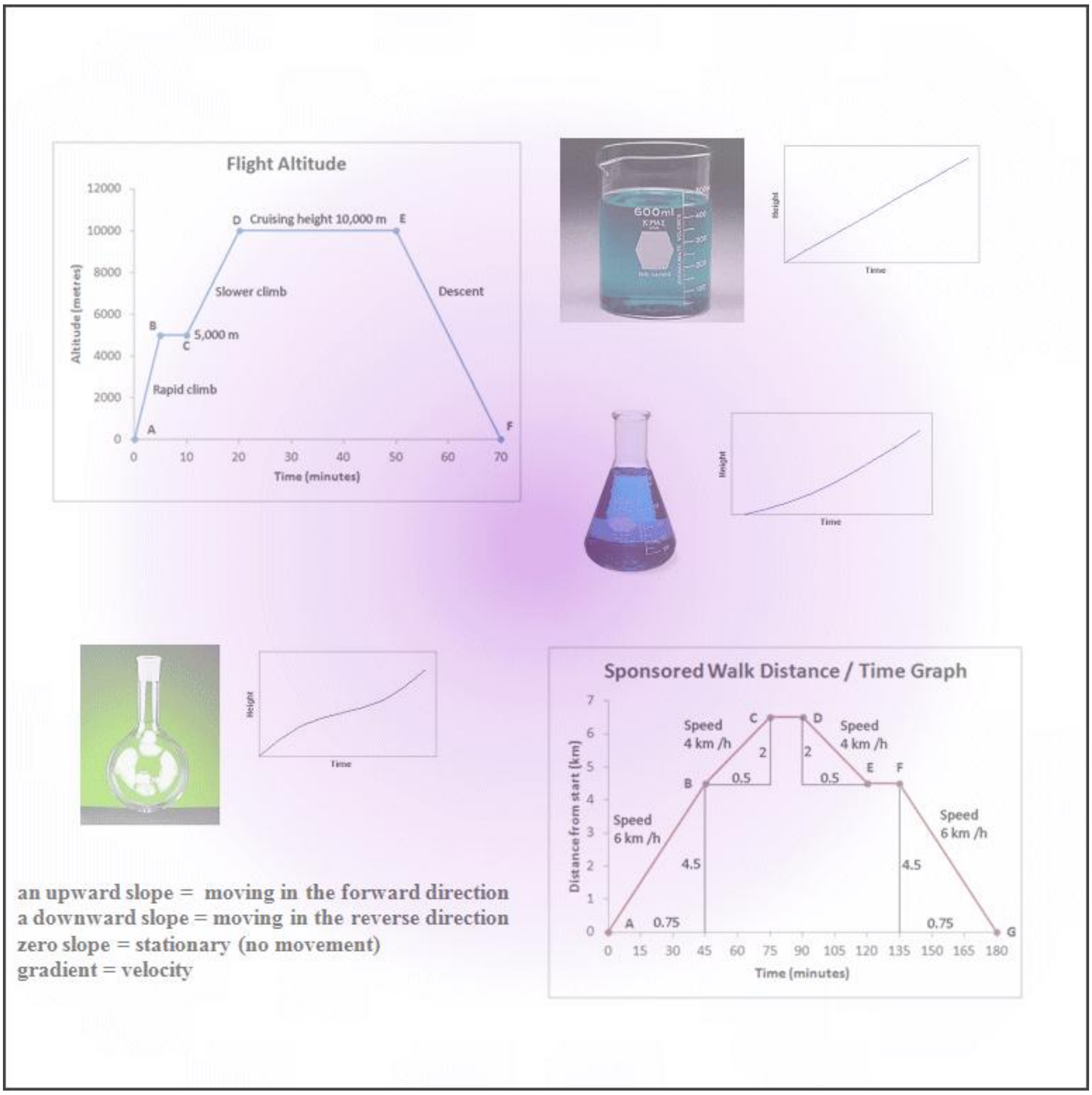


M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

REAL LIFE GRAPHS



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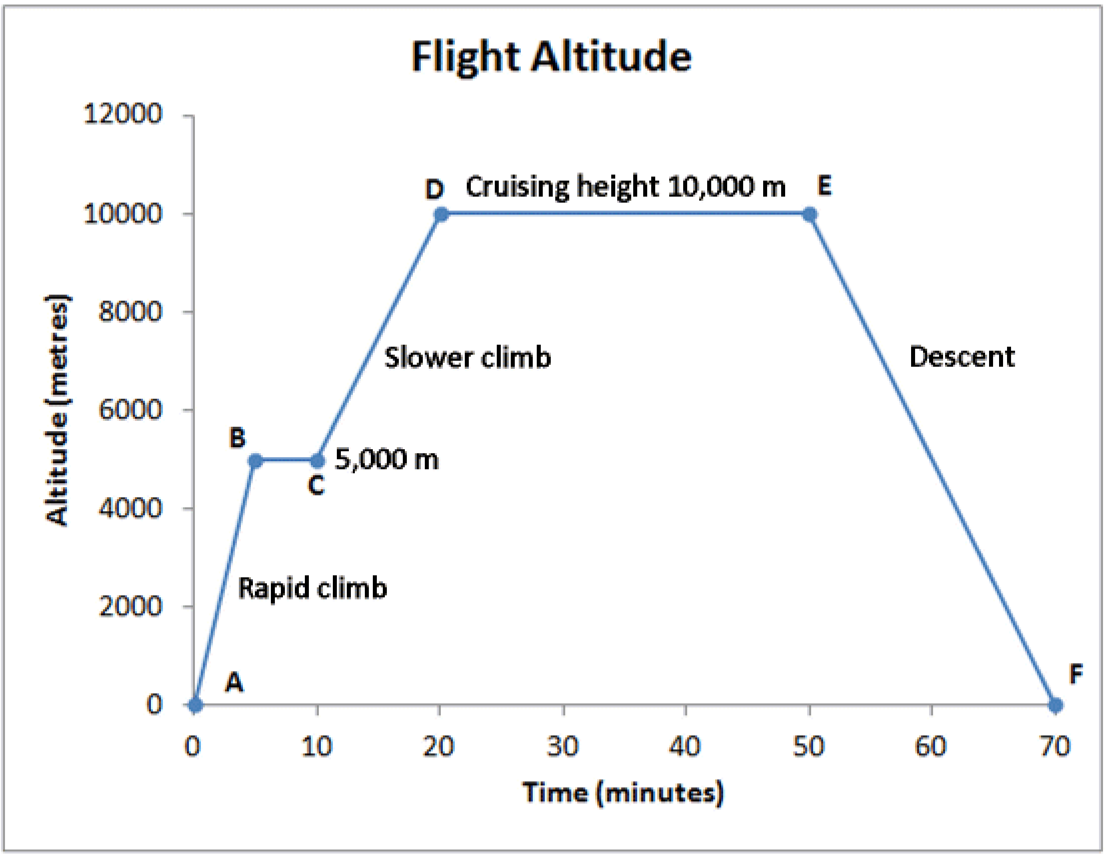
REAL-LIFE GRAPHS

Many real-life situations lend themselves to graphical representation. The most important among them are **distance-time** graphs (more correctly **displacement-time** graphs).

Example (1): A plane takes off from Manchester and climbs for five minutes to a height of 5,000 metres, stays at that height for another five minutes, and then takes ten minutes to reach its cruising altitude of 10,000 metres. It remains at this cruising height for half an hour until beginning a twenty-minute descent, landing at Paris after a 70-minute flight.

Draw a distance-time graph to represent the plane's altitude during the flight, assuming steady climbing and descent.

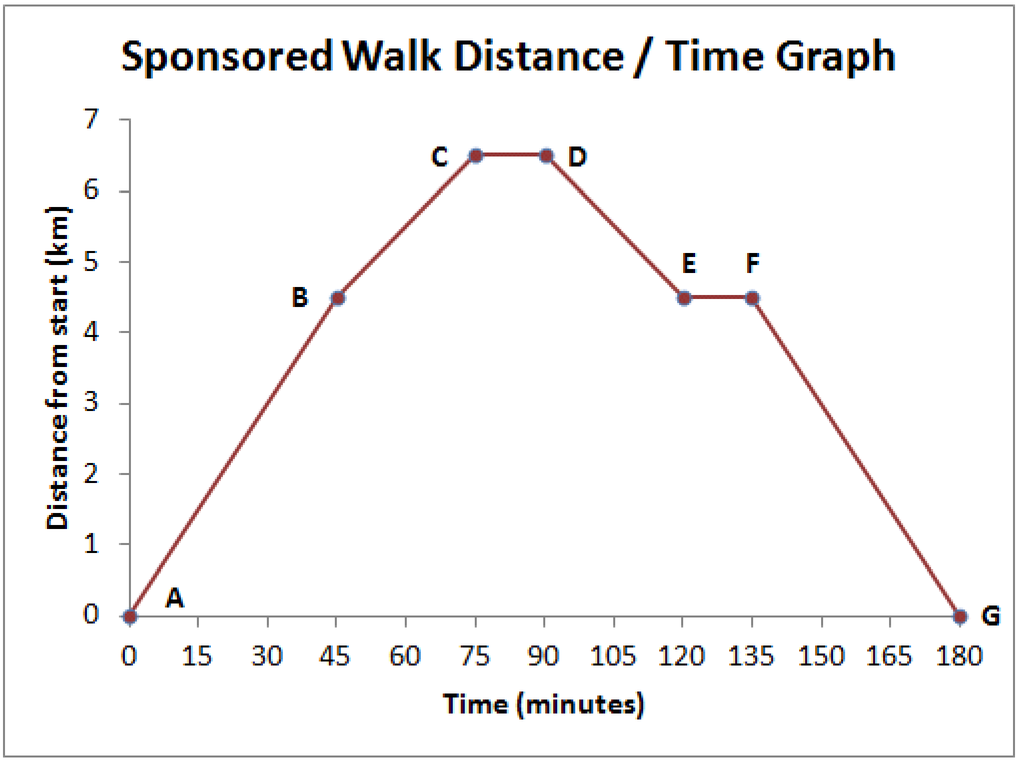


The section from **A** to **B** represents a rapid climb from ground level at Manchester to 5,000 metres in the space of five minutes, followed by a short five-minute spell at that intermediate height from **B** to **C**.

From **C** to **D** the plane gains height again in ten minutes, but at a slower rate than it did from **A** to **B**, as can be seen by the gentler slope. The next 30 minutes from **D** to **E** have the plane at its cruising height, until it begins its 20-minute descent into Paris at point **F**.

This was an example was of a distance-time graph with altitude on the *y*-axis.

Example (2): Sue takes part in a sponsored walk and keeps a record of her distance from the start as follows. She then plots her progress on the displacement-time graph below.



The resulting graph is also often termed a travel graph.

i) At what points did Sue take rest breaks ?

ii) The course of the walk was on both paved roads and on footpaths in open country.
Which sections of the walk were on the paved road , given that the paved roads allowed quicker progress ?

iii) At what point did Sue begin her return walk ?

iv) Compare Sue's walking speed on the paved roads with that on the footpaths.

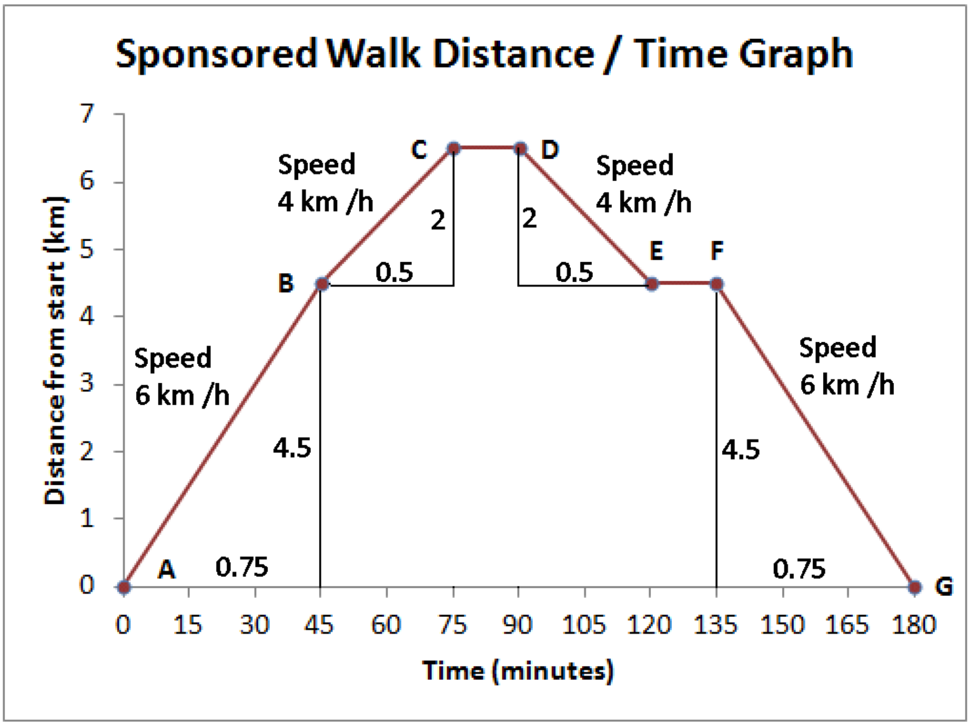
i) The slope of the graph gives a clue in each case, as it represents speed. When the slope is zero, or the graph is flat, then Sue is not walking. Her rest stops are therefore between points **C** and **D**, and also between points **E** and **F.**

ii) The paved roads allow for faster walking speed than the footpaths, and so the faster speed will be shown by the steeper sections of the graph. The sections from **A** to **B** and from **F** to **G** are steeper than **B** to **C**, or **D** to **E**.

iii) The return walk began at the point where Sue's distance from home started to decrease, namely point **D**.

In short, on a displacement-time graph:
an upward slope = moving in the forward direction
a downward slope = moving in the reverse direction
zero slope = stationary (no movement)
gradient = velocity (if the section of the graph is curved, then use the gradient of the tangent)

There is an important difference between **velocity** and **speed** (see later !)



Finding speed from displacement-time graphs.

Example (3): Work out Sue's speed between points **A** and **B** and also between points **C** and **D**.

Sue's speed over various sections of the walk can be worked out by finding gradients.
(we must remember to convert minutes into decimal fractions of hours here).

For example, she covers the 4.5 km between points **A** and **B** in the space of 45 minutes (0.75 hour), which makes her speed over that section $\frac{4.5}{0.75}$ km/h or 6 km./h.

Her speed over the section from **B** to **C** similarly works out at $\frac{2}{0.5}$ km/h or 4 km./h.

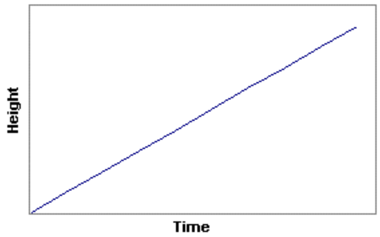
Other applications.

Another example of real-time graphs concerns rates of flow into vessels (such as flow of rain into a gauge). The example below illustrates the pattern:

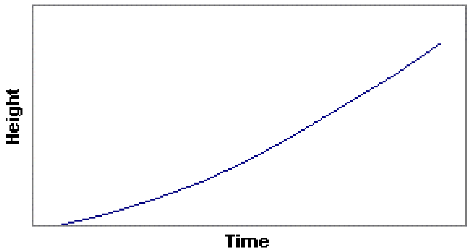
Example (4): Water is poured at a constant rate into i) a beaker of constant width; ii) a conical flask; iii) a round-bottomed flask.

Sketch graphs showing the change in the height of the water in each container with increasing time.

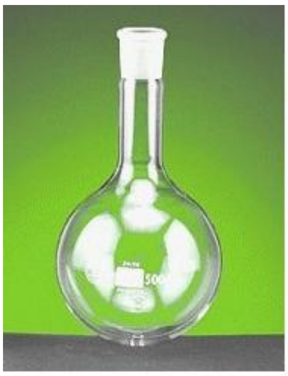
The increase in water level depends on both the flow rate and the surface area of the water-air boundary. In other words, the smaller that surface area, the greater the increase in the height.

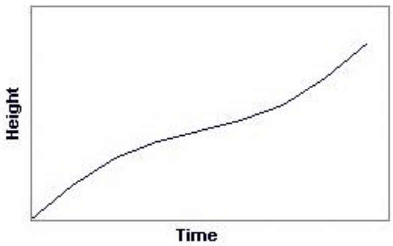


The beaker has constant width, and so the increase in the height of the water follows a straight-line graph. There is no change in the surface area of the water / air boundary.



The conical flask narrows towards the neck, and therefore the water level increases at a faster rate as the surface area of the water / air boundary decreases. (We have not lncluded flow into the neck.)





With the round flask, the surface area of the water / air boundary starts off small, then grows to a maximum, and finally becomes small again. The flow rate therefore follows this trend in reverse; fast, then slow, then fast again. (Again, we have not included flow into the neck.)

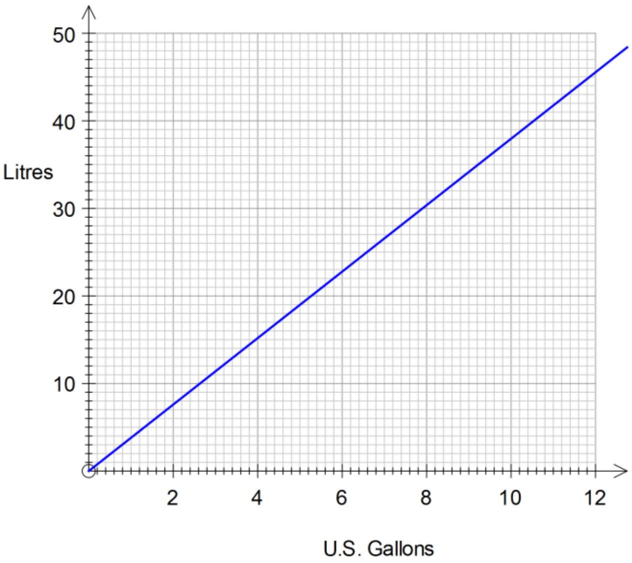
Conversion graphs.

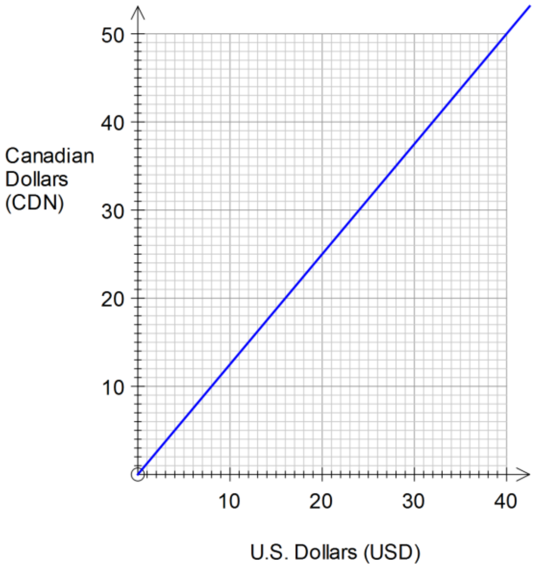
Example (5):

Bill is on a motoring tour of North America, close to the border of the U.S.A. and Canada.

Petrol in the U.S.A. costs \$3.30 (U.S.) per gallon; in Canada, \$1.25 (Canadian) per litre.

He wants to fill up with 8 gallons of petrol, but is not sure whether to do so in the U.S. or in Canada. Use the conversion graphs below to find which country offers the better deal.





With petrol at \$3.30 per gallon in the U.S. the cost of the petrol would be \$3.30 × 8 = **\$26.40 U.S.**
By reading off the "U.S. Gallons to Litres" graph, it can be seen that 8 U.S. gallons = 30 litres.

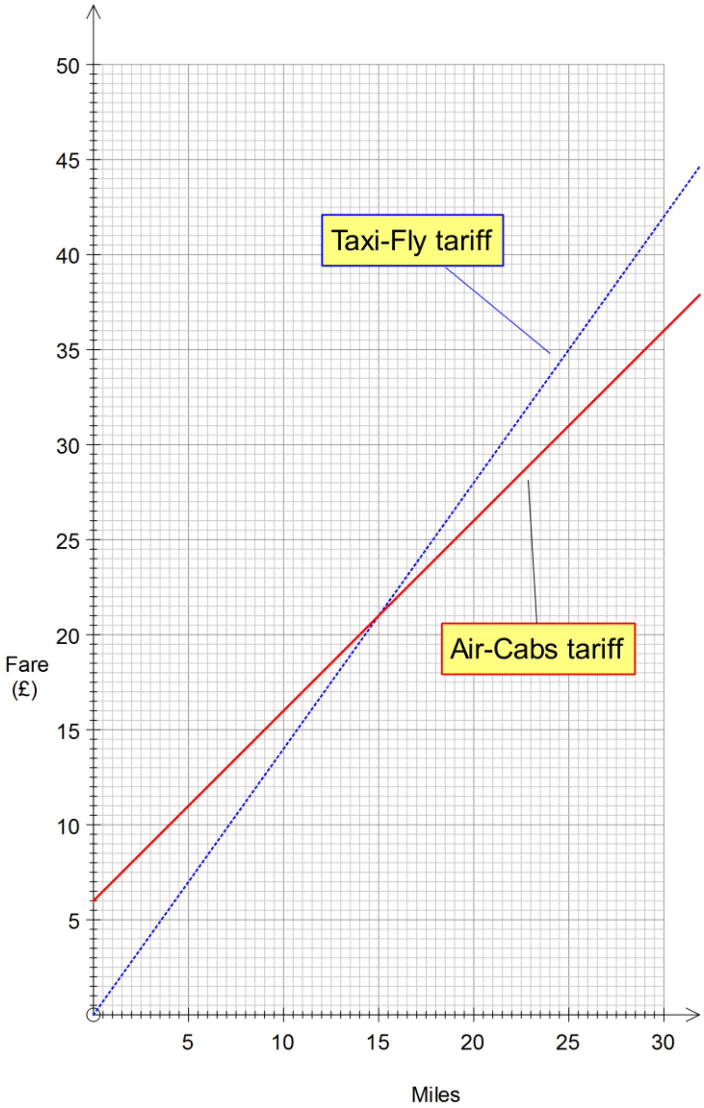
With petrol in Canada costing \$1.20 per litre, those 30 litres would cost \$1.20 × 30 = **\$36 Canadian**.

We need to convert from Canadian to U.S. dollars, and by reading off the "U.S. to Canadian Dollars" graph, we see that **\$36 Canadian = \$29 U.S.**

It is therefore cheaper for Bill to fill up in the U.S.A. rather than in Canada.

"Work and Rates" Graphs.

Example (6): Two taxi firms operate out of Manchester Airport, Air-Cabs and Taxi-Fly. The graphs show their tariffs.



i) For what travel distance are the fares for Air-Cabs and Taxi-Fly the same ? Explain.

ii) Which firm offers the better fare deal from Manchester Airport to
a) Bolton, 20 miles away, and
b) Stockport, 10 miles away ?

Show how you obtained your answers.

iii) Which firm charges a fixed initial charge of £6 ?

iv) Calculate the fare **per mile** for each taxi firm. You can use the results from parts ii) and iii).

i) The two firms charge the same fare at the distance where their tariff graphs cross, i.e. at 15 miles.

ii) a) Bolton is 20 miles away from Manchester Airport, and for that distance, Air-Cabs charge £26 and Taxi-Fly £28.

$\therefore$ Air-Cabs offer the better deal from Manchester Airport to Bolton.

ii) b) Stockport is 10 miles away from Manchester Airport – for that distance, Air-Cabs charge £16 and Taxi-Fly £14.

$\therefore$ Taxi-Fly's deal from Manchester Airport to Stockport is better.

iii) The Air-Cabs tariff graph meets the vertical (fares) axis at £6 for zero miles travelled. This represents the fixed initial charge of £6.

iv) Taxi-Fly charge £14 for the 10-mile trip to Stockport, and zero fixed initial charge. For every mile travelled, the fare increases by $£\frac{14-0}{10-0} = £1.40$

(We have worked out the gradient between the points (0,0) and (10,14) on Taxi-Fly's graph).
$\therefore$ Taxi-Fly's fare per mile is £1.40 with no initial charge.

Air-Cabs, on the other hand, charge £26 for the 20-mile trip to Bolton, but with a £6 fixed initial charge. For every mile travelled, the fare increases by $£\frac{26-6}{20-0} = £1.00$

(We have worked out the gradient between the points (0,6) and (20,26) on Air Cabs' graph).
$\therefore$ Air-Cabs' fare consists of a fixed charge of £6 **plus** £1.00 for every mile travelled.