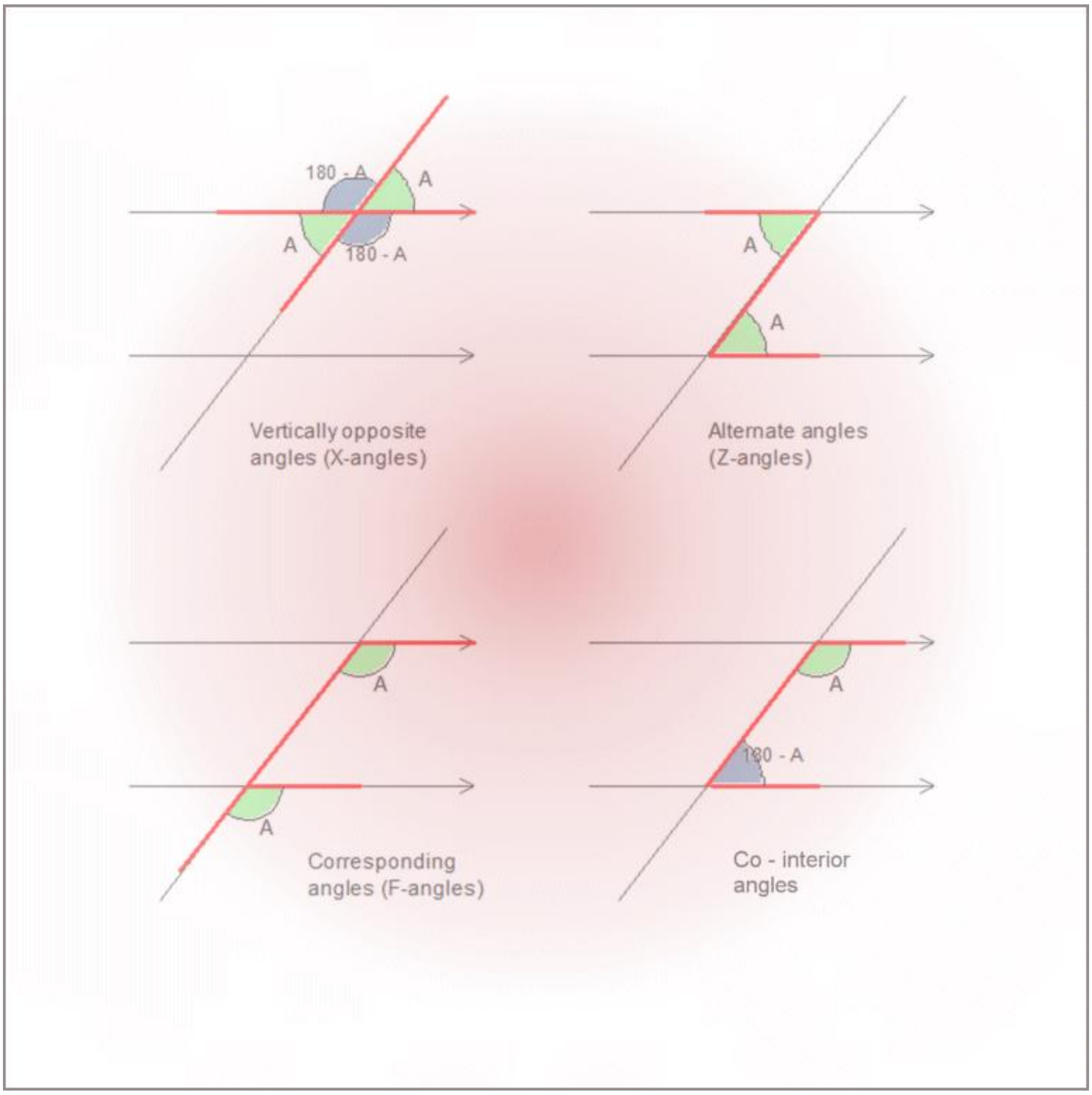
M.K. HOME TUITION

Mathematics Revision Guides

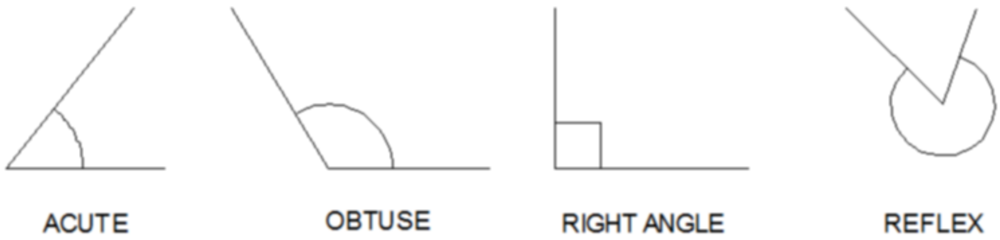
Level: GCSE Foundation Tier

SOLVING ANGLE PROBLEMS



Version: 2.2 Date: 02-12-2013

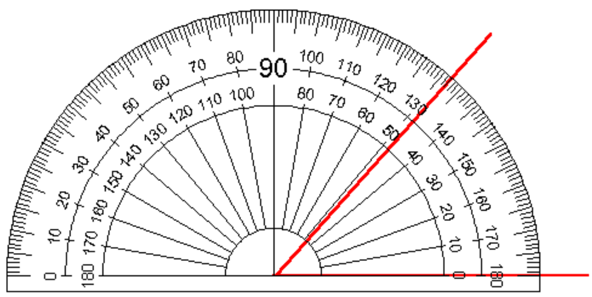
OVERVIEW – TYPES OF ANGLE



- An **acute** angle is an angle greater than 0° but less than 90°, and appears 'sharp' to the eye.
- An **obtuse** angle is greater than 90° but less than 180°, and appears more 'blunt'.
- A **right** angle is **exactly** 90°.
- An angle of 180° is merely a straight line.
- An angle greater than 180° but less than 360° is 'bent back' on itself and termed a **reflex** angle.

Using a protractor to measure angles.

The diagram shows how to align a protractor correctly.



Note that there are two scales – one clockwise and the other anticlockwise.

Line up the point of the angle against the centre of the semicircle in the protractor.

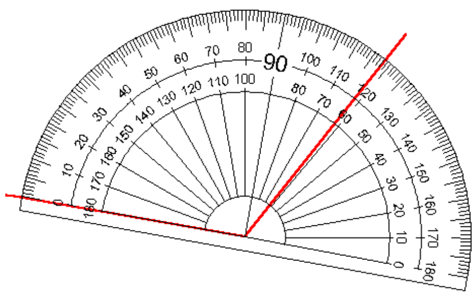
Then, line up one of the line segments making up the angle against one of the zero lines on the protractor (call it the base). Do not use the edge of the protractor for this !

Next, read the angle against the other line segment, making sure that the correct scale is used.
Here the angle can be either 48° or 132°, but we can avoid errors by noticing that it is **anticlockwise** from the base, so we read off the anticlockwise scale to obtain 48°.

(Alternatively we could have recognised the angle as being acute, and therefore between 0° and 90°, or 48°).

The next example is of an obtuse angle.

Here, the other line containing the angle is clockwise of the base line, and so we read the **clockwise** scale of the protractor to obtain 118° and not 62°.



(Again, recall that obtuse angles are between 90° and 180°)

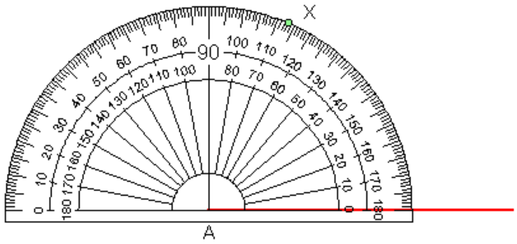
To measure a **reflex** angle using a 180° protractor, we must measure the **difference** from 360°.

If the angle in the last example were reflex, then we would have needed to subtract 118° from 360° to get 242°.

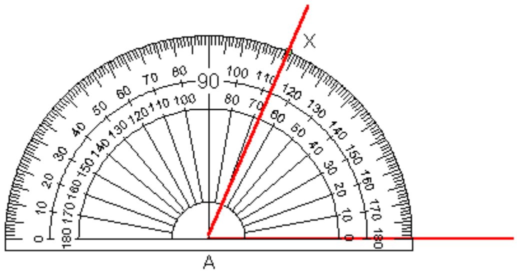
Constructing accurate angles using a protractor.

Here we are given a base and are required to construct a 67° angle at point A, anticlockwise to the base.

We therefore line up point A with the 'centre' of the protractor and mark a point X at 67° on the anticlockwise scale.

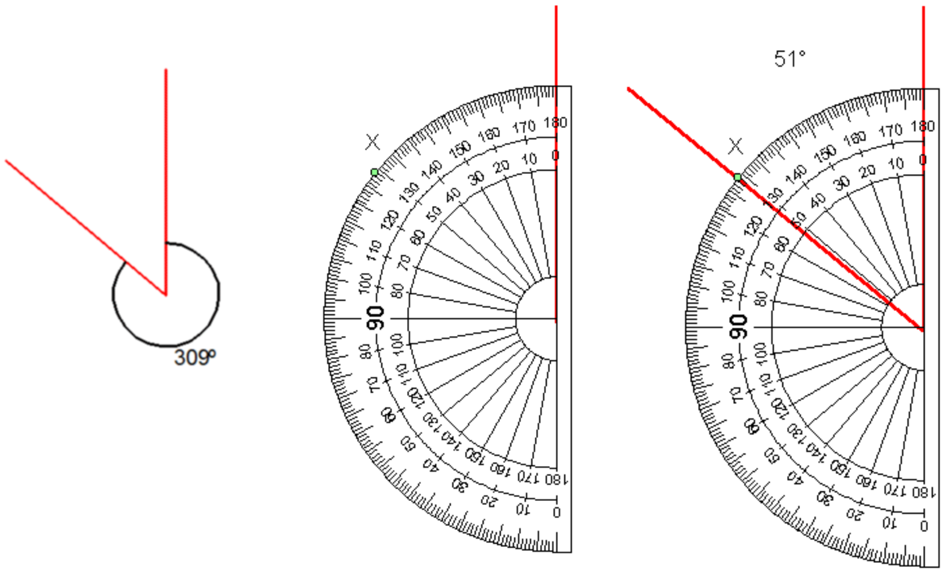


Then, we join A to X and complete the angle.



This method is needed to construct bearing diagrams and to draw triangles when angles are given.

Example: This angle is not drawn accurately. Construct an accurate angle here, using the same vertical line.

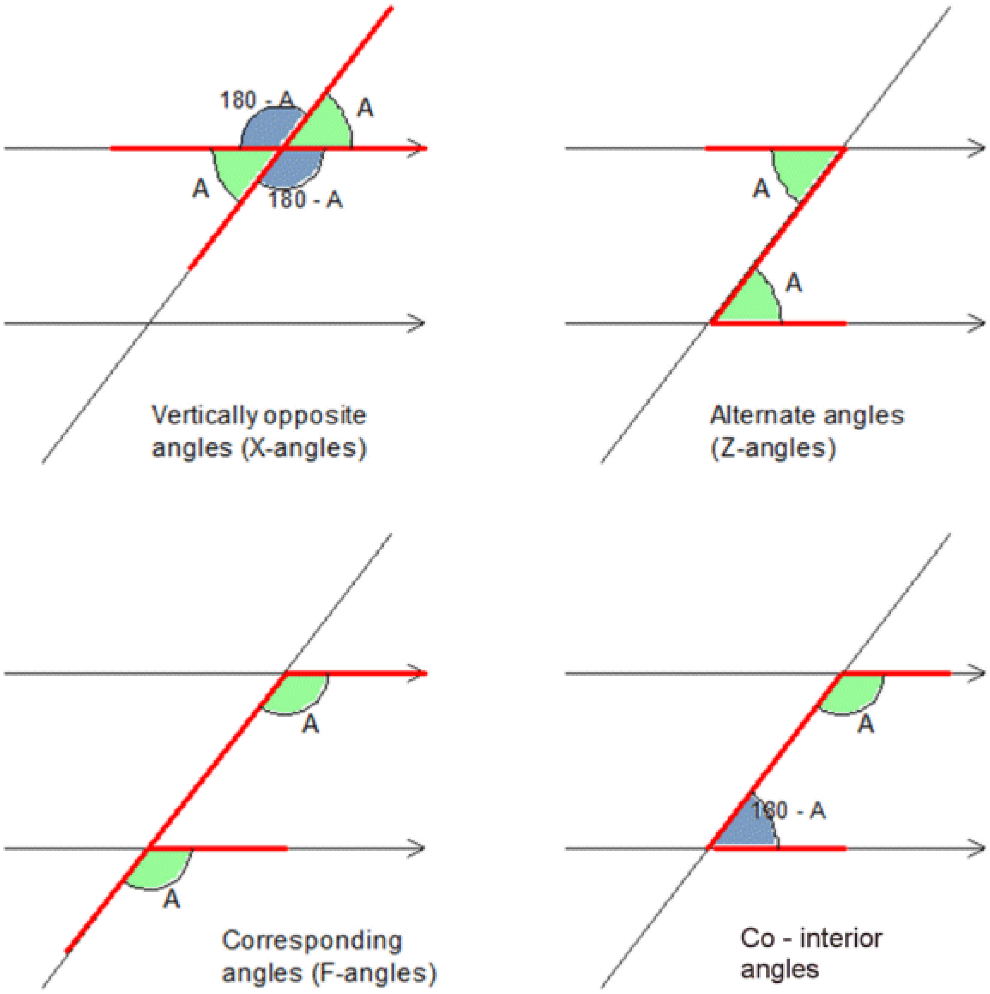


Because this is a reflex angle measured **clockwise** from the vertical, we must construct its difference from 360° **anticlockwise** to obtain the same angle.

Since 360 – 309 = 51, so we are effectively drawing a 51° angle anticlockwise from that vertical. marking point X at 51°.

(See diagrams above right for the construction method .)

SOLVING ANGLE PROBLEMS.

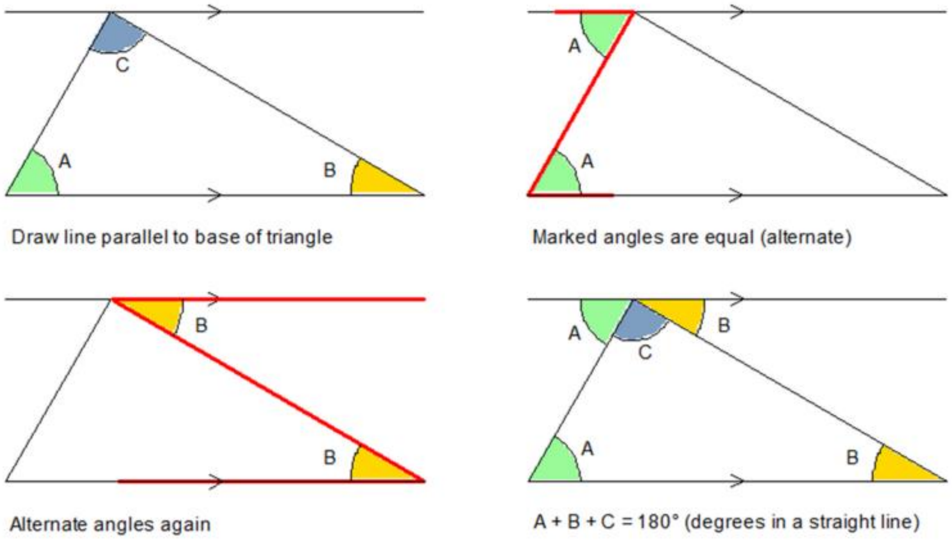


The diagrams above are the key to solving many angle problems in plane geometry, especially those involving parallel lines.

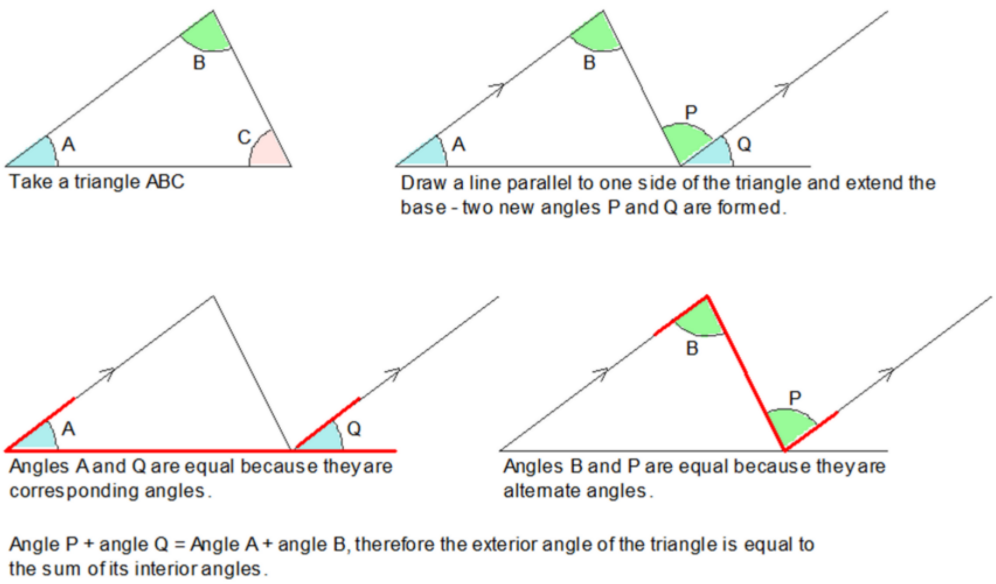
In each diagram, angles marked *A* are identical.

Remember that the angle in a straight line is 180° and that angles around a point sum to 360°.

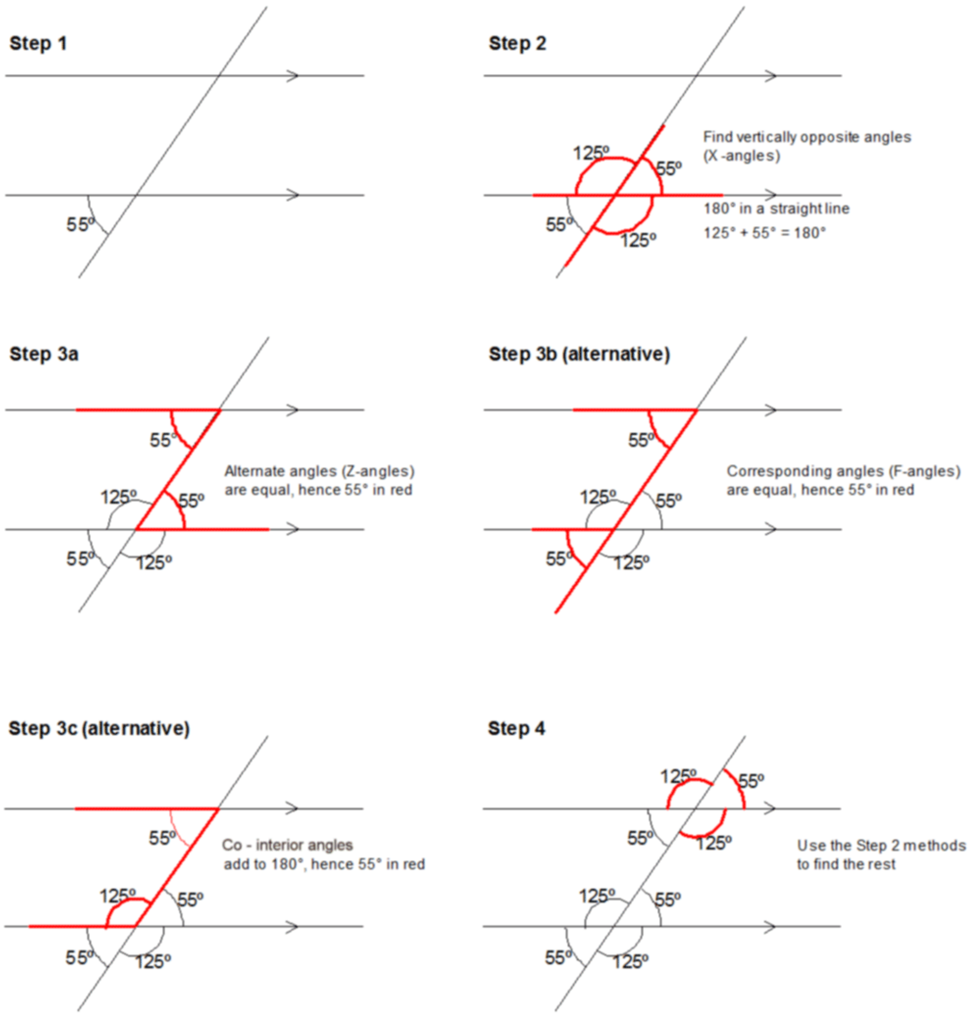
Example 1. Prove that the angles of a triangle sum up to 180°.



Example 2. Prove that the exterior angle of a triangle is equal to the sum of the other two interior angles.



Example (3): Find all the unknown angles in the "Step 1" diagram. The arrowed lines are parallel.



The original diagram in Step 1 only gives us an angle of 55° around one intersection.

In Step 2, we use the fact that vertically opposite angles (X-angles) are equal, and also the fact that there are 180° in a straight line, to determine all the angles around that point. Note: 125° + 55° = 180°.

In Step 3, we can choose various strategies to determine the angles around the other intersection.
We can use the equality of alternate angles as in 3a, the equality of corresponding angles (F-angles) as in 3b, or co-interior angles summing to 180° as in 3c.

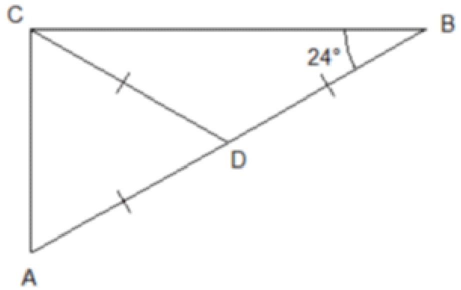
In Step 4, Having found one angle in Step 3, we can use vertically opposite angles and 180° in a straight line to find the other three. Note as well how all four angles around the upper intersection correspond to all four angles around the lower one.

Example (4):

In the triangle on the right (not drawn accurately) , $\angle$ *CBD* = 24°.

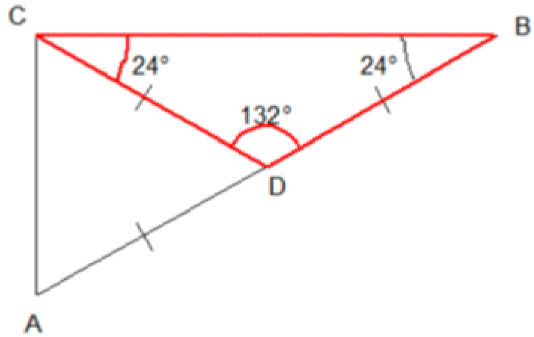
Lengths *AD*, *BD* and *CD* are also all equal.

Find $\angle$ *ACD*, and hence show that $\angle$ *ACB* is a right angle.



Since *CD* = *DB*, Δ*CDB* must be isosceles, and so $\angle$ *DCB* = 24°.

Therefore $\angle$ *CDB* = 180° – (2 × 24)° = 132°

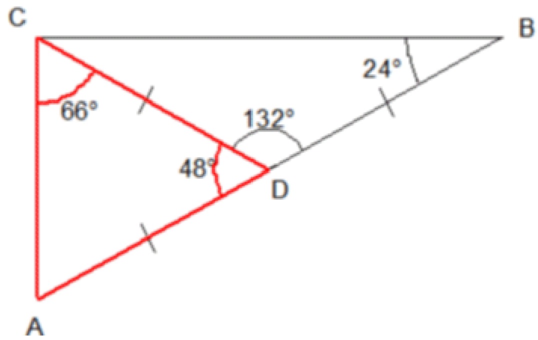


Having found $\angle$ *CDB*, we know that $\angle$ *ADC* = (180 – 132)° = 48°.
(180° in a straight line !)

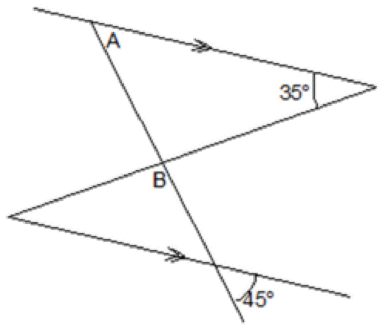
Since *CD* = *AD*, Δ*ACD* must be also be isosceles, and so $\angle$ *CAD* and $\angle$ *ACD* must also be equal.

Thus $\angle$ *ACD* = ½(180-48)° = 66°.

Finally, $\angle$ *ACB* = $\angle$ *ACD* + $\angle$ *DCB*
= 66° + 24° = 90°.

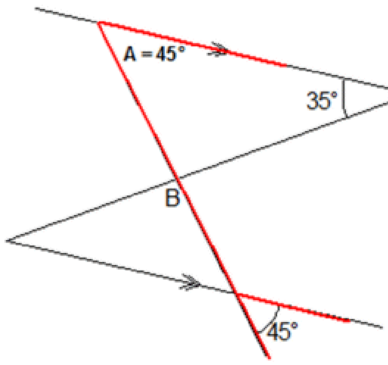


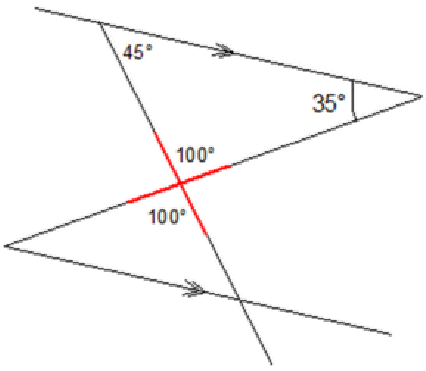
Example (5a): Find angles *A* and *B* in the diagram below.



We can find angle *A* by noticing that it and the 45° angle are corresponding angles.

Thus *A* = 45° (corresponding angles are equal). (see lower left)

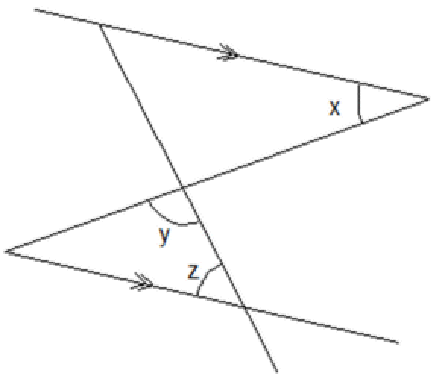


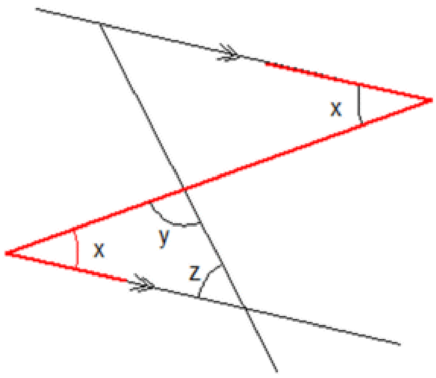


To find angle *B*, we can subtract 45° + 35° from 180° to find the last angle of the upper triangle, i.e. 100°.

Angle *B* is vertically opposite, so it is also equal to 100°. (see lower right).

Example (5b): Prove that $x + y + z = 180°$ in the diagram below.



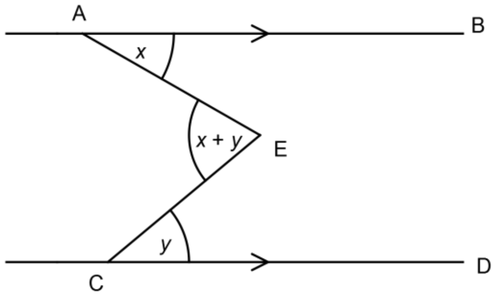


The proof is easily found using alternate angles (right).

We now have the lower triangle complete, so $x + y + z = 180°$ (angle sum of triangle).

Example (5c): Lines *AB* and *CD* are parallel, and point *E* lies between the lines.
Let angle *BAE* = x and angle *DCE* = y.

Prove that angle *AEC* = $x + y$.

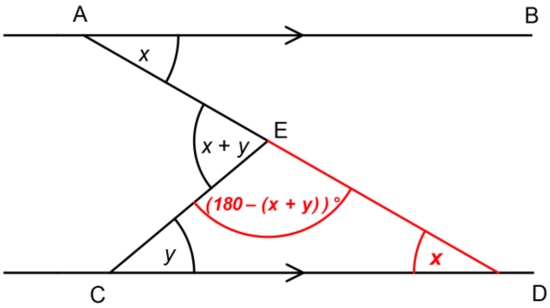


Proof using angle sum of triangle.

Extend line *AE* until it meets line *CD* at point *D*.

By alternate angles, $\angle CDE = x$.
Hence $\angle CED = (180 - (x + y))°$ and
$\angle AEC = (180-(180-(x + y)))°$
$= x + y$ as requested.

Also. the exterior angle *AEC* is the sum of the interior angles *ECD* and *EDC.*



Proof using alternate angles only.

Draw a line parallel to *AB* and *CD*, passing through point *E*, and thus dividing angle *AEC* into two parts, angles *AEP* and *CEP*.

By alternate angles, $\angle AEP = \angle BAE = x$.
Also, $\angle CEP = \angle DCE = y$.

Hence $\angle AEC = \angle AEP + \angle CEP = x + y$.
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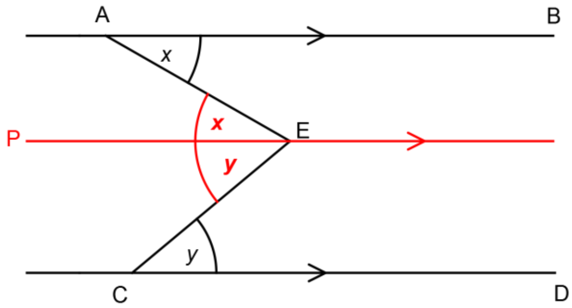
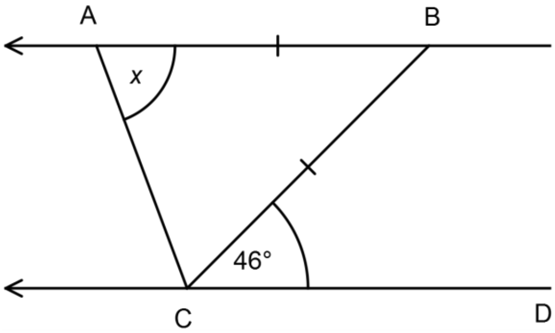
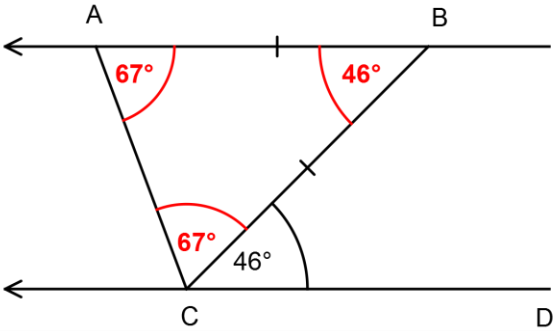


Example (6): Find the size of angle *CAB*, labelled *x* in the diagram.





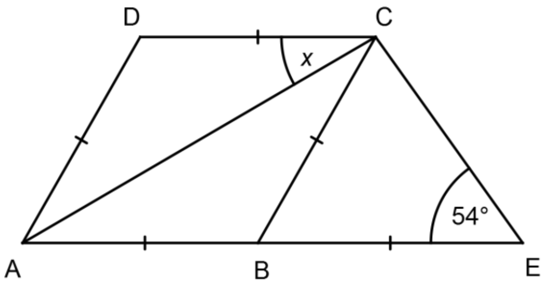
Angles *BCD* and *ABC* form a pair of alternate angles, so $\angle ABC = \angle BCD = 46°$.
Also, *AB* = *BC*, so Δ *ABC* is isosceles, and ∠ *ACB* and ∠ *CAB* must also be equal.

Thus $\angle CAB = \frac{1}{2}(180-46)° = 67°$.

The following examples assume knowledge of the properties of quadrilaterals.

Example (7): Figure *ABCD* is a rhombus, the triangle *BEC* is isosceles, and *ABC* is a straight line.

Work out the size of angle *DCA*, labelled *x.*
(Diagram not drawn accurately)

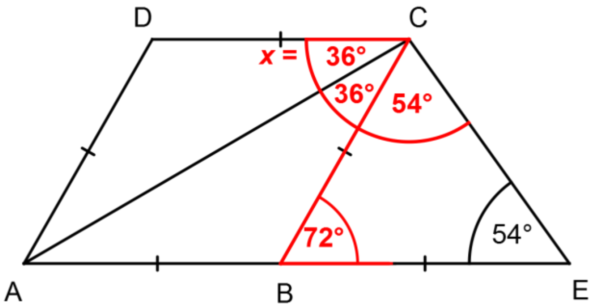


Because the triangle *BEC* is isosceles with *BC* and *BE* as the two equal sides, ∠ *BCE* = 54° and the third angle, ∠ *CBE* = 72°.

Next, we deduce that ∠ *DCB* = ∠ *CBE* = 72°, using alternate angles.

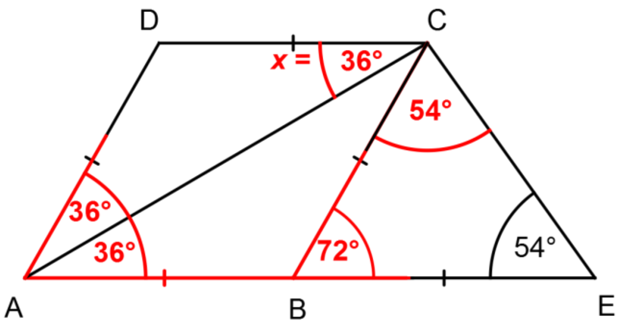
The diagonals of a rhombus are also its lines of symmetry, therefore they bisect its angles.

Hence angle *DCA*, labelled *x*, is half of 72° or **36°**.



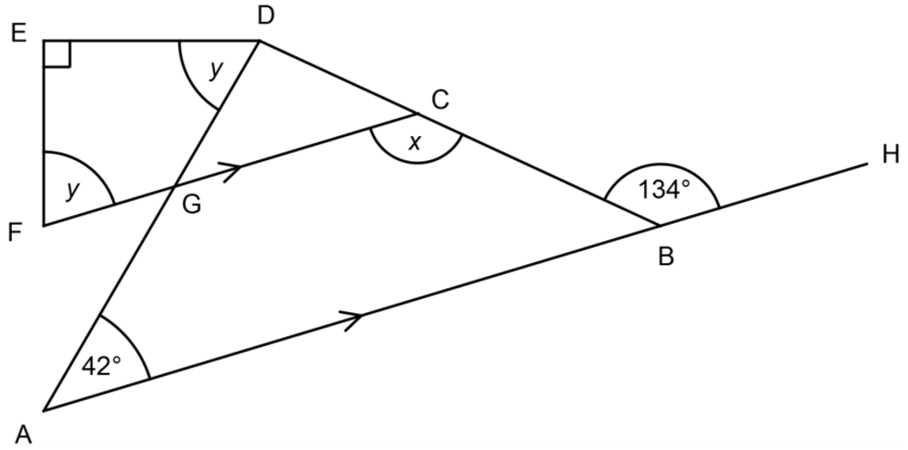
Alternatively, we could have deduced that ∠ *DAB* = ∠ *CBE* = 72°, using corresponding angles and then continued with ∠ *DCB* = ∠ *DAB* = 72° because opposite angles of a rhombus are equal.

Many angle problems can be solved in such a variety of ways, all equally valid. Just don't waste time on unnecessary detours !



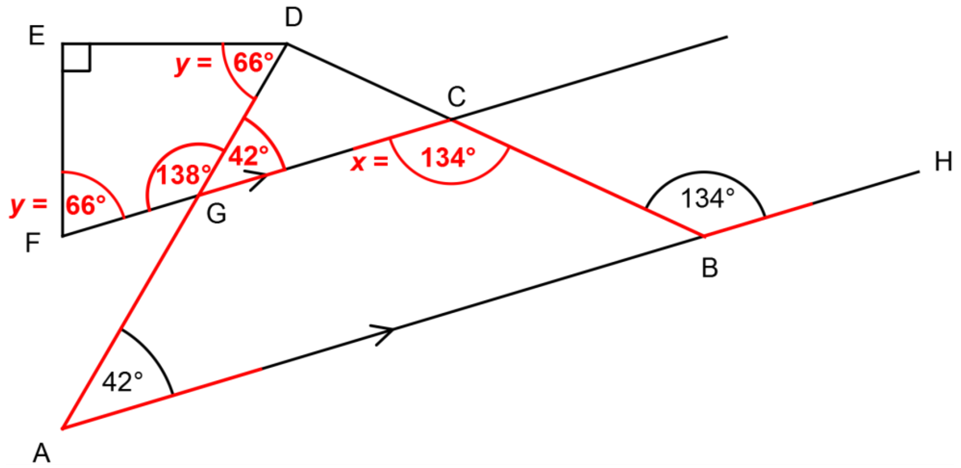
Example (8): Lines *ABH* and *FGC* are parallel and angle *DEF* is a right angle.
The angles *GFE* and *GDE*, labelled *y*, are equal. (Diagram not drawn accurately).

i) State the size of angle *x*, giving a reason for your answer.
ii) Work out the size of angle *y*, showing clear working and reasons.



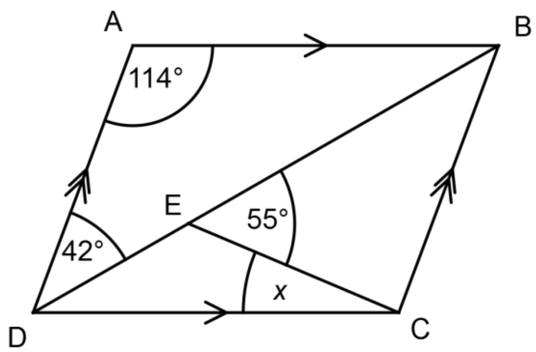
i) Angle *DCB* (marked *x*) = **134°** because it and angle *CBH* are alternate angles and therefore equal.

ii) By corresponding angles, $\angle DGC = \angle DAB = 42°$, so $\angle FGD = (180 - 42)° = 138°$.
We now have two angles of the quadrilateral *DEFG*, so the two angles labelled *y* must sum to $(360 - (138 + 90))°$ or 132°. Hence each angle labelled *y* is half of 132°, or **66°**.



Example (9): Figure *ABCD* is a parallelogram, where ∠ *ADB* = 42°, ∠ *CEB* = 55° and ∠ *DAB* = 114°.

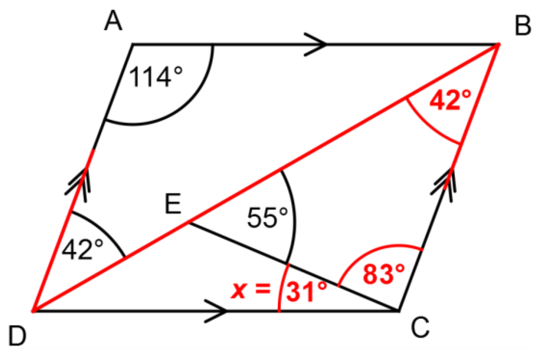
Calculate the size of angle ∠ *DCE*, labelled *x* in the diagram (not accurately drawn).



Because *ABCD* is a parallelogram, sides *AD* and *BC* are parallel, so angles *ADB* and *DBC* form a pair of alternate angles, each of size 42°.

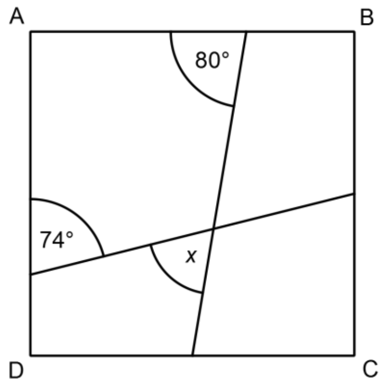
We now have two angles of triangle *EBC*, so the missing angle *ECB* = (180 – (55 + 42))° = 83°. (Interior angle-sum of a triangle = 180°).

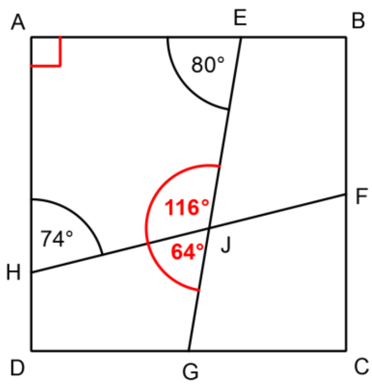
Because opposite angles of a parallelogram are equal, ∠ *DCB* = ∠ *DAB* = 114°.
Hence the missing angle *DCE* (labelled *x*) = ∠ *DCB* - ∠ *ECB* = 114° - 83° = **31°**.



Example (10): *ABCD* is a square.

Work out the size of angle x, showing all working.
(Not drawn accurately)





We cannot use corresponding angles to say that $\angle HJG = x = 80°$ because the line *HF* is not parallel to line *AB.*

Neither can we use alternate angles to say that $\angle HJG = x = 74°$ since lines *EG* and *AD* are not parallel.

Because *ABCD* is a square, $\angle DAB = 90°$. Also, the angles of a quadrilateral add to 360°, so $\angle HJE = (360 – (90 + 80 + 74))°$ or 116 °.

Finally, $\angle HJE = x = (180\text{-}116)° =$ **64°** because there are 180° in a straight line.