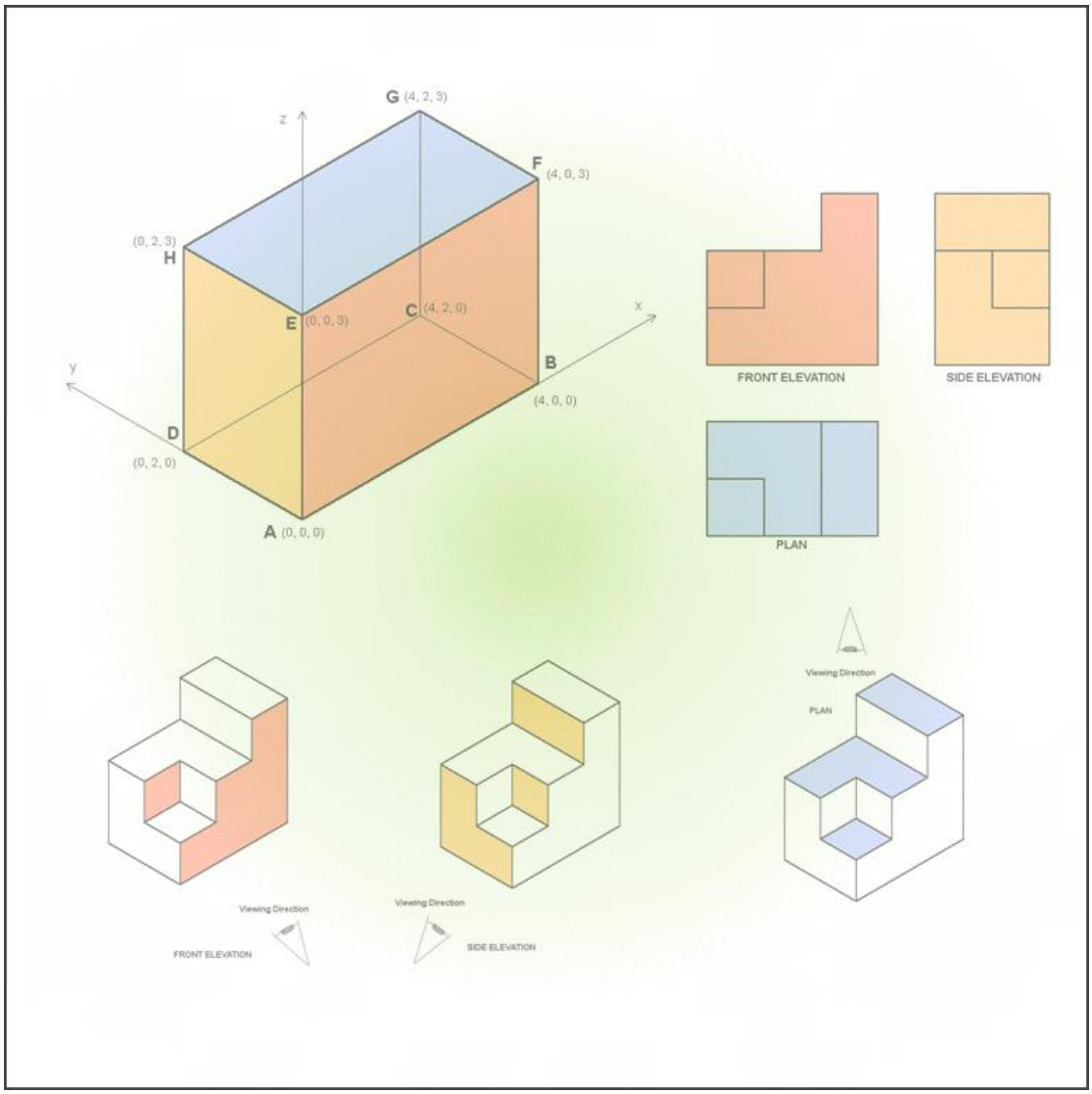
M.K. HOME TUITION

Mathematics Revision Guides

Level: GCSE Foundation Tier

THREE-DIMENSIONAL PROJECTION



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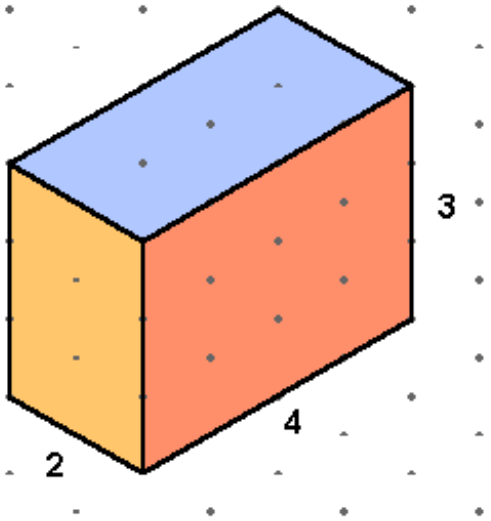
THREE-DIMENSIONAL PROJECTION

There are various ways of representing a three-dimensional figure in two dimensions.

One method is to display it in **isometric projection**, where lengths are true to scale and the object is viewed from equal angles to all the three axes. The dots on an isometric grid coincide with the corners of the tessellation of equilateral triangles.



Example (1): Draw a 4 × 3 × 2 cuboid using isometric projection. (The blank grid would normally be supplied with the exam question.)

The diagram on the right shows one possible alignment.

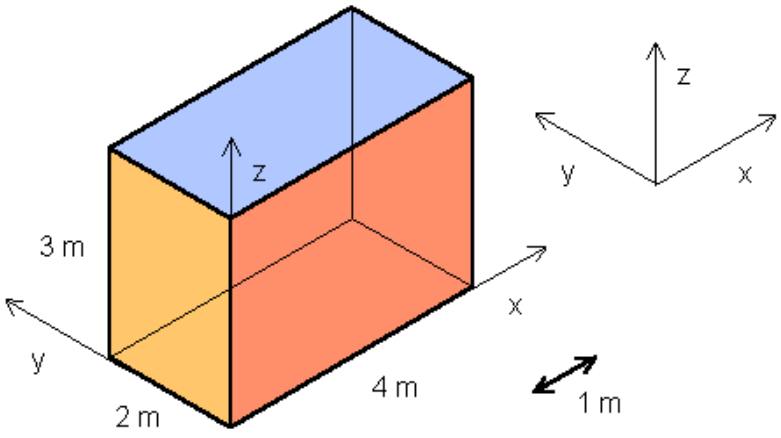
Three-dimensional coordinates.

The coordinate system in three dimensions makes use of three axes, *x*, *y* and *z*, all at right angles to one another.

In the example on the right, the lowermost corner of the block is taken to be the origin of the coordinate system.

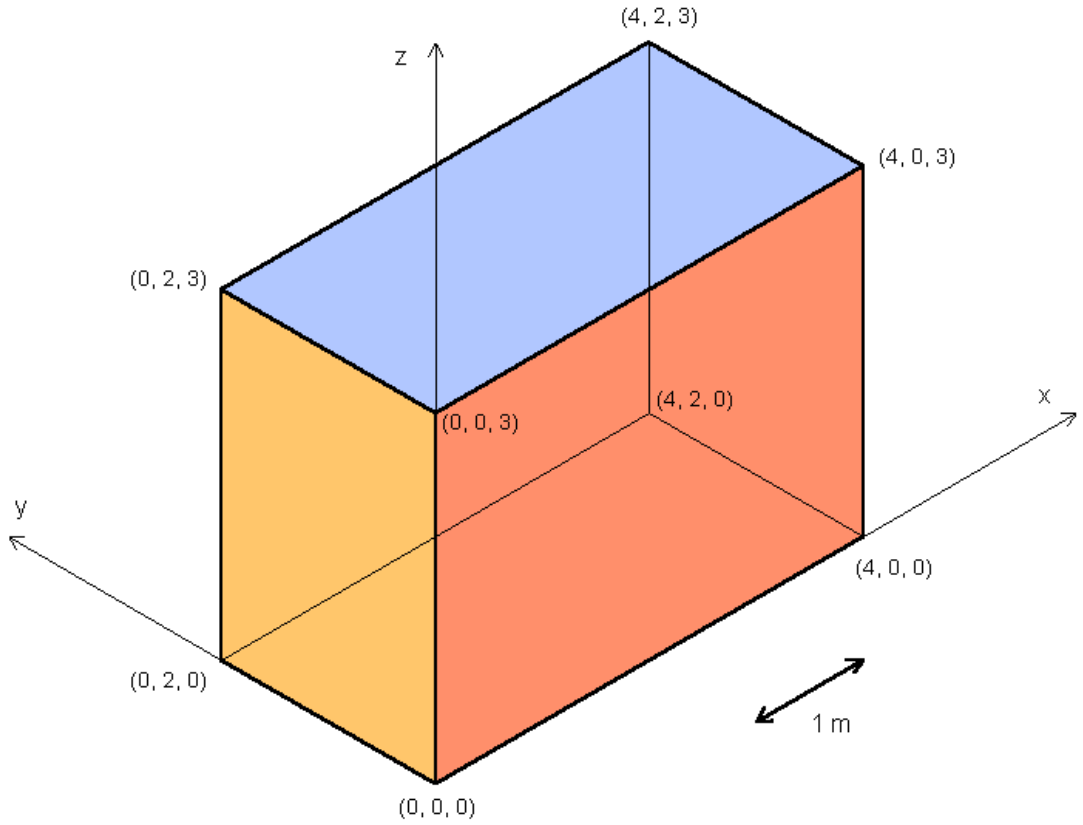
If we were to view the block by looking directly at the blue face, we would find the *x*- and *y*-axes in their expected position , whilst the *z*- axis would appear to point out vertically from the origin.

Points in three-dimensional space have (*x*, *y*, *z*) coordinates, where the origin is taken as (0, 0, 0).



The diagram on the next page will demonstrate the 3-D coordinate system.

Isometric view:



(Such detail will not be expected in exam questions – this is just an example to show the workings of the 3-D coordinate system).

Notice the pattern in the coordinates of each face of the cuboid:

All the points on the front face (in red) have their *y*-coordinate fixed at zero.
(On the hidden rear face, all points have *y*-coordinates of 2).

All the points on the side face (in gold) have their *x*-coordinate fixed at zero.
(On the hidden opposite side face, all points have *x*-coordinates of 4)

All the points on the top face (in blue) have their *z*-coordinate fixed at 3.
(On the hidden bottom face, all points have *z*-coordinates of 0).

Projections.

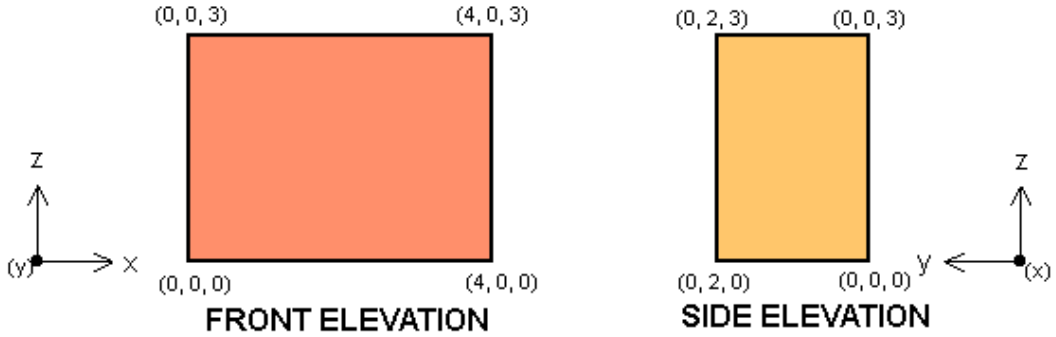
The isometric view of the cuboid in the last example displayed three faces in one diagram, from one particular angle.

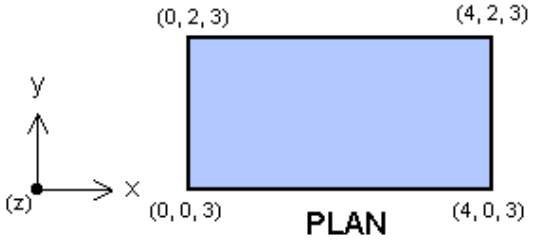
The idea of the projection method is to view the cuboid from three different angles.

In the **front elevation**, the eye is looking directly at the front of the cuboid (red), with the *y*-axis foreshortened to a point.

In the **side elevation**, the eye is looking at the visible side of the cuboid (gold), with the *x*-axis now 'invisible'.

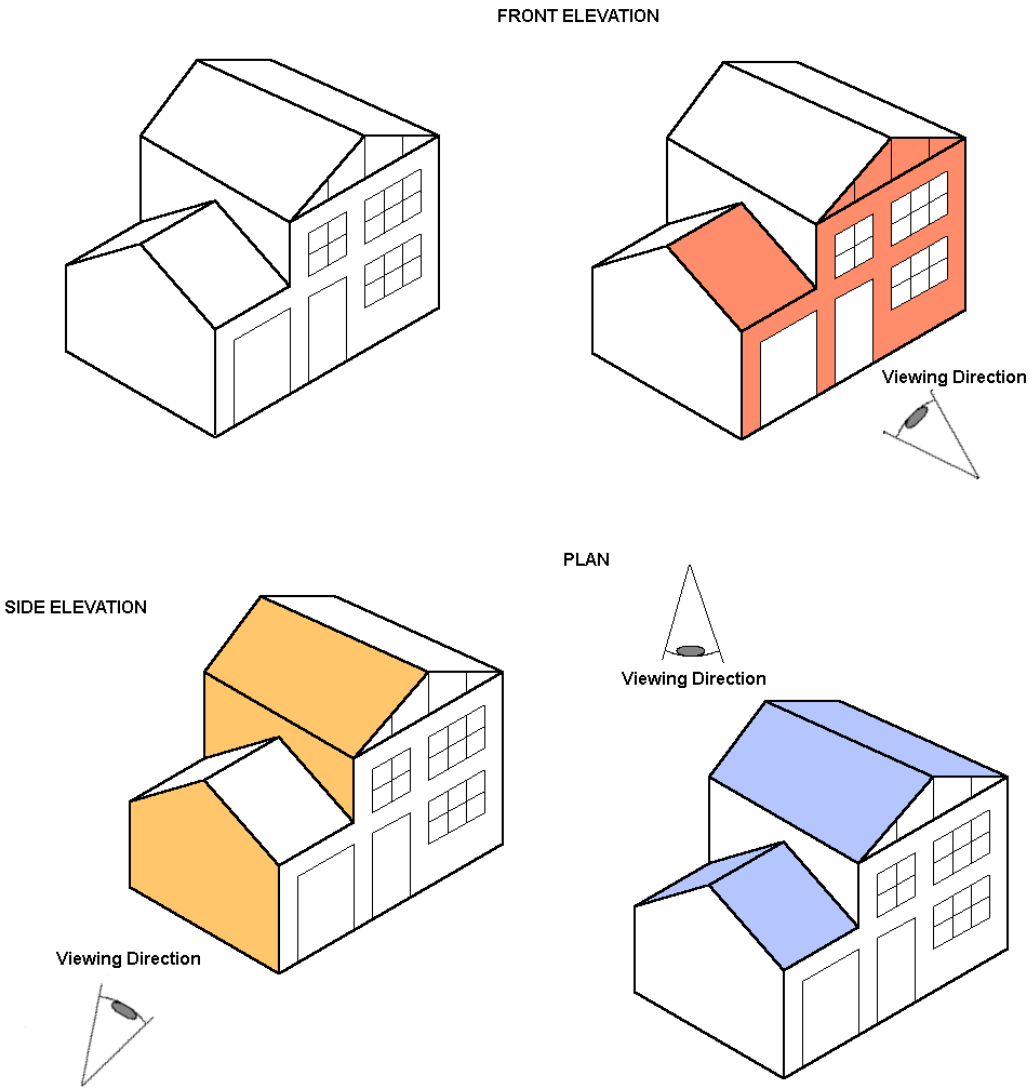
Finally, in the **plan,** the eye is looking at the cuboid from above (blue). Here it is the turn of the *z*-axis to become foreshortened out of view.





The coordinates of each point have been given here to match with the isometric projection, but again this detail is not needed in exams.

The house below is a more complex example of an isometric projection and will form the basis of the next detailed example.



Although the figure of the house is more complex than the earlier example, the same ideas apply when it comes to drawing the projections.

Again, the eye is looking ‘face-on’ for the front elevation (in red), ‘side-on’ for the side elevation (in gold), and from ‘up above’ for the plan (in blue).

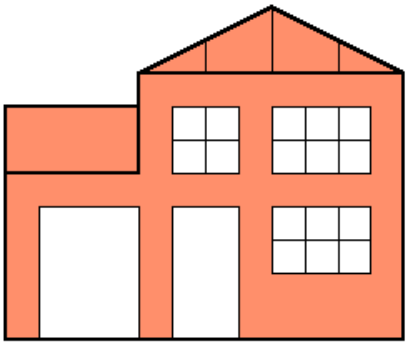
See the projections on the following page.

Notice how some features, i.e. the roofs, are visible on more than one projection.

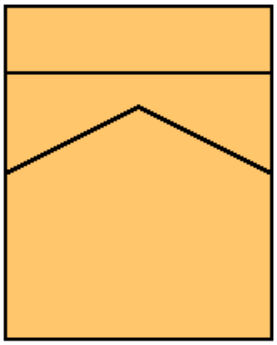
For instance, the front garage roof is visible on both the plan and the front elevation, and the left side house roof on both the plan and the side elevation.

Notice also how there is no visible sense of 'depth' through perspective.

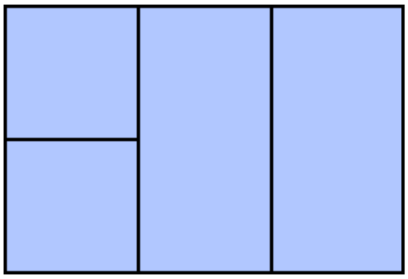
Thus, on the plan, there is no way of seeing that the main roof is on a higher level than the garage roof, for example.



FRONT ELEVATION

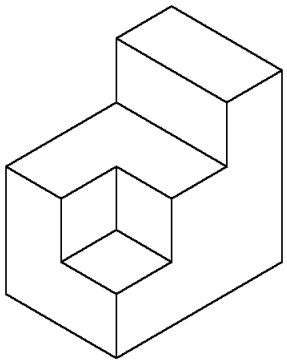


SIDE ELEVATION



PLAN

Examination questions on projections will usually be restricted to simpler arrangements of cubes, as in the next example.

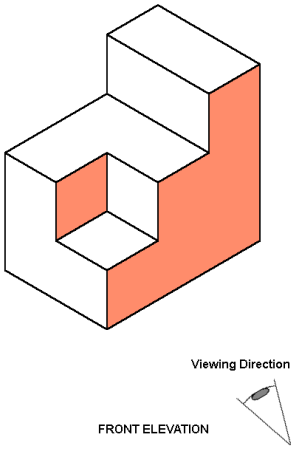


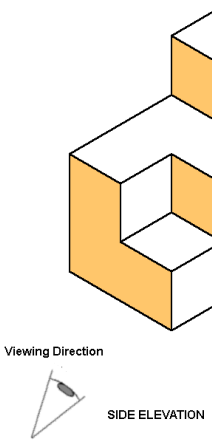
Example (2): The figure on the right is composed entirely of unit cubes.

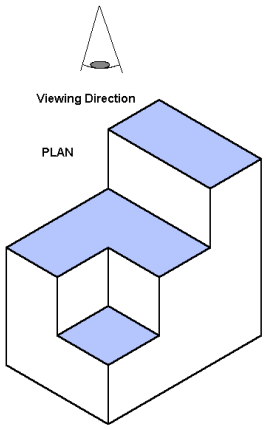
Draw the front elevation, side elevation and plan of the figure.
(An exam question would generally be accompanied by a square grid.)

The diagrams below show the parts of the figure in view for each particular projection.

The eye is looking ‘face-on’ for the front elevation (in red), ‘side-on’ for the side elevation (in gold), and from ‘up above’ for the plan (in blue).







The projections of the figure are shown on the right.

Notice again how there is no visible sense of ‘depth’ through perspective.