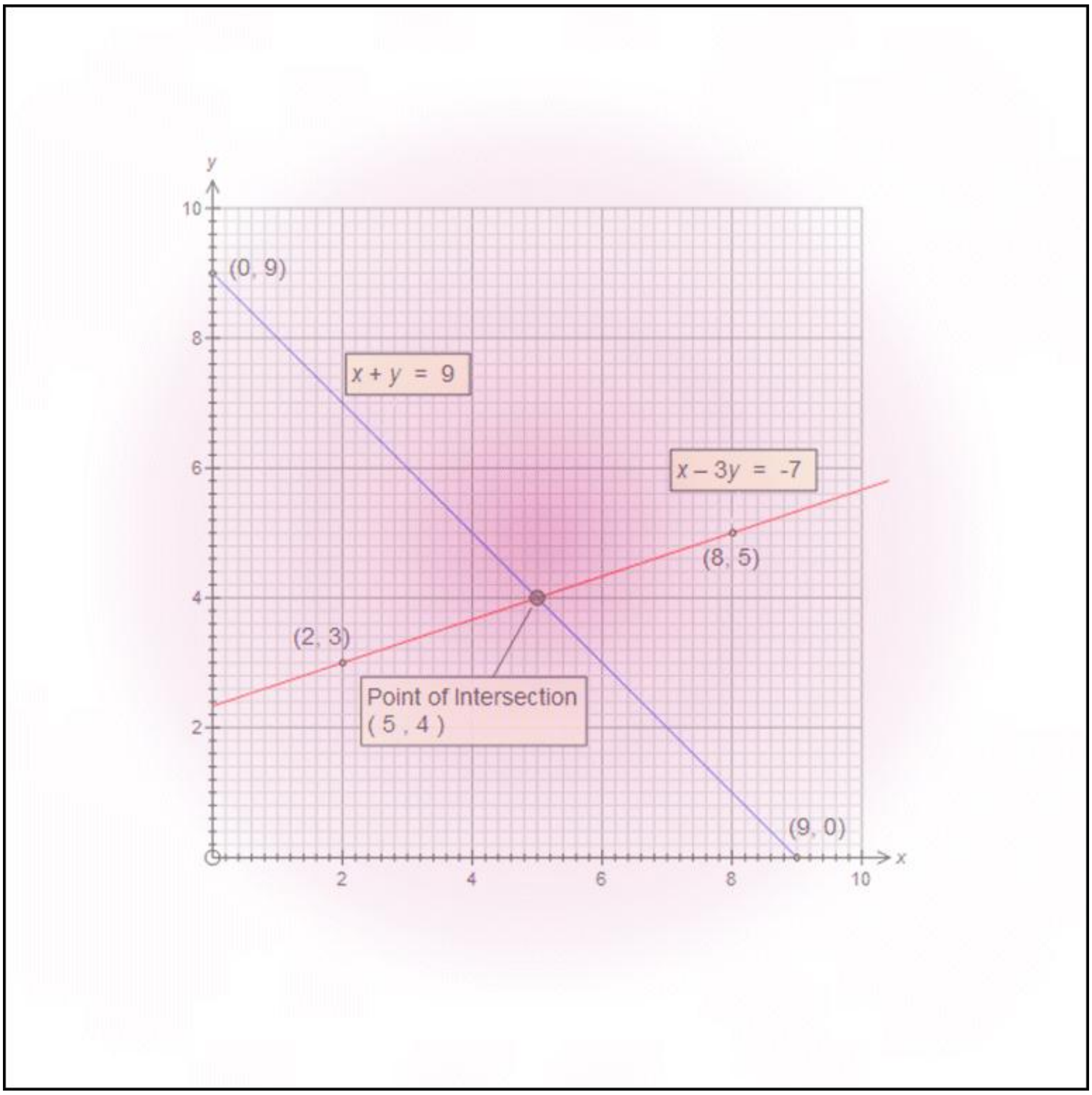


M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

SIMULTANEOUS EQUATIONS



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SIMULTANEOUS EQUATIONS

We begin this section with a 'tea-time' problem.

Example (1) : An apple and a banana cost 50p in total, and the apple costs 10p more than the banana. Find the cost of each fruit.

We can use trial and error to find two numbers that add to 50, but which also differ by 10.
This requirement can be rewritten in algebraic terms as a pair of equations.

If a is the cost of an apple, and b the cost of a banana, we can say

$a + b = 50$
$a - b = 10$

What we have here is a pair of **simultaneous equations** in two variables, a and b.

Each equation has an infinite number of solutions by itself.
Thus $a + b = 50$ has possible solutions of $a = 40$, $b = 10$ or $a = 30$, $b = 20$ or infinitely many others.
Similarly $a - b = 10$ has solutions $a = 40$, $b = 30$ or $a = 30$, $b = 20$ and so on.

The combination, though, has only one solution, namely $a = 30$, $b = 20$.
Hence an apple costs 30p and a banana costs 20p.

There are various methods of solving linear simultaneous equations, as will be shown in the next examples.

Graphical methods of solving simultaneous equations.

This method involves plotting straight-line graphs corresponding to each equation, and simply reading off the coordinates of the point where the two lines intersect.

Example (2) :

i) Show that the equations $2x + y = 13$ and $y - 2x = 1$
can be rewritten as $y = 13 - 2x$ and $y = 2x + 1$ respectively.

ii) Plot two linear graphs, substituting $x = 0$, 2 and 4 in each case.

iii) Hence solve the simultaneous equations $2x + y = 13$ and $y - 2x = 1$.

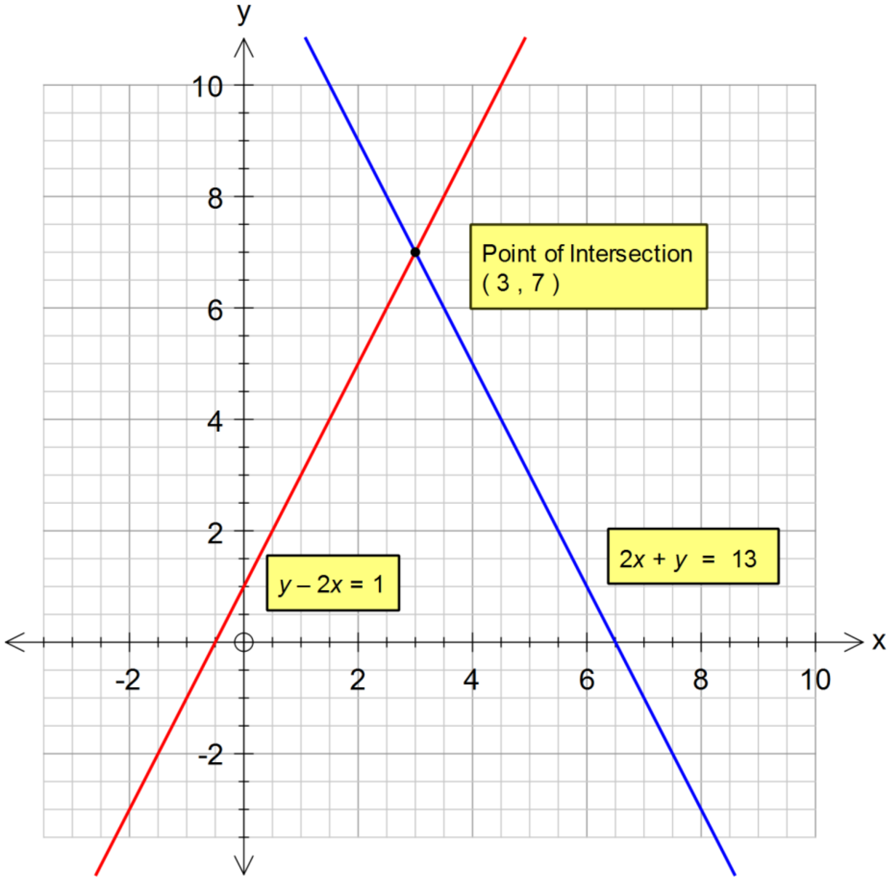
i) $2x + y = 13$ can be rewritten as $y = 13 - 2x$ by subtracting $2x$ from both sides.
Similarly $y - 2x = 1$ can be rewritten $y = 2x + 1$ by adding $2x$ to both sides.

ii) The tabled results are :

x	0	2	4
$y = 13 - 2x$	13	9	5

x	0	2	4
$y = 2x + 1$	1	5	9

We then plot the resulting graphs :



iii) The two graphs intersect at the point (3, 7).

Hence the solutions to the simultaneous equations are $x = 3$, $y = 7$.

Example (3):

Use graphs to solve the simultaneous equations

$x - 3y = -7$
$x + y = 9$

The equations can be rewritten:

$x - 3y = -7 \longrightarrow -3y = -7 - x \longrightarrow 3y = x + 7 \longrightarrow y = \frac{x+7}{3}$

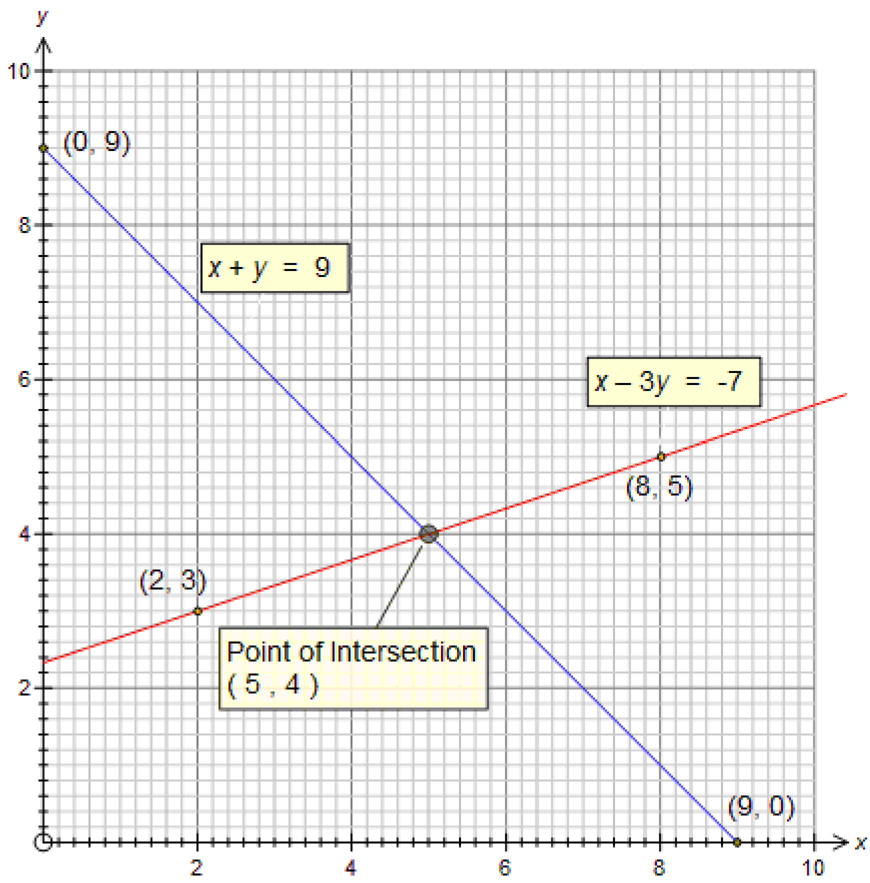
Two points to plot are (-7, 0) and (2, 3)

$x + y = 9 \longrightarrow y = 9 - x$

Two points to plot are (0, 9) and (9, 0)

The two graphs meet at the point (5, 4) , therefore the solution is

$\boldsymbol{x = 5, \ y = 4}$



Graphical methods are useful for solving easier simultaneous equations, but there are also algebraic methods available.

Elimination method of solving simultaneous equations.

We begin this section by repeating Example (1) using algebra rather than by trial and error.

Example (4) : An apple and a banana cost 50p in total, and the apple costs 10p more than the banana. Find the cost of each fruit.

We need to find two numbers that add to 50, but which also differ by 10.
This requirement can be rewritten in algebraic terms as a pair of equations.

If a is the cost of an apple, and b the cost of a banana, we can say

$a + b = 50$; $a - b = 10$.

Returning to Example (1), we have the equations

$a + b = 50$ $\quad A$
$a - b = 10$ $\quad B$

If we were to add equations A and B to form a new equation, we would be left with

$2a = 60$ $\quad A+B$

Since $2a = 60$, $a = 30$, and therefore the apple costs 30p.
Because the apple and the banana cost 50p in total, the banana costs 20p.

What we have done here is **eliminate** the variable b by adding the two equations and deducing $a = 30$. Then, we substituted $a = 30$ into the equation $a + b = 50$ to obtain $b = 20$.

Example (5): A football league uses a rather peculiar points scoring system, whereby teams obtain w points for a win, d points for a draw and no points for a loss.

Dynamics have won 5, drawn 3 and lost 2 games and are on 31 points.
Locos have won 6, drawn 1 and lost 3 games and are on 32 points.

How many points does this league award for a win, and how many does it award for a draw ?

We begin by forming a pair of simultaneous equations out of the given data. Because a win is worth w points and a draw is worth d points, we can form the equation $5w + 3d = 31$ from Dynamics' statistics. Similarly, we can form the equation $6w + d = 32$ from Locos' statistics. We can ignore the lost games, as a loss is worth zero points here. Therefore :

$5w + 3d = 31$ $\quad A$
$6w + d = 32$ $\quad B$

We can multiply equation B by 3, and subtract equation A from the result :
$5w + 3d = 31$ $\quad A$
$18w + 3d = 96$ $\quad 3B$

$13w = 65$ $\quad 3B - A$ $\qquad \therefore$ $\boldsymbol{w = 5}$

Substituting 5 for w in the first equation gives $25 + 3d = 31$, and thus $3d = 6$ and $\boldsymbol{d = 2}$.
This league therefore awards 5 points for a win and 2 points for a draw.

Example (6): Use the elimination method to solve the simultaneous equations
$x + 4y = 2$; $x - 3y = -5$

$x + 4y = 2$ *A*
$x - 3y = -5$ *B*

It is possible to eliminate x by subtracting equation *B* from equation *A*.

$7y = 7$ *A-B*

This gives $y = 1$, and so the value could be substituted into either of the original equations.
Substituting into equation *A* gives $x + 4 = 2$, therefore $x = -2$.

The solution to these equations is therefore $x = -2$, $y = 1$.

Example (7):

Use the elimination method to solve the simultaneous equations
$3x - 4y = 17$; $2x + 5y = -4$

$3x - 4y = 17$ *A*
$2x + 5y = -4$ *B*

We need to multiply both equations by a suitable number so that one of the variables can be eliminated. (The choice of x as the variable to eliminate is arbitrary - we could have used y.)

Note that the multiples of x in the two equations are 2 and 3. We need these multiples of x to equal the L.C.M . of 2 and 3, namely 6, in order to eliminate, hence the choice of multipliers. Note also that, because the multiples have the same sign, we eliminate by subtraction.

$6x - 8y = 34$ 2*A*
$6x + 15y = -12$ 3*B*
$23y = -46$ 3*B* - 2*A*

This makes $y = -2$. Substituting into equation *A* gives $3x - (-8) = 17$, and thus $x = 3$.

$\therefore$ The solution is $\mathbf{x = 3, y = -2}$.

Example (7a): Repeat the last example, but eliminate y first.

Had we chosen to eliminate y, the working would have been

$3x - 4y = 17$ *A*
$2x + 5y = -4$ *B*

The L.C.M. of the coefficients of y is 20 (ignoring the signs), hence the multipliers. Also, since the signs of the multiples of y are different, we eliminate by addition.

$15x - 20y = 85$ 5*A*
$8x + 20y = -16$ 4*B*
$23x = 69$ 5*A* + 4*B*

This makes $x = 3$. Substituting into equation *A* gives $9 - 4y = 17$, and thus $y = -2$.

$\therefore$ The solution is $\mathbf{x = 3, y = -2}$.