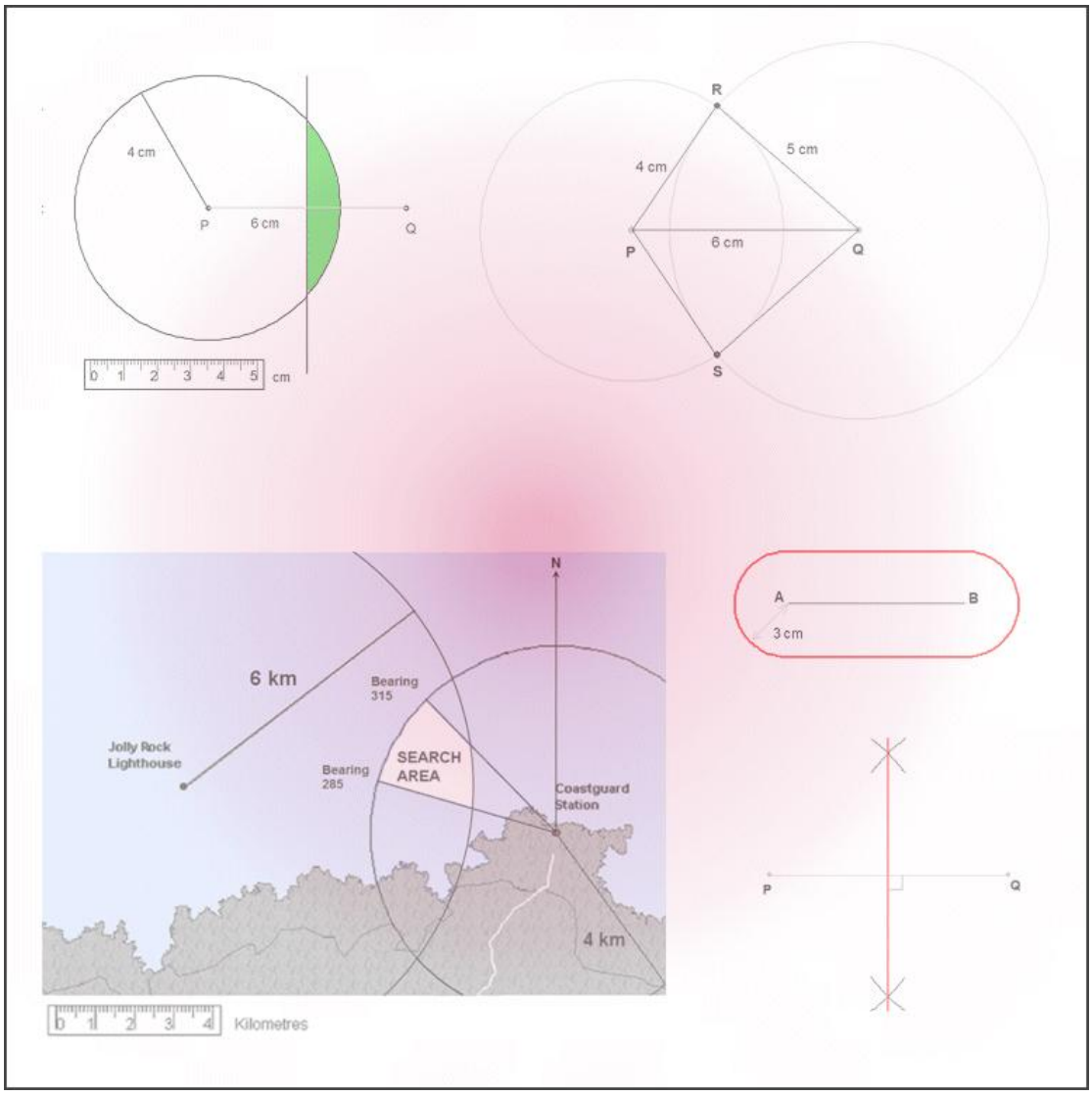
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

LOCI



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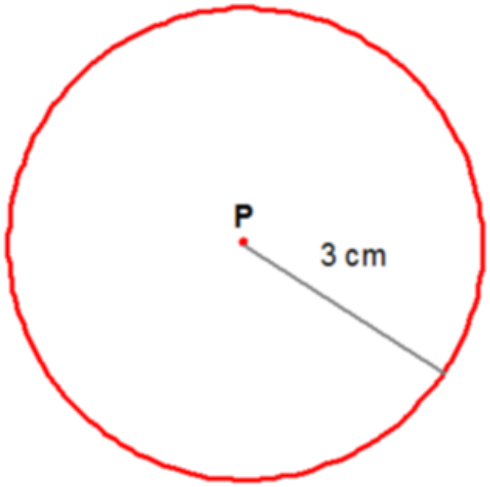
LOCI

Don't be put off by this word ! A **locus** (plural: **loci**) is a set of points satisfying certain given conditions, usually connected with distance from a given point, line or shape.

The locus of points equidistant from a fixed point is a circle whose centre is the fixed point and whose radius is the distance.

Example (1): Draw the locus of points 3 cm from the given point P.



The locus is merely a circle with radius 3 cm centred on P.

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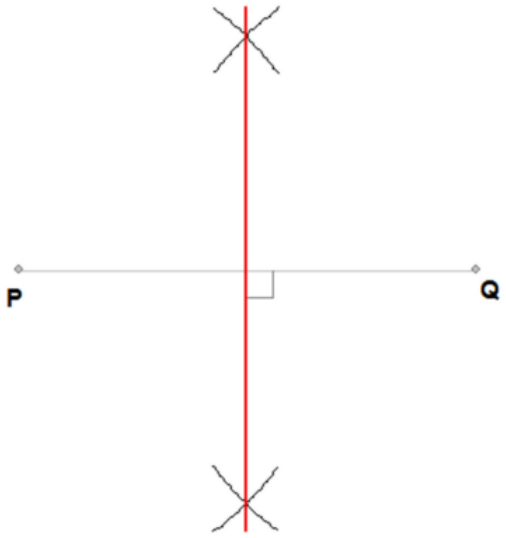
The locus of points equidistant from two points is the perpendicular bisector of the line joining them.

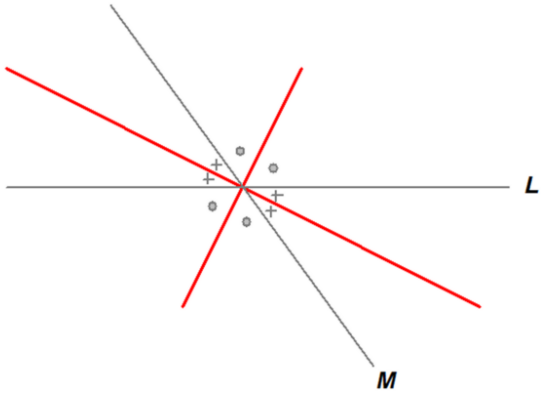
See the example on the right, with construction arcs included.

Although shown as a finite line, the complete locus is infinite in length.



The locus of points equidistant from two non-parallel lines consists of the two bisectors of the angles formed by the lines.

As the diagram on the right shows, these bisectors are also perpendicular.

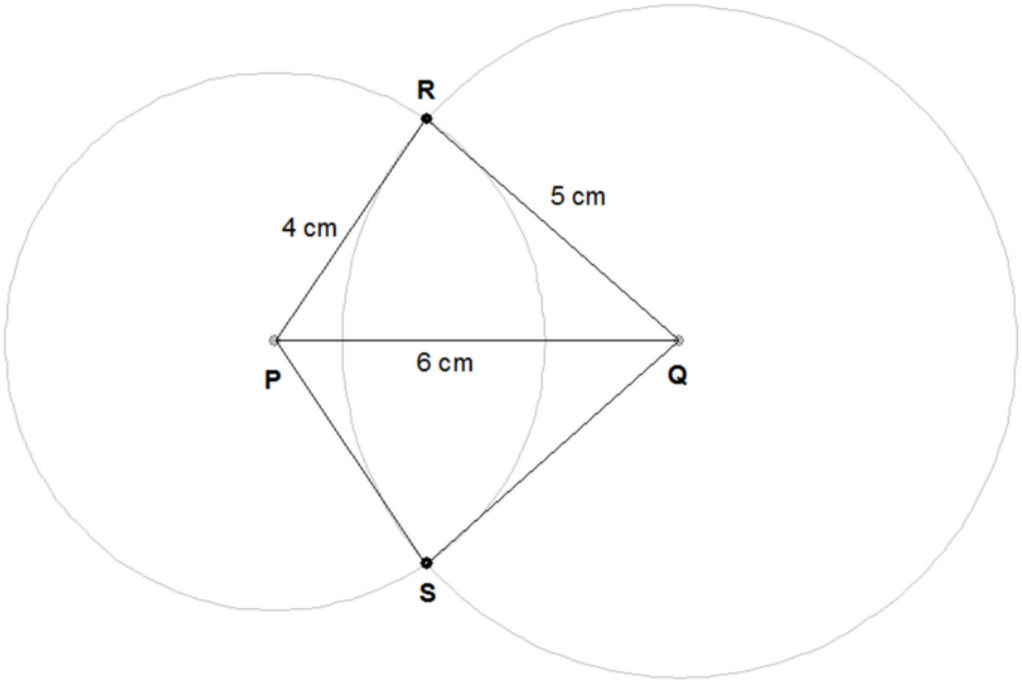


Example (2): Points P and Q are 6 cm apart.

What is the locus of points 4 cm from P and 5 cm from Q ?

The locus of points 4 cm from P is a circle centred there, and of radius of 4 cm.
Similarly, the locus of points 5 cm from Q is a circle whose centre is Q and whose radius is 5 cm.

We therefore draw two circles as in the diagram below.



The set of points where **both** conditions are satisfied, i.e. the required locus, is the pair of points where the two circles intersect, shown as R and S in the diagram.

(This construction is the same as the one used in drawing a triangle given the three sides.)

The locus of points equidistant from a finite straight line is an oval curve made up of two line segments and two semicircles.

Example (3): Draw the locus of all points 3 cm away from the line segment AB.

We begin by drawing two line segments parallel and equal in length to AB, and 3 cm away from it. The ends of the line segments must also be aligned, as in the left-hand diagram.

This is not complete, because if we take the end points A and B of the line, we can still include points that are a radius of 3 cm away from those points. These additional points describe semicircles, with the result that the complete locus forms an oval curve, as in the diagram on the right.

Example (4) (Harder):

Points P and Q are 6cm apart.

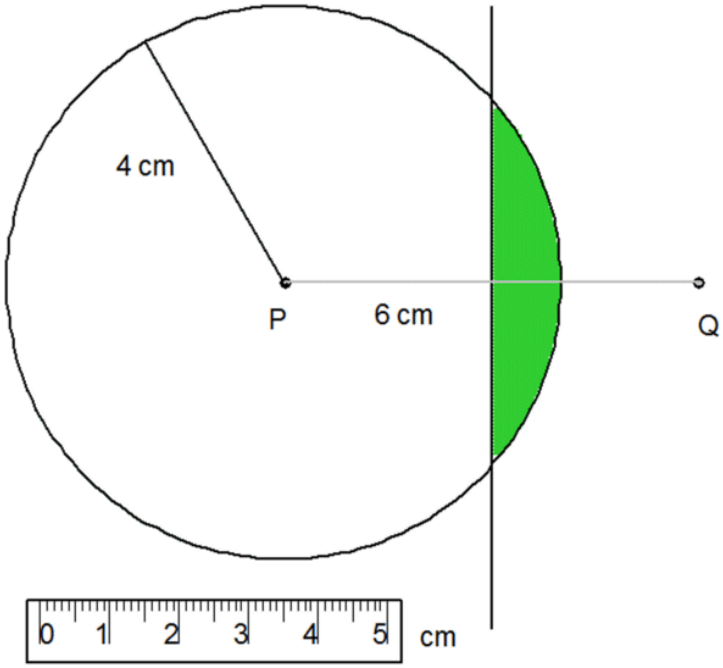
Draw the locus of points less than 4 cm away from P, but also closer to Q than P.

The points less than 4 cm away from P would lie inside the circle with 4 cm radius centred on P.

The points closer to Q than P would lie to the right of the perpendicular bisector of PQ.

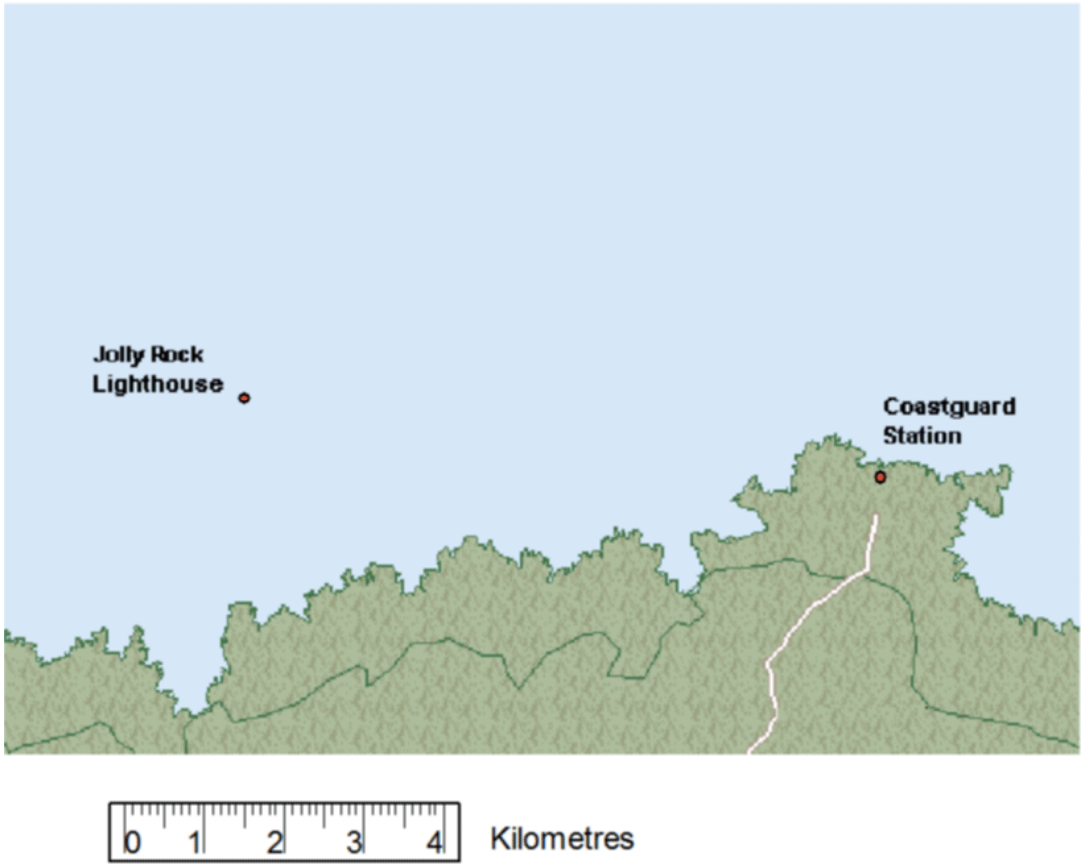
Combining both conditions gives the shaded segment on the right.

Note that points on the edges of the segment do not satisfy the strict rules of the two conditions



Example (5).
A sailing boat crew are in difficulty and send out a signal, which is picked up by operators at the coastguard's office and at the Jolly Rock lighthouse.

The lighthouse keepers work out that the boat is no more than 6 km away from their site. The coastguards also know that the boat is no more than 4 km from their office, and at a bearing of between 285 and 315 from it.



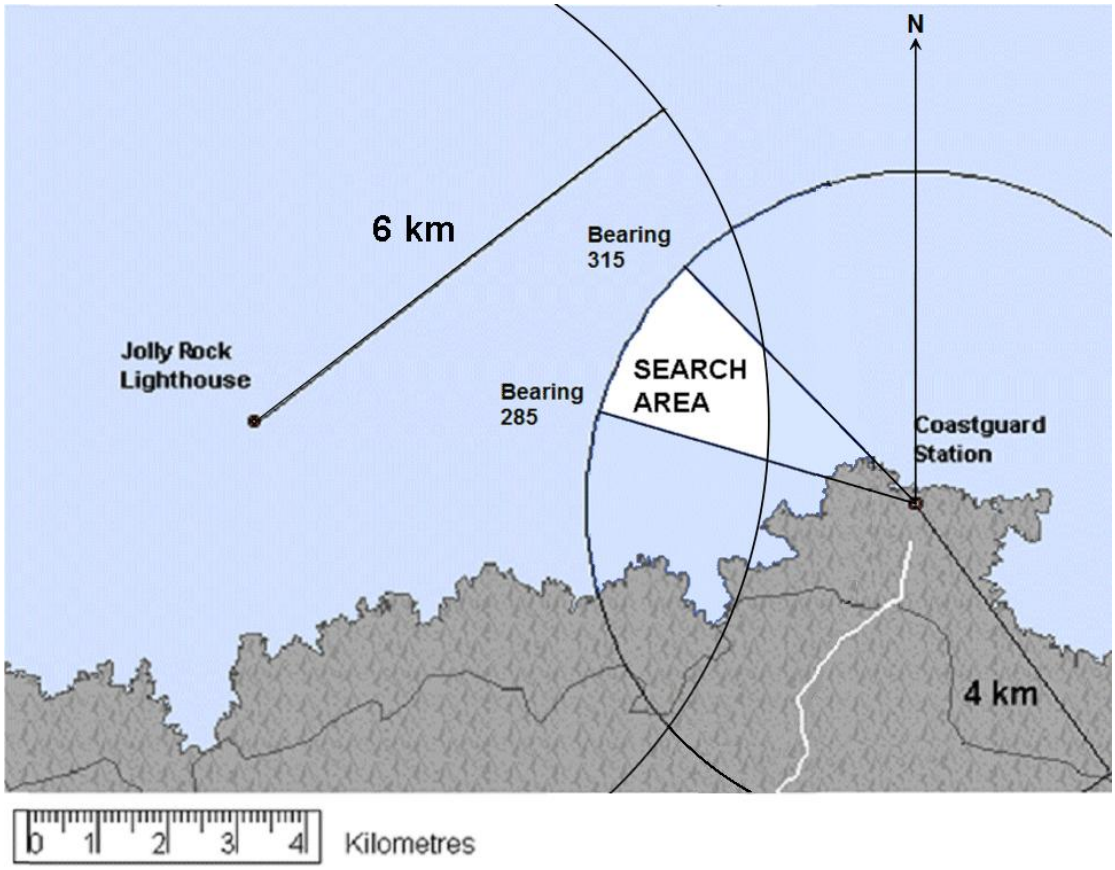
Shade in the possible region where the boat could be located.

We begin by looking for a region of overlap between two circles. One has a radius of 6 km (scaled as required) and is centred on the lighthouse; the other has a radius of 4 km and is centred on the coastguard station.

In addition we must add two bearing lines from the coastguard station; one at 285 and one at 315. To do this we draw in a northline, measure the two angles with a protractor, and draw radii from the coastguard station.

(Since both 285° and 315 °are reflex, we subtract from 360° and mark anticlockwise angles of 75° and 45°. Another hint is that a bearing of 270 is due west and a bearing of 315 is due northwest.)

The region of overlap (shaded) is shown overleaf.


N
6 km
Bearing
315
Jolly Rock
Lighthouse
SEARCH
AREA
Bearing
285
Coastguard
Station
4 km
0 1 2 3 4
Kilometres

Example (6): A rectangular garden is 16 metres long and 12 metres wide.

There is an apple tree at **A**, 3 m from the west wall and 6 m from the south wall, and a birch tree at **B**, 2 m from the east wall and 8 m from both the north and the south walls.

Charlie wants to plant a cherry tree, but is very particular about the location.

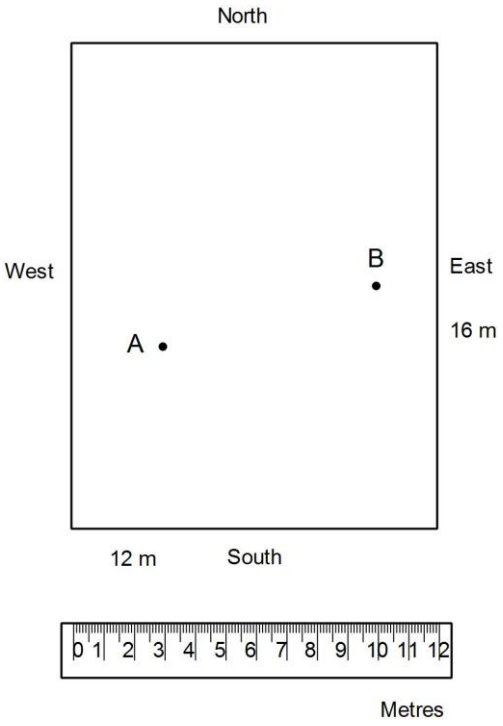
a) It must be at least 5 m from the south wall.

b) It must be closer to the south wall than the west wall.

c) It must be at least 4 m away from the apple tree.

d) It must be at least 3 m away from the birch tree.

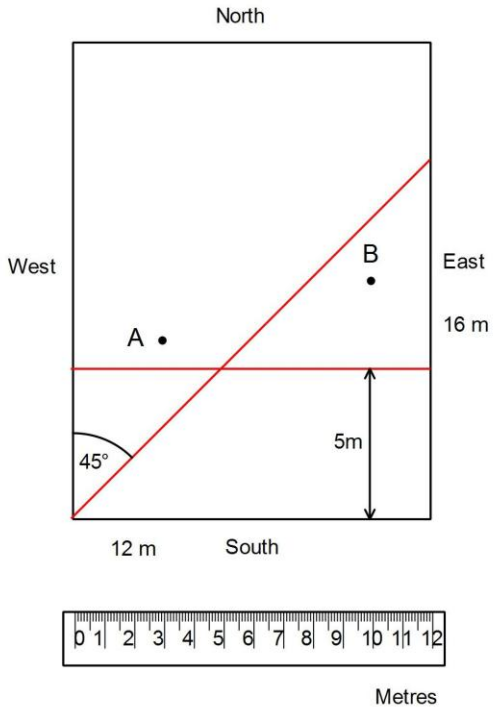
Shade in the regions where the cherry tree can be planted.

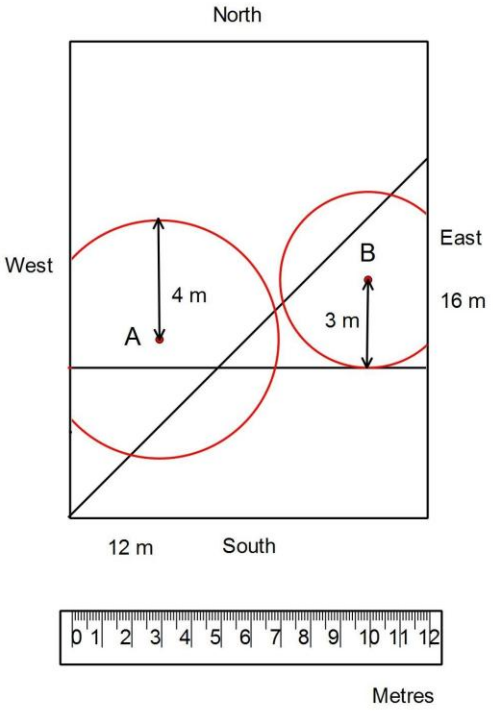


a) We begin by drawing a line parallel to the south wall and 5 m (scaled as required) away from it.

b) Next, we draw a line through the south-west corner, at 45° to the south and west walls, bisecting the right angle between them. (left).

c) and d) We then draw circles of radius 4 m (scaled) centred on **A**, and of radius 3 m (scaled) centred on **B.** (right)





We now have drawn all the required lines and circles to determine where the cherry tree can be planted.

It must be at least 5 m away from the south wall, so it cannot be planted below the horizontal line.

It must be closer to the south wall than the west wall, so it cannot be planted anywhere above and left of the diagonal line.

It must be at least 4 m from the apple tree at **A** and 3 m from the beech tree **B**, so it cannot be planted inside either of the circles.

This leaves three regions below (in green) where Charlie can plant the cherry tree.