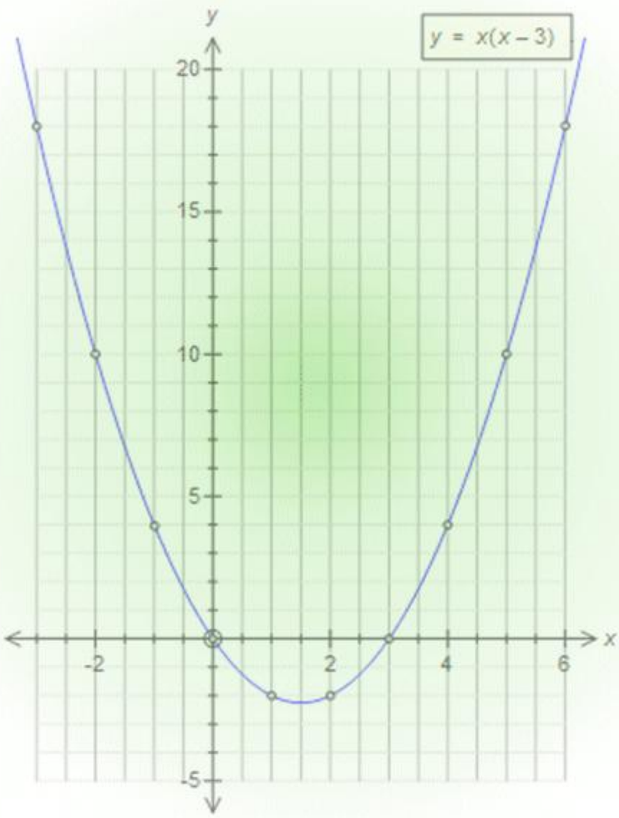
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

NON-LINEAR GRAPHS

x	-3	-2	-1	0	1	2	3	4	5	6
$(x - 3)$	-6	-5	-4	-3	-2	-1	0	1	2	3
$y = x(x - 3)$	**18**	**10**	**3**	**0**	**-2**	**-2**	**0**	**3**	**10**	**18**



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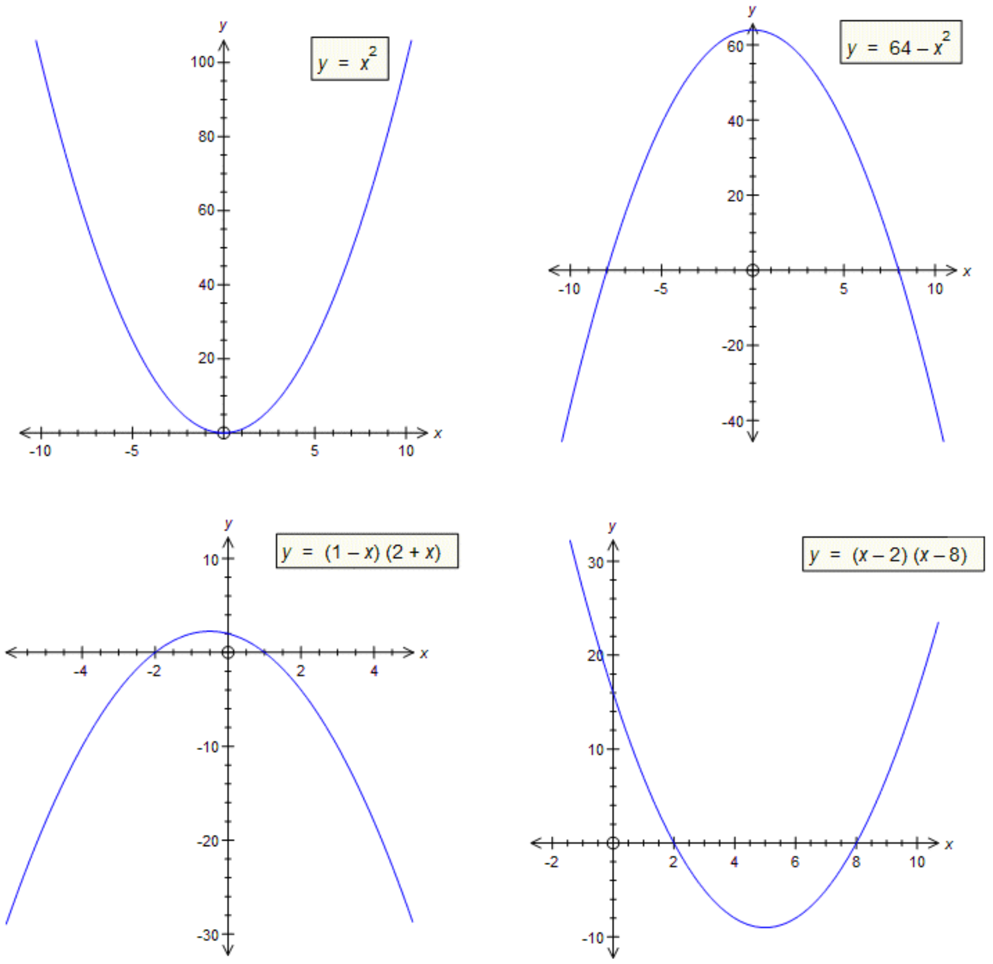
NON-LINEAR GRAPHS.

Quadratic graphs.

These graphs are of the form $y = ax^2 + bx + c$ where a, b and c are constants, and a is not zero. The highest power of x is 2 (the square of x). The basic graph of $y = x^2$ is shown upper left.



These graphs are 'bucket-shaped'.

When the x^2 term is positive, the graphs point downwards at a trough and the function takes a minimum value. The expansion of $y = (x - 2)(x - 8)$ is $y = x^2 - 10x + 16$.

On the other hand, they point upwards at a crest and have a maximum value when the x^2 term is negative. The expansion of $y = (1 - x)(2 + x)$ is $y = 2 - x - x^2$.

The 'depth' of a quadratic graph can vary, but this is as dependent on the scaling of the graph axes as on the actual function.

Example (1). Plot the graph of $y = x(x - 3)$ for x from -3 to 3, in steps of 1 unit.

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
$(x - 3)$	-6	-5	-4	-3	-2	-1	0	1	2	3
$y = x(x - 3)$	**18**	**10**	**3**	**0**	**-2**	**-2**	**0**	**3**	**10**	**18**

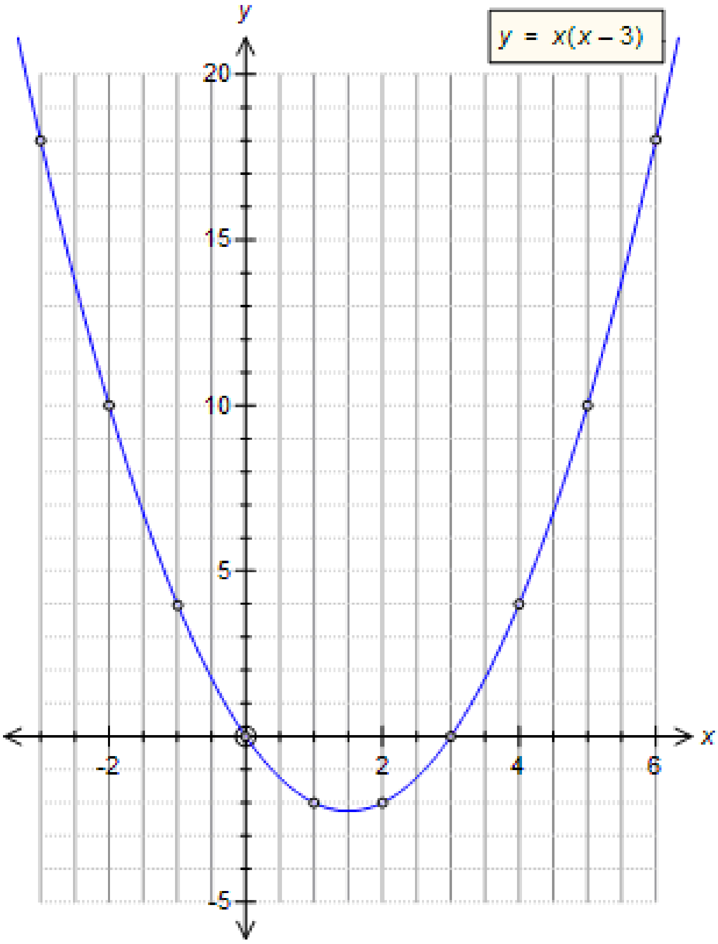
This is a quadratic graph, and so will have a ‘bucket’ shape.

We therefore plot the points (-3, 18), (-2, 10), (-1, 3), (0, 0), (1, -2) and so forth, finally drawing a smooth curve through them all.

Any computing errors will betray themselves by points being off the path of the expected curve.

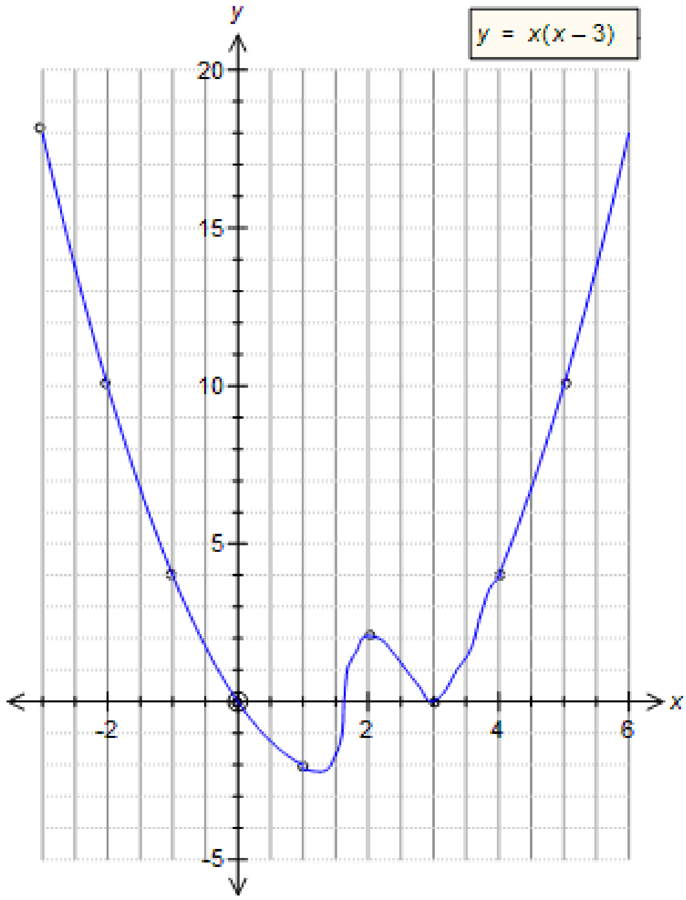
Notice how the points (1, -2) and (2, -2) are still connected by a curve, and not a straight line segment !

A way of avoiding such an error would be to plot an extra point at (1.5, -2.25).



Another horror would be an attempt to plot a curve through an obviously incorrect point.

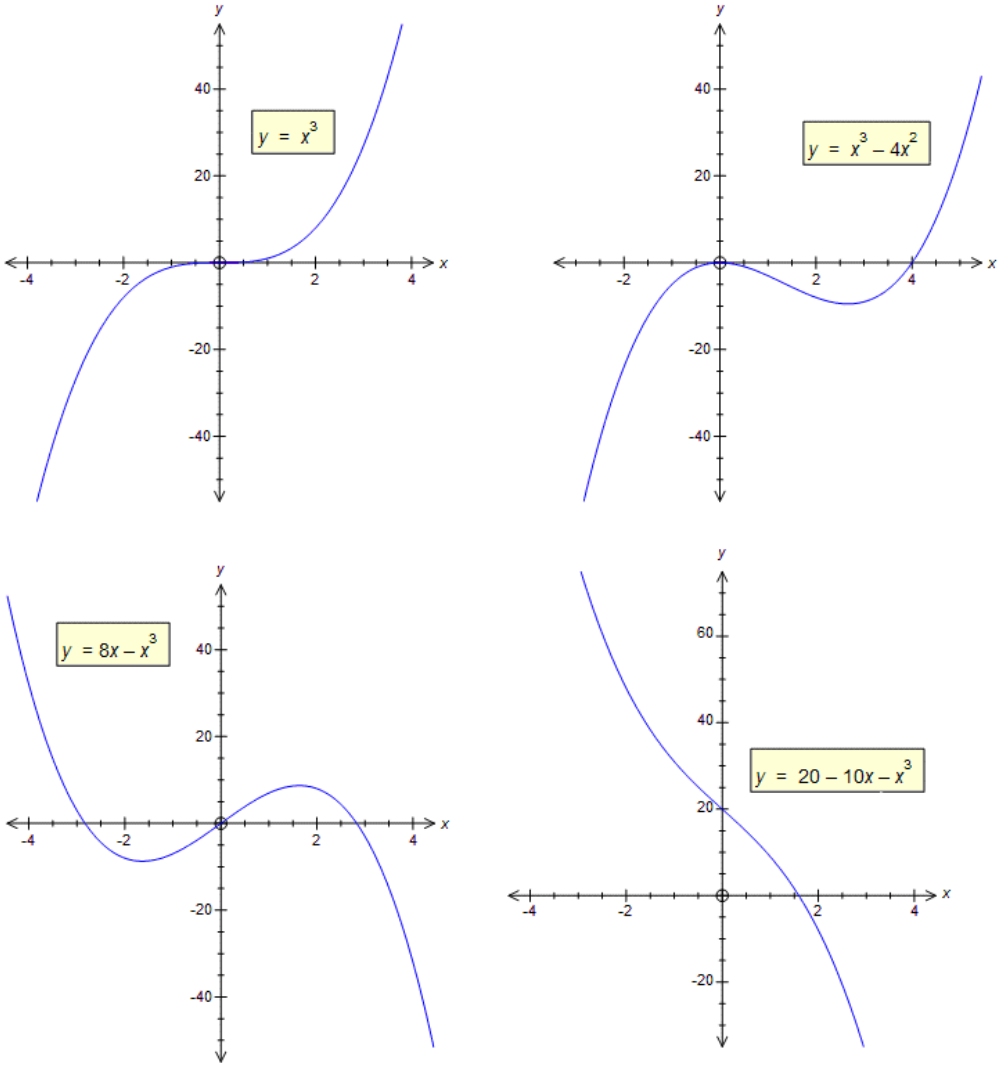
Here, the point (2, -2) has been incorrectly plotted as (2, 2), giving a graph with more twists and bends than is should have.



Cubic graphs.

These are a little more complicated than quadratic graphs.
Their functions are of the form $y = ax^3 + bx^2 + cx + d$ where a, b, c and d are constants, and a is not zero. The highest power of x is 3 (the cube of x).

The basic graph of $y = x^3$ is shown upper left.



These graphs are characterised by a 'double bend'.

If the term in x^3 is positive, the general slope is upward from lower left, but if the x^3 term is negative, the general slope is downward from upper left.

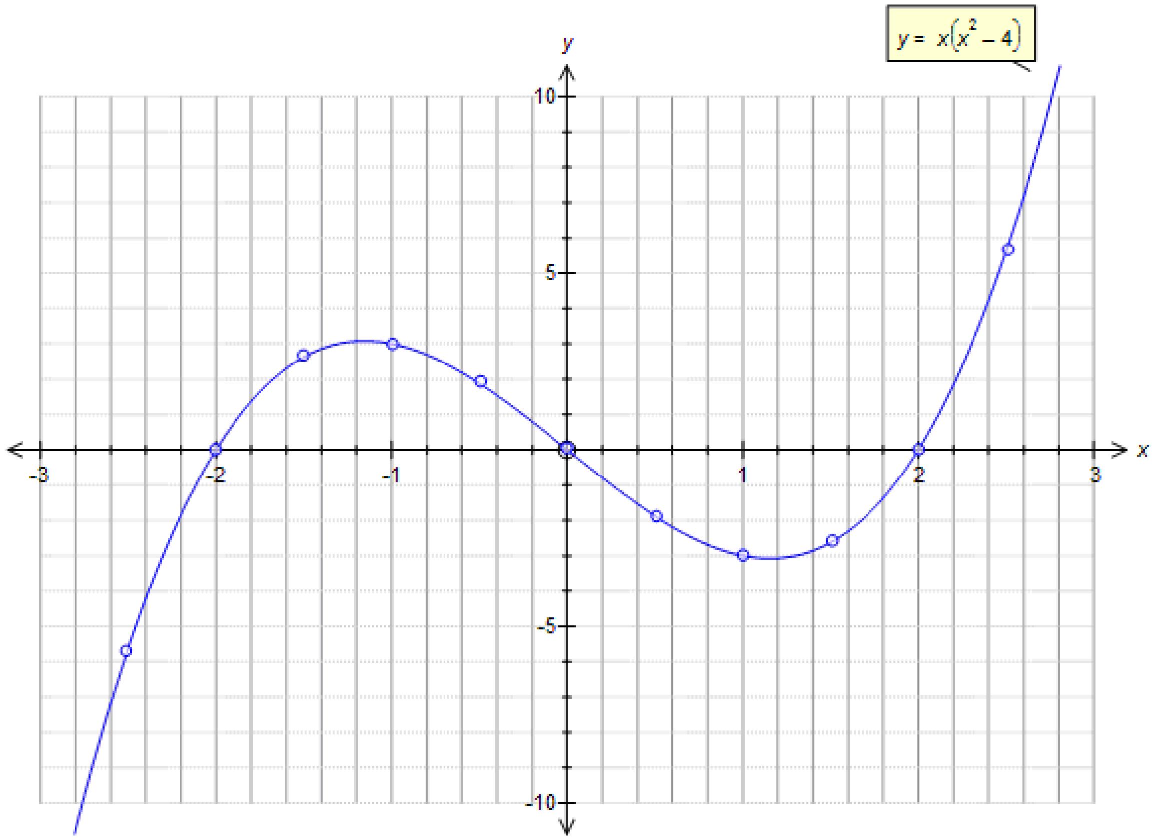
The 'bend' can also vary in severity - the graph on lower left has sharper 'bends' than the one on lower right.

Example (2). Plot the graph of $y = x(x^2 - 4)$ for x from -2.5 to 2.5 in steps of 0.5, using a calculator.

Round your results to 1 decimal place.

x	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
$y = x(x^2 - 4)$	-5.6	0	2.6	3	1.9	0	-1.9	-3	-2.6	0	5.6

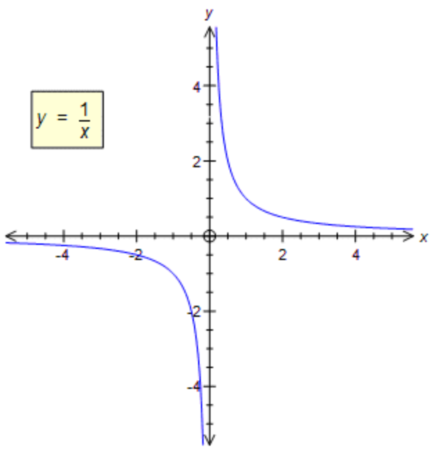
(Intermediate calculations not shown)

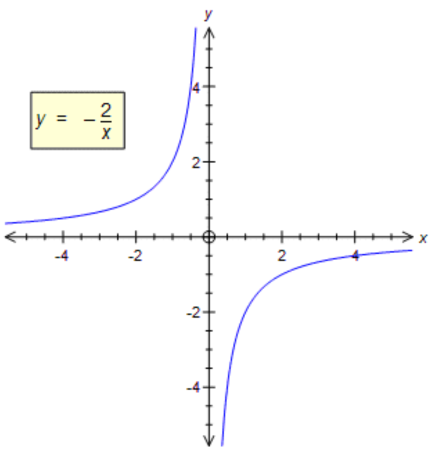


The resulting graph is a cubic, and has the characteristic 'double bend' shape.

Remember $x(x^2 - 4) = x^2 - 4r$.

Reciprocal graphs.





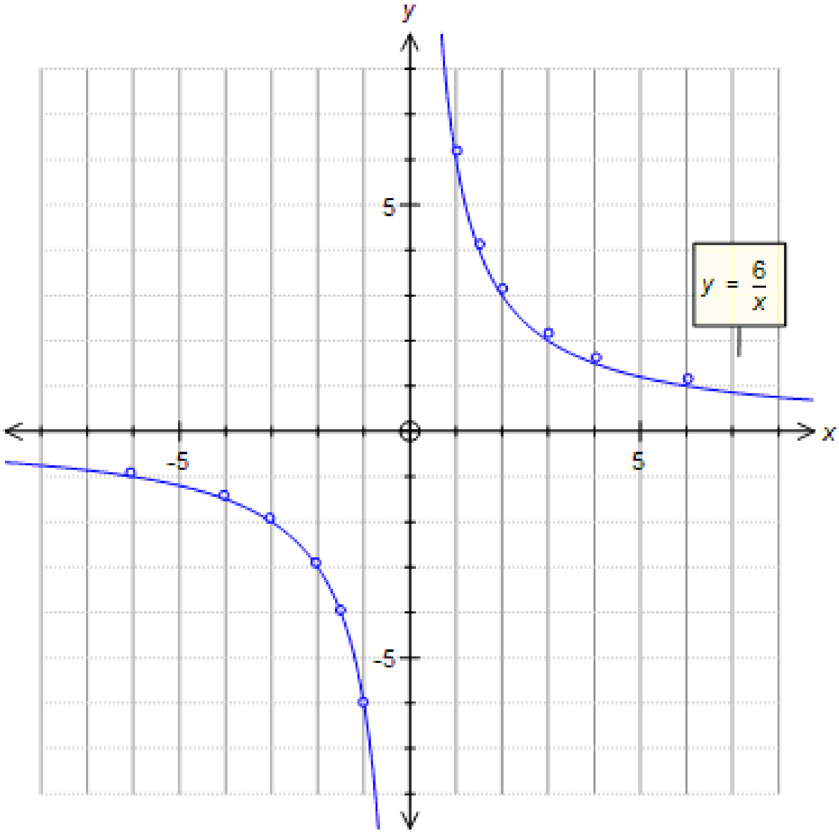
These graphs are shared by functions of the form $y = \frac{k}{x}$, where *k* is a non-zero constant. They differ from previous examples in that they seem to be in two unconnected parts: if *k* is positive, the two sections are in the upper right and lower left, but if *k* is negative, the sections are in the upper left and lower right.

Note the following features of the standard graph $y = \frac{1}{x}$.

As *x* becomes large and positive, *y* stays positive but approaches zero.
As *x* becomes large and negative, *y* stays negative but approaches zero.

Example (3): Plot the graph of $y = \frac{6}{x}$ for x between -6 and 6, but excluding x = 0.

x	-6	-4	-3	-2	-1.5	-1	1	1.5	2	3	4	6
$y = \frac{6}{x}$	-1	-1.5	-2	-3	-4	-6	6	4	3	2	1.5	1



The graph is a reciprocal graph, consisting of two separate and unconnected parts.

In fact, the function $y = \frac{6}{x}$ is undefined when x = 0.