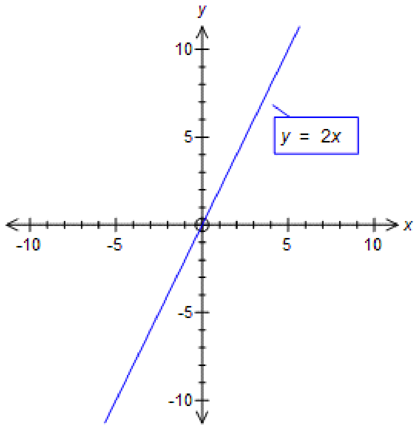
When $y = x$, the graph shows a **positive gradient**, i.e. ***y* increases as *x* increases**.
When $y = -x$, the graph shows a **negative gradient**, i.e. ***y* decreases as *x* increases**.

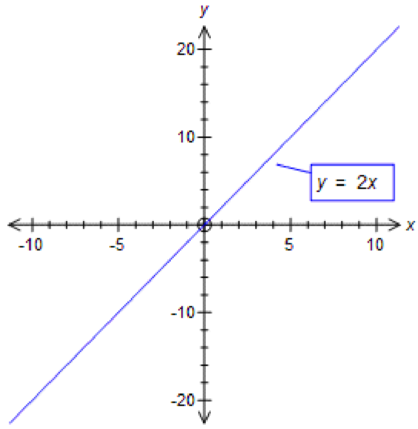




The next sets of graphs also pass through the origin, but they are of the form $y = kx$ where k is a non-zero constant.

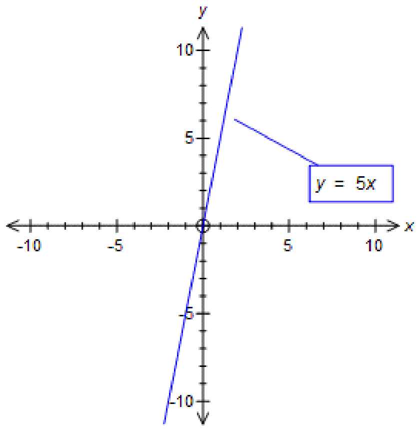
Two graphs of $y = 2x$ are shown below.

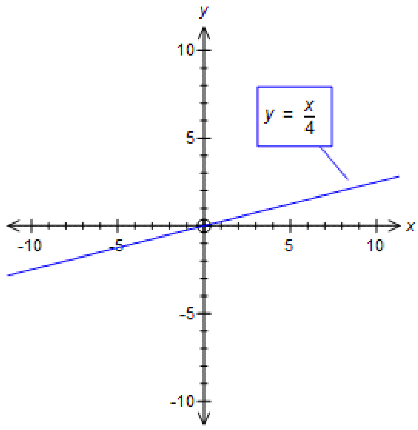




The graph on the left is steeper than that of $y = x$, but the one on the right appears different. This is because the scales of the axes are not uniform. It is important to bear this in mind in later sections, particularly when finding gradients.

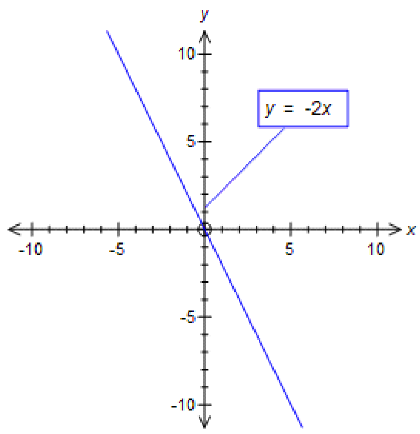
For any graph of $y = kx$ on a uniform scale, where k is positive, the larger k is, the steeper the graph. Conversely, when k is positive and less than 1, the shallower the resulting graph.

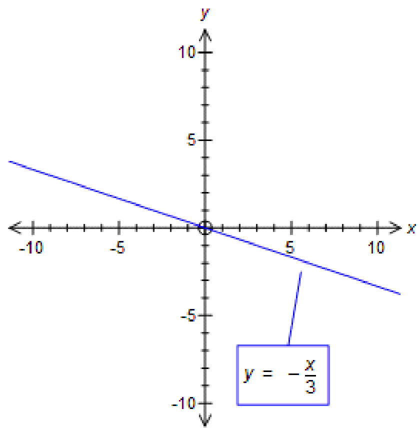




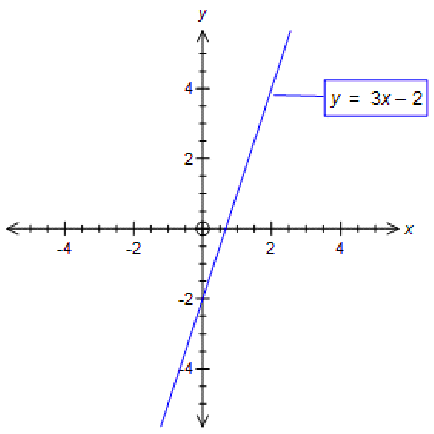
Notice how the gradients of the graphs are still positive; as x increases, so does y.

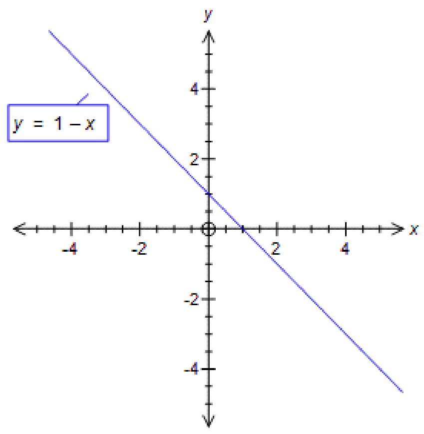
When the value of k for a graph of $y = kx$ happens to be negative, the overall results are similar to the previous examples, but this time the gradients of the graphs are negative.





All of the straight line graphs so far have passed through the origin. However, the vast majority do not !





The graph of $y = 3x - 2$ (above left) does not pass through the origin, but appears to cross the y-axis at the point (0, -2). It also has a positive slope as might be expected of the positive multiple of x, here 3.

The graph of $y = 1 - x$ (above right) appears to cross the y-axis at the point (0, 1). It also has a negative slope, given the negative multiple of x, here -1.

The point where a linear graph cuts the y-axis is also known as the y-intercept, or simply the intercept.

The point where the graph cuts the x-axis is sometimes called the x-intercept, but is more often called the root (as in the solution of an equation).

Plotting a Linear Graph.

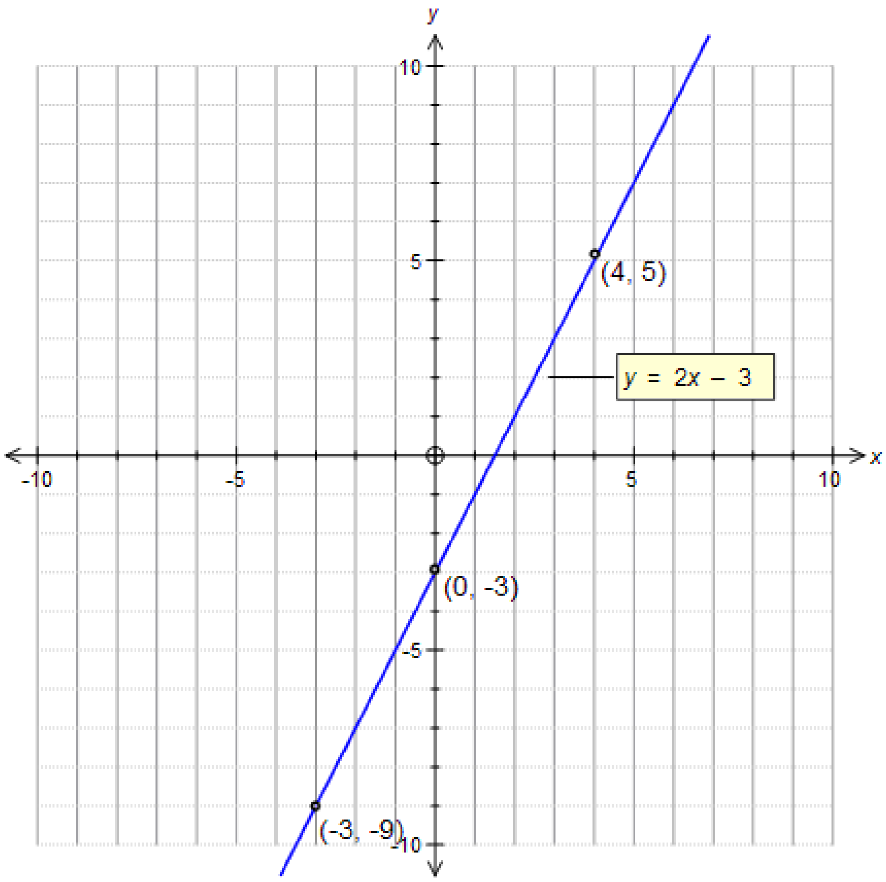
There are two main ways of plotting a graph of a linear function.
We can produce a table of three values by substituting certain values for x, as in the examples below.

Example (1): Plot the graph of $y = 2x – 3$ for values of x between –3 and 5.

We first substitute three values of x into the equation and tabulate the result:

x	-3	0	4
y	-9	-3	5

Then, we plot the three points and draw the straight line passing through them, as in the diagram below.

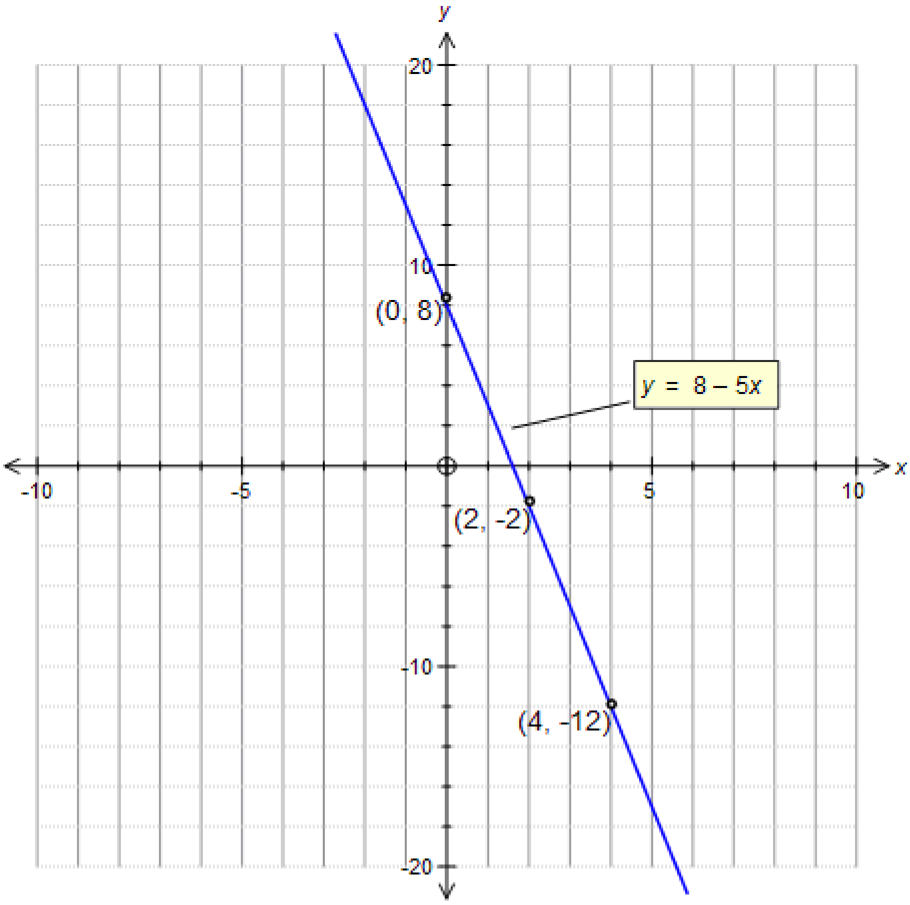


Although two points are sufficient to define a linear graph, it is good practice to plot three, because it would show up any errors in calculating the values of y.

If the three points are not on a straight line, then it is time to check the calculated values of y !

Example (2): Plot the graph of $y = 8 - 5x$ for values of x between –1 and 4.

x	0	2	4
y	8	-2	-12

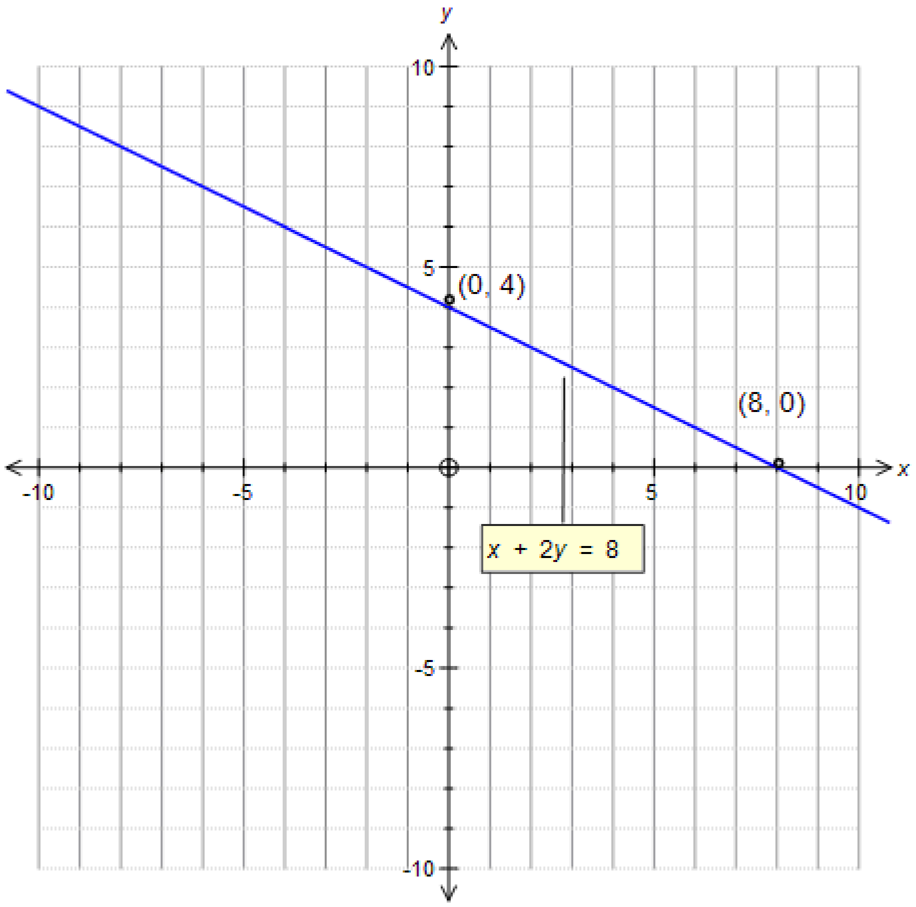


Sometimes the equation might be given in a different form, such as the next example:

Example (3): Plot the graph of $x + 2y = 8$ for values of x between –5 and 15.

Here the easier method of plotting the graph is to substitute $x = 0$ and $y = 0$ into the equation.
When $x = 0$, $2y = 8$, and $y = 4$, so we plot the point (0, 4) – the y-intercept.
When $y = 0$, $x = 8$ and therefore we plot the point (8, 0) – the x-intercept.

Joining the two points gives the graph below.



The Gradient of a Line.

To find the gradient of a linear graph, all we need is to choose two points on it.

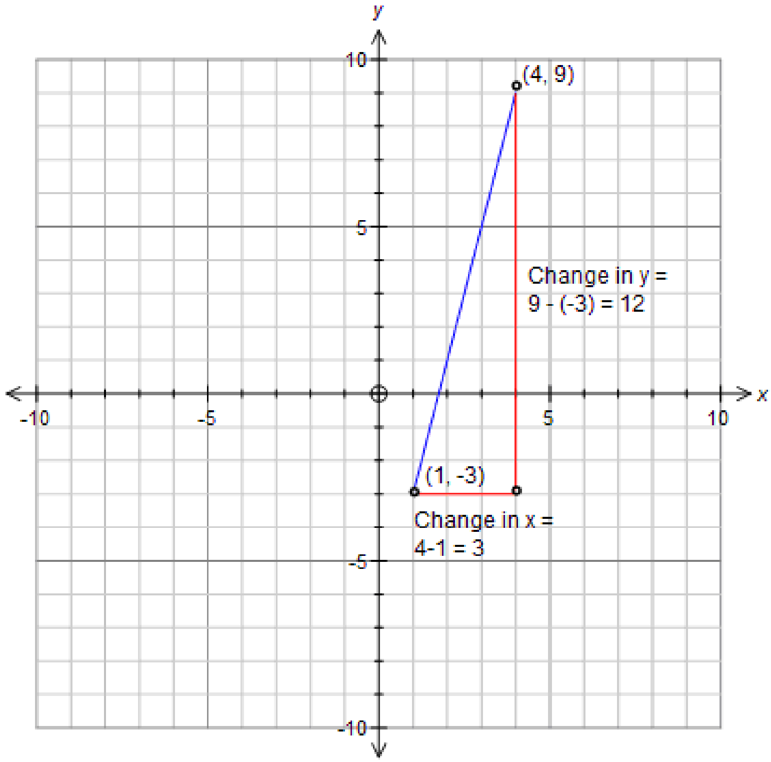
The gradient of a line connecting two points (x_1, y_1) and (x_2, y_2) is given by the formula $\frac{y_2 - y_1}{x_2 - x_1}$ - it is the change in the value of y divided by the change in the value of x.

Example (4): Find the gradient of the line passing through the points (1, -3) and (4, 9).

Taking (1, -3) as (x_1, y_1) and (4, 9) as (x_2, y_2), the gradient of the line above is therefore

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-(-3)}{4-1} = \frac{12}{3} = 4.$$

It does not matter which point is taken as (x_1, y_1) – the calculated gradients will be the same.



Example (5): Find the gradient of the line $y = 2x - 3$, using two points in the table from Example 1.

Choosing (0, -3) as (x_1, y_1) and (4, 5) as (x_2, y_2), the gradient works out at $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{5-(-3)}{4-0} = \frac{8}{4} = 2.$

Example (6): Find the gradient of the line $y = 8 - 5x$, using two points in the table from Example 2.

Choosing (0, 8) as (x_1, y_1) and (2, -2) as (x_2, y_2), the gradient works out at $\frac{-2-8}{2} = -\frac{10}{2} = -5.$

Example (7): Find the gradient of the line $x + 2y = 8$, from Example 3.

Choosing (0, 4) as (x_1, y_1) and (8, 0) as (x_2, y_2), the gradient works out at $\frac{0-4}{8} = -\frac{1}{2}.$

Example (8): Find the gradient of the line passing through the points (1, 3) and (1, 7).

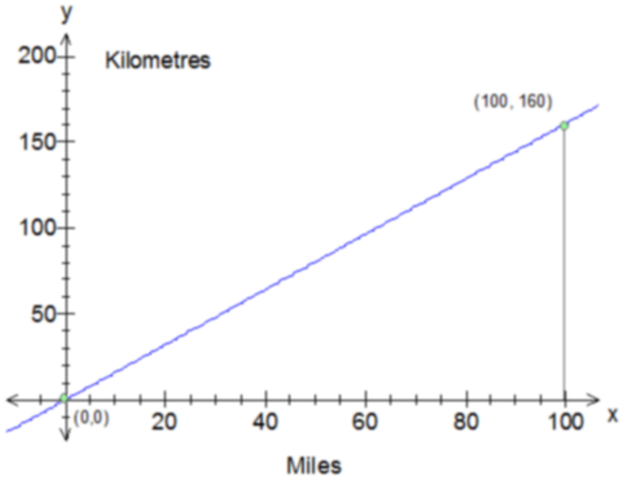
Here we run into trouble, since $x_1 = x_2 = 1$, and substituting into the formula would lead to division by zero, which is undefined. (The line is in fact parallel to the y-axis, and its equation is $x = 1$).

In general, if a line is parallel to the y-axis, its gradient is undefined.

Practical Gradients.

Many practical situations can be approximated or illustrated using straight-line graphs.

The graph on the right is a conversion graph miles on the horizontal axis and kilometres on the vertical.



By selecting two points on the graph, namely (0, 0) and (100,160), we can work out the gradient.

The gradient is therefore the change in *y* (km) divided by the change in *x* (miles) .

$\frac{160-0}{100-0}$ or 1.6 - the number of km in a mile.

Similar graphs could be used for currency conversion (e.g. £ to €).

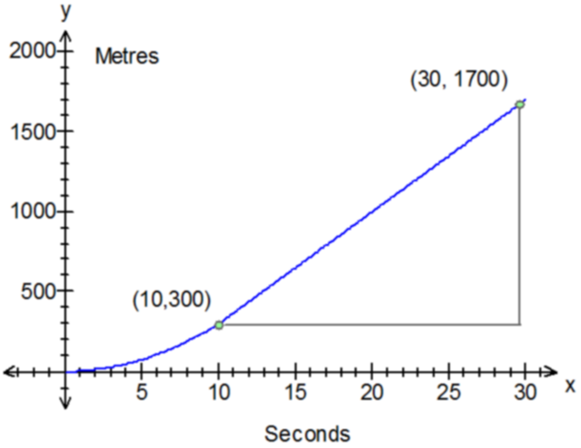
Other examples (note how they all have **time** on their horizontal axis) :

Real-life Situation	**Horizontal axis**	**Vertical Axis**	**Gradient**
Aircraft climbing after take-off	Time (sec)	Altitude (metres)	Climb rate in metres /s
Water pumped into a reservoir	Time (sec)	Volume (litres)	Flow rate in litres / s
Car travelling in a straight line	Time (hrs)	Distance (km)	Speed in km / h
People entering a stadium via turnstiles	Time (min)	Number of people through turnstile	People per minute

Sometimes a real-life graph might have curved sections in it. In such cases, we would only be asked to find and interpret the straight sections.

Example (9): The graph below shows the distance travelled by a racing car on a straight track for 30 seconds. Find the final steady speed of the car.

This time, we have to find two points on the **straight** section of the graph, so the origin (0,0) is of no use here.



We can see that the straight section begins at 10 seconds, so we can use the points (10,300) and (30,1700) to find the gradient.

The gradient of that section is therefore the change in *y* (metres) divided by the change in *x* (seconds)

$\frac{1700-300}{30-10}$ or $\frac{1400}{20}$ or 70 .

Because this is a distance-time graph, the gradient represents speed, i.e. **70 m/s.**

Forms of the straight-line equation.

The commonest way of expressing linear equations at GCSE is $\boldsymbol{y = mx + c}$, known as the **gradient-intercept** form.

The gradient-intercept form cannot be used for equations of the "$x = c$ " type as attempting to find the gradient of such a line would mean dividing by zero, which is illegal.

Using the results from previous examples, we found that the graph of $y = 2x – 3$ passed through the point (0, -3) and that it had a gradient of 2.

Similarly the graph of $y = 8 - 5x$ had a gradient of –5 and passed through the point (0, 8).
If we rewrite the last example as $y = -5x + 8$, we can see that the gradient is evidently the multiple of x, and that the graph crosses the y-axis where y takes the value of the constant.

- Any graph of the form $\boldsymbol{y = mx + c}$ has a gradient of m and a y-intercept at (0, c).

Sometimes a little algebraic manipulation is needed to put an equation into gradient-intercept form.

Example (10): Rewrite the following equations in "$\boldsymbol{y = mx + c}$" form:
i) $x + y = 5$; ii) $5y = 4x – 3$

i) $x + y = 5$ can be rewritten as $y = 5 – x$ (or $y = -x + 5$).
ii) $5y = 4x – 3$ needs to be divided by 5 on each side to give $y = \frac{4}{5}x - \frac{3}{5}$.

Finding the equation of a straight-line graph.

The equation of a straight-line graph can be determined by just one point and the gradient. If the gradient is not known, two points will also suffice.

Example (11): Find the equation of the line with gradient 4 passing through the point (0, -7).
The y-intercept is –7, and so the equation of the line is $y = 4x-7$.

Example (12): Find the equation of the line with gradient 2 passing through the point (3, 1).
This time we do not have the y-intercept, so we can only say that the equation of the line is $y = 2x + c$ where c must be determined.

Substituting $x = 3$ and $y = 1$ into the equation gives $1 = 6 + c \rightarrow c = -5$.
The equation of the graph is therefore $y = 2x – 5$.

Example (13): Find the equation of the straight line passing through the points (-1, 2) and (4, 17).
Here we are not given the gradient, m, so we have to work it out first.

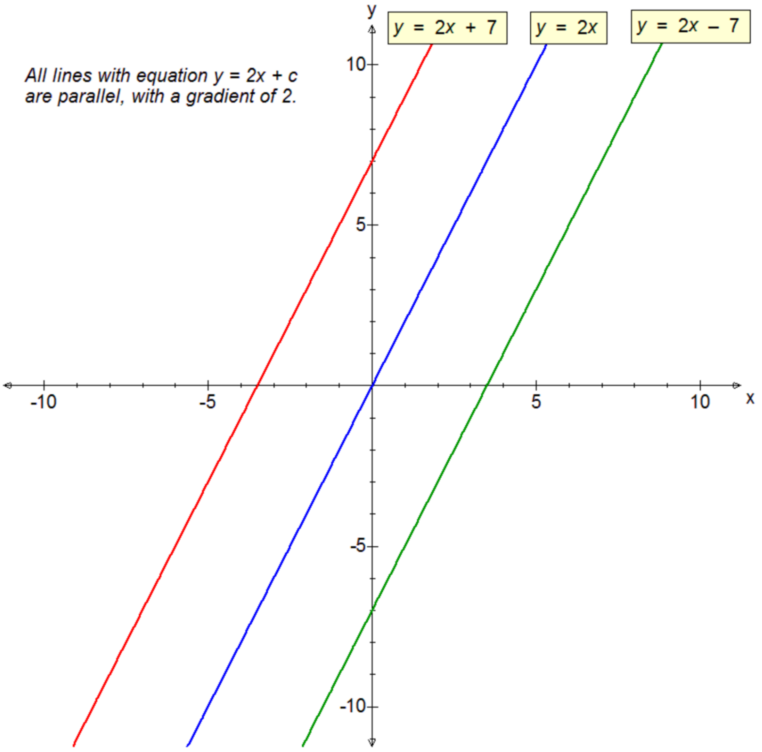
Taking (-1, 2) as (x_1, y_1) and (4, 17) as (x, y) we obtain

$$m = \frac{y - y_1}{x - x_1} \rightarrow m = \frac{17-2}{4-(-1)} \rightarrow m = 3.$$

The gradient of the line is 3, and its equation is $y = 3x + c$.

Substituting $x = 4$ and $y = 17$ into the equation gives $17 = 12 + c \rightarrow c = 5$.
The equation of the graph is therefore $\boldsymbol{y = 3x + 5}$.

Finding the equation of a line parallel to a given line, passing through a specified point.



Parallel lines all have the same gradient, as the graphs above show.

The graphs of $y = 2x + 7$, $y = 2x - 7$ and $y = 2x$ are all parallel, as are the graphs of $y = 2x + c$ where c is any constant.

Example (14): Find the equation of the straight line parallel to $y = 10 - 3x$, and passing through the point (-2, 7).

The required line must have a gradient of -3, so its equation must be $y = -3x + c$ or $c = y + 3x$
.
Substituting $y = 7$ and $x = -2$ gives $c = 7 - 6$, $\rightarrow c = 1$.

$\therefore$ the equation of the required line is $\boldsymbol{y = -3x + 1}$ (or $\boldsymbol{y = 1 - 3x}$).

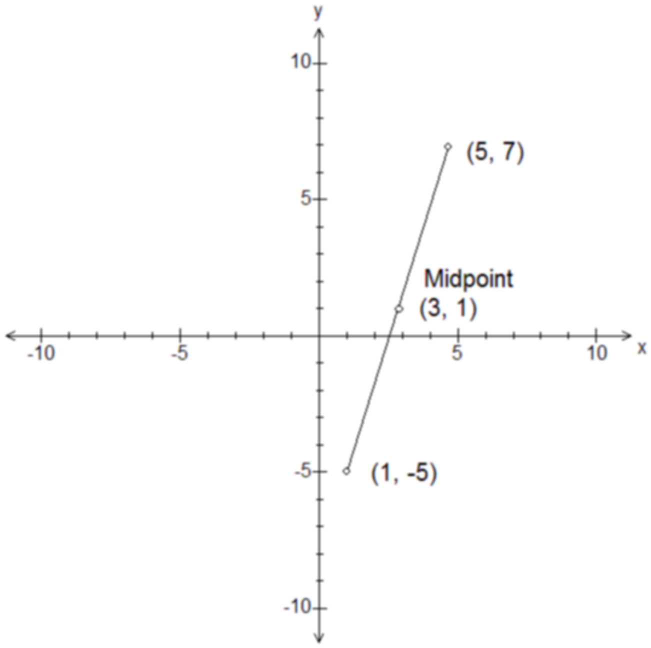
Midpoint of a line.

If a point P has coordinates (x_1, y_1) and a point Q has coordinates (x_2, y_2) , then the midpoint of the line PQ has the coordinates

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In the above example, the midpoint of the line joining the points (1, -5) and (5, 7) is the point (3, 1).

Note : 3 is halfway between 1 and 5, and 1 is halfway between –5 and 7.



Example (15): Find the midpoint of the line joining the points (-4, 5) and (6, 1).

The coordinates of the midpoint of the line are given as $\left(\frac{(-4)+6}{2}, \frac{5+1}{2}\right)$, simplifying to **(1,3)**.