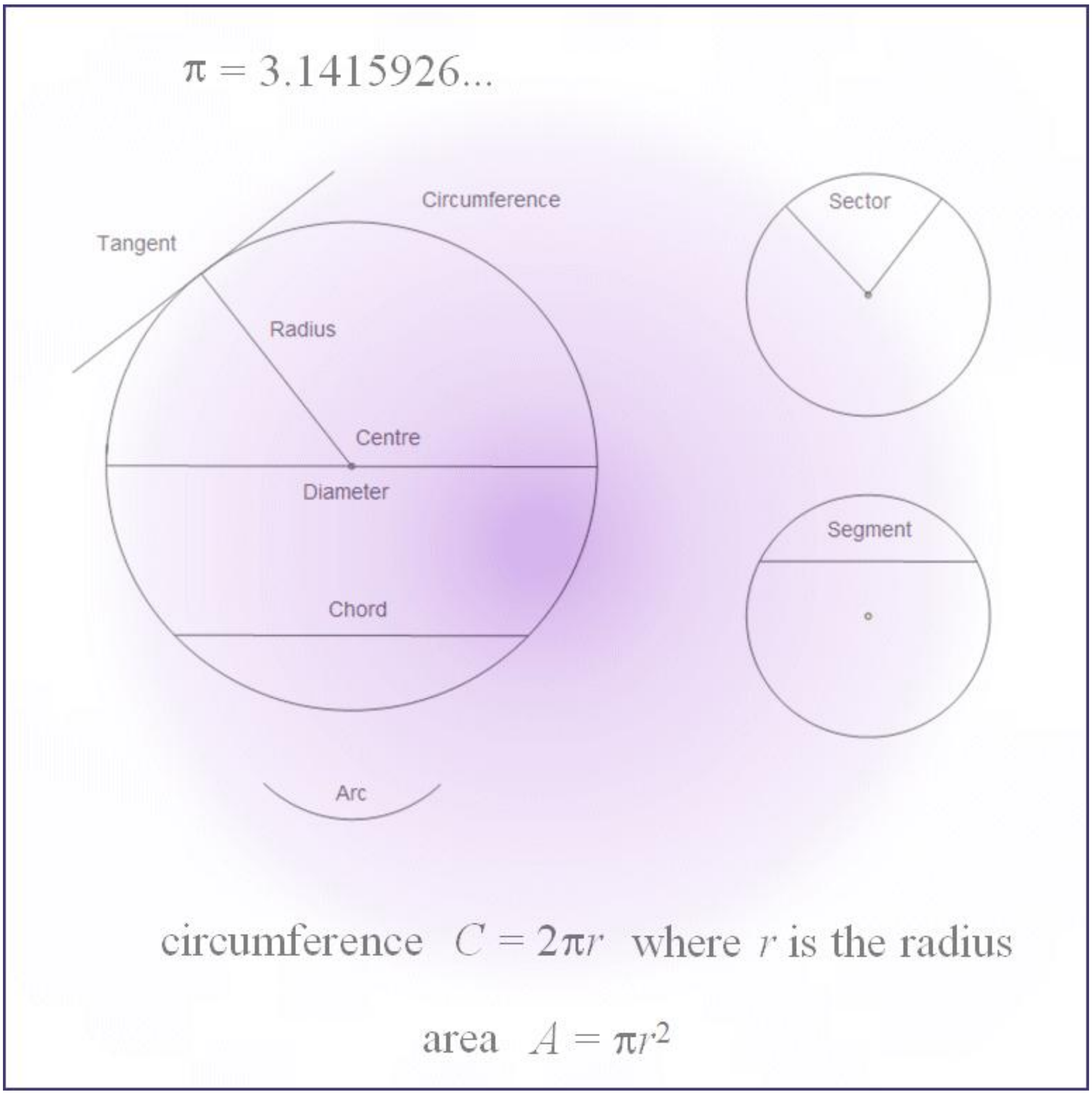
Author: Mark Kudlowski

M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

CIRCLE PROPERTIES



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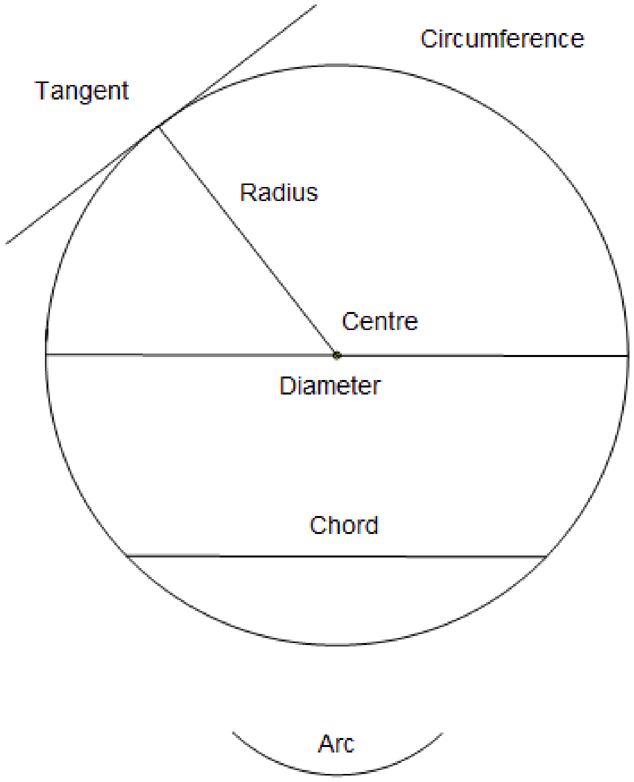
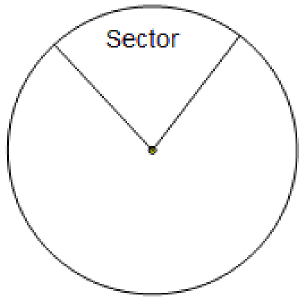
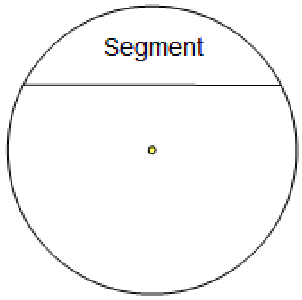
CIRCLE PROPERTIES

Recall the following definitions relating to circles:







A circle is the set of points at a fixed distance from the **centre**.
The perimeter of a circle is the **circumference**, and any section of it is an **arc**.
A line from the centre to the circumference is a **radius** (plural: **radii**).
A line dividing a circle into two parts is a **chord**.
If the chord passes through the centre, then it is a **diameter**.
A diameter divides a circle into two equal parts, and its length is twice that of a radius.
A **tangent** is a line that touches a circle at one point.

If the chord is not a diameter, the two resulting unequal regions of the circle are the (major and minor) **segments**.

A **sector** is a region of a circle bounded by two radii. The smaller one is the minor sector, the larger one the major sector.

To complete the list, arcs can be termed major and minor arcs, depending on the sector or segment in question.

Circular Measure.

Introductory Example (1): Ashley measured the diameters and circumferences of various circular household objects by using a flexible tape rule, and in each case he divided the circumference by the diameter. His results were tabulated as follows :

Item	Circumference (cm), C	Diameter (cm), d	Quotient, C/d
Dinner plate	85	27	3.15
Coffee tin	31	10	3.10
CD	38	12	3.17
Pill box	22	7	3.14

The measurements may have been a little rough and ready, but we can see that dividing the circumference of each circular object by its diameter gave the same order of result in each case.

This result, which is a little over 3, is a constant quantity, given the Greek letter symbol of π (pronounced 'pie'), which is the ratio of a circle's circumference to its diameter. It is not a whole number, nor is it an exact fraction or decimal.

Its value is 3.1415926... but is often taken as 3.14 or $\frac{22}{7}$ in non-calculator questions.

- To find the circumference of a circle, use the formula $C = 2\pi r$ where r is the radius (or πd where d is the diameter). Remember that a diameter is twice as long as a radius.

Example (2): A cycle wheel has a diameter of 70cm. Find the distance travelled after the wheel has made 500 complete revolutions. (1 revolution = 1 circumference). Use $\pi = \frac{22}{7}$.

The diameter d is 70 cm, so the circumference is $\pi d = \frac{22}{7} \times 70 = 220$ cm = 2.2m.
Since 1 revolution = 2.2m, 500 revolutions = (2.2×500)m = 1100m = 1.1km.

Hint: Non-calculator questions often ask for answers to be left in terms of π.
If the question asked for the result to be left as a multiple of π, we would write:

The diameter d is 70 cm, so the circumference is $\pi d = 70\pi$ cm = 0.7π m.
Since 1 revolution = 0.7π m, 500 revolutions = $(0.7\pi \times 500)$ m = 350π m.

Example (3): Another cycle has a rev counter on its front wheel counting the number of revolutions made from a starting point.

A cyclist resets the rev counter to zero before a trial race of 4km. The rev counter reads 1852 at the end of the trial. What is the diameter of the front cycle wheel ? (Use the π key on your calculator).

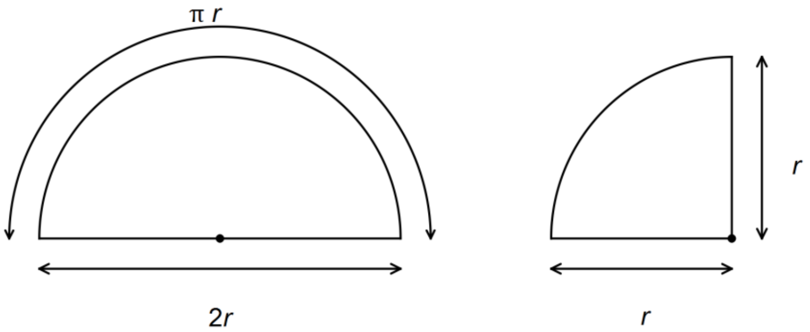
Here, 1852 revolutions = 4000m, so one revolution = 1 circumference = $\frac{4000}{1852}$ m or 2.160m
$\therefore$ Diameter of wheel = $\frac{2.160}{\pi}$ m = 0.687m = 69cm to nearest cm.

- To find the area of a circle, use formula $A = \pi r^2$ where r is the radius.

Example (4): A circular ornamental lake in a park has a diameter of 38m. Work out its area, using the π key on your calculator.

We are given the diameter here, so we need to halve it first to obtain the radius, namely 19m.
The area is πr^2 , or $\pi \times 19^2$, or 361π m^2 or 1130 m^2 to 3 s.f.

The formulae for the area and circumference of a circle can be readily adapted for semicircles and quadrants (quarter-circles).



The area of a semicircle is given by $\frac{\pi r^2}{2}$ (half that of a circle !) and the arc length by πr. Its perimeter also includes a 'straight' of length $2r$, so its value is $\pi r + 2r$ or $(\pi + 2)r$.

The corresponding formulae for the quarter-circle are :

Area = $\frac{\pi r^2}{4}$; Arc length = $\frac{\pi r}{2}$; Total perimeter = $\frac{\pi r}{2} + 2r$.

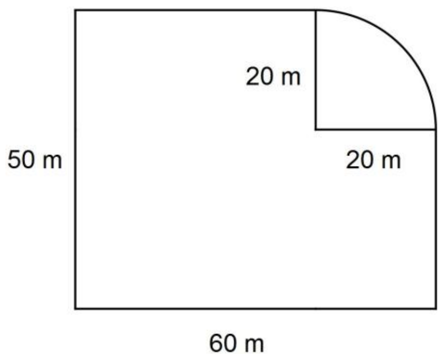
Example (5): A semicircular table top has a straight edge of 1.26 m. Calculate its area and perimeter, giving answers to 3 significant figures.

The straight edge of the table top is its diameter, so the radius is 0.63 m.

The area is $\frac{\pi r^2}{2} = \frac{\pi(0.63^2)}{2}$ = 0.623 m^2.

The perimeter is $\pi r + 2r$ = (0.63π + 1.26) m = 3.24 m .

Example (6): A school's playing field has the following plan, where the section in the top right-hand corner is a quarter-circle. Find its area to the nearest square metre and perimeter to the nearest metre.



The playing field is basically a rectangle measuring 60 m by 50 m, but with a square of side 20 m removed and replaced with a quarter-circle of radius 20 m.

Therefore, area of original rectangle = 60 m × 50 m = 3000 m^2.
Area of square (to be subtracted) = 20^2 m^2 = 400 m^2.

Area of quarter-circle (to be added) = $\frac{\pi r^2}{4} = \frac{\pi(20)^2}{4}$ = 100π m^2 = 314 m^2.

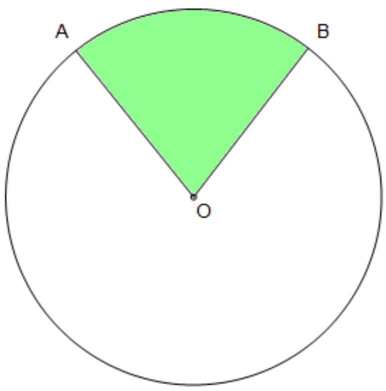
Hence the area of the playing field = (3000 – 400 + 314) m^2 = **2914 m^2.**

The straight line sections of the playing field total 60 + 50 + (60 – 20) + (50 – 20) m, or 180 m.

The quarter-circular arc is $\frac{\pi r}{2}$ = 10π m, or 31 m to the nearest metre.

The perimeter of the playing field is 180 + 31 = **211 m** .

Sectors.



We can generalise the cases of the semicircle and quarter-circle to find areas and perimeters of sectors of any angle.

To recall, a sector is a part of a circle bounded by two radii and an arc. In the diagram, *OA* and *OB* are radii and *AB* is the corresponding arc.

The shaded region is the sector *AOB.*

A whole circle can be said to be a sector with an angle of 360°, and its area is $A = \pi r^2$.

It can be seen that if a sector has an angle $\theta°$, this angle is $\frac{\theta}{360}$ of a circle.

We can therefore find the area of the sector by multiplying the area of the circle by $\frac{\theta}{360}$.

The area of a sector of angle $\theta°$ and radius *r* is therefore $\frac{\pi r^2\theta}{360}$.

Similarly, the arc length of a sector can be obtained from the formula for the circumference and multiplying it by $\frac{\theta}{360}$.

$\therefore$ arc length of a sector of angle $\theta°$ and radius *r* is given by $\frac{(2\pi r)\theta}{360} = \frac{\pi r\theta}{180}$.

Example (7): Find i) the area and ii) the perimeter of a 60° sector of a circle of radius 24 cm.

i) We substitute $\theta = 60°$ and $r = 24$ cm into the formula $\frac{\pi r^2\theta}{360}$ to obtain the area of the sector :

$\frac{\pi \times 576 \times 60}{360}$ cm^2 = 96π cm^2 or **302 cm^2.**

ii) We similarly substitute $\theta = 60°$ and $r = 24$ cm into the formula $\frac{\pi r\theta}{180}$ to obtain the arc length :

$\frac{\pi \times 24 \times 60}{180}$ cm = 8π cm or 25.1 cm. We must also add the lengths of the two radii to obtain the full perimeter of (24 + 24 + 25.1) cm or **73.1 cm**.

Note:. We could have realised that 60° is one sixth of a circle and simplified the calculations:

Area : $\frac{\pi \times 576}{6}$ cm^2 = 96π cm^2 ; arc length: $\frac{\pi \times 24}{3}$ cm = 8π cm and proceeded from there.