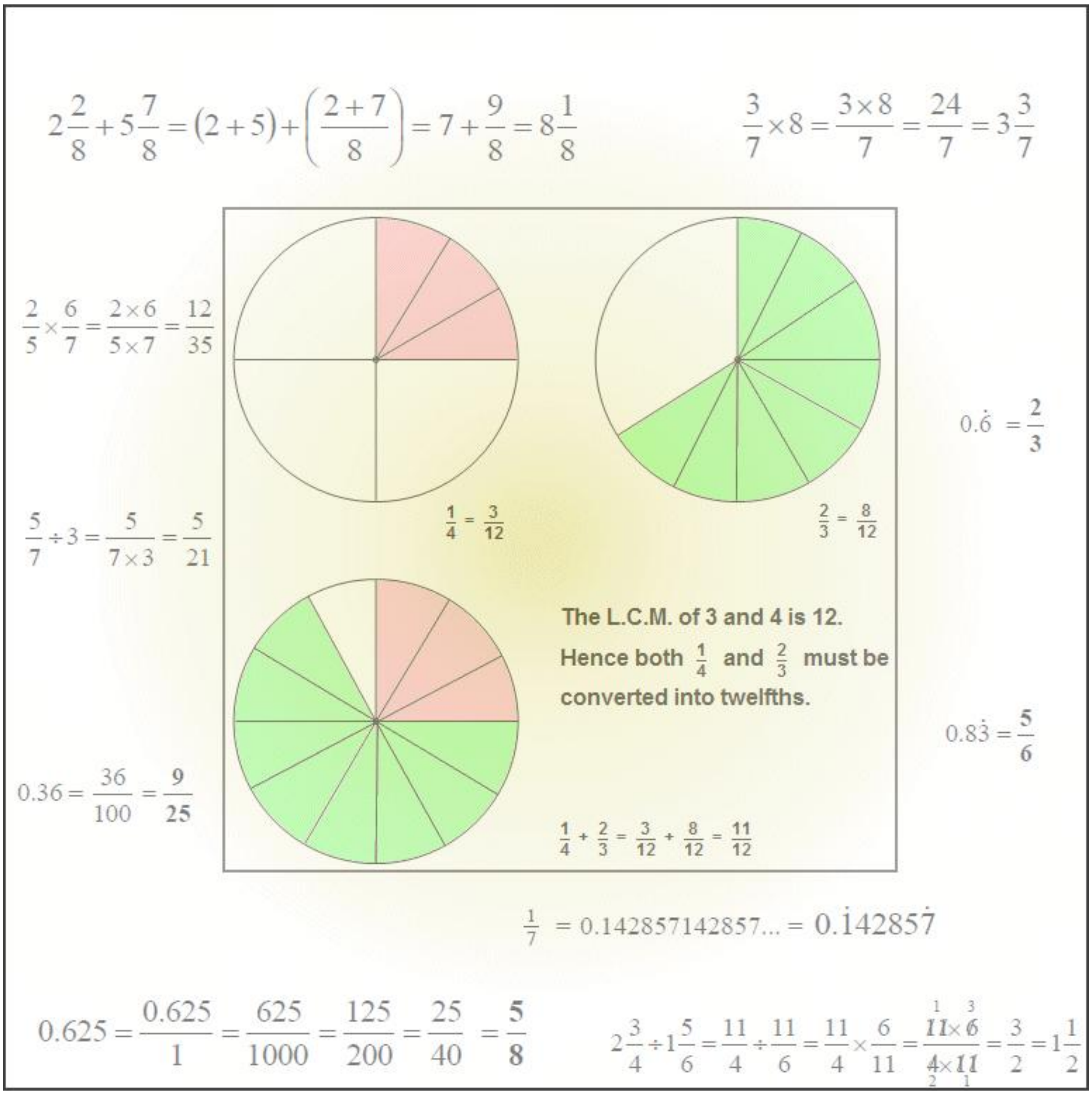
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

FRACTIONS AND DECIMALS



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FRACTIONS AND DECIMALS

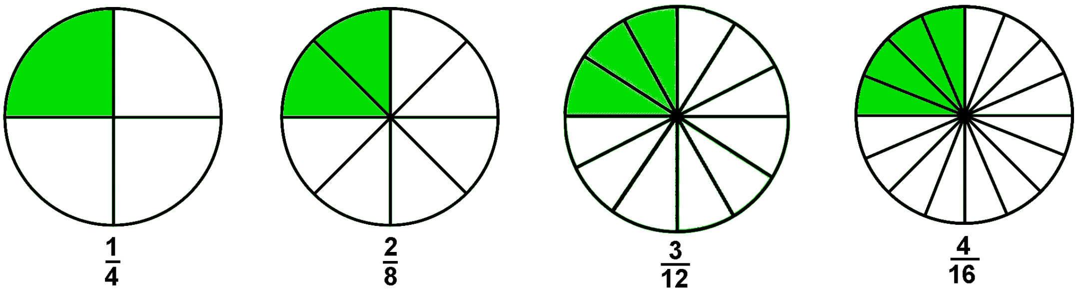
Types of fractional numbers.

A fractional number can either be a **proper** fraction, an **improper** fraction, or a **mixed number**.
In a proper fraction, the top line (**numerator**) is less than or equal to the bottom line (**denominator**).
Examples of proper fractions are $\frac{3}{4}$, $-\frac{1}{8}$ and $\frac{8}{10}$.

In an improper fraction, or a top-heavy fraction, the numerator is greater than the denominator.
Examples: $\frac{22}{7}$, $-\frac{8}{5}$, $\frac{18}{3}$.

Equivalent fractions.

Look at the 'pie' diagrams below:



Each pie has been cut into differing numbers of equal parts; the first one into quarters, the second one into eighths, the third one into twelfths and the fourth one into sixteenths.

One slice of the four has been shaded in the leftmost diagram, i.e. one quarter has been shaded.

Two slices of the eight have been shaded in the second diagram, but the shaded area is still one quarter of the pie, so one quarter = two eighths, or $\frac{1}{4} = \frac{2}{8}$.

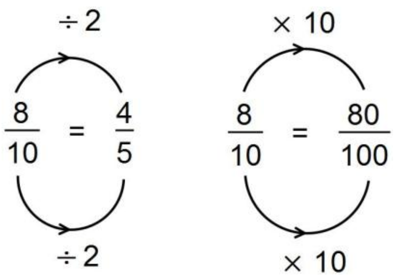
By looking at the other two diagrams, we can see that $\frac{1}{4} = \frac{3}{12}$ and also that $\frac{1}{4} = \frac{4}{16}$.
Generally, a fraction can be converted into an equivalent one by **multiplying (or dividing) the top and bottom by the same number**.

Thus, $\frac{1}{4}$ was re-expressed as $\frac{3}{12}$ by multiplying both the top and bottom by 3.

Example (1): Re-express the fraction $\frac{8}{10}$ in fifths and in hundredths.

To express $\frac{8}{10}$ in fifths, we must change the denominator (bottom line) from 10 to 5. We therefore divide both top and bottom by 2, thus giving $\frac{4}{5}$.

To express $\frac{8}{10}$ in hundredths, we must change the denominator from 10 to 100, which we do by multiplying both top and bottom by 10, giving $\frac{80}{100}$.



Percentages (introduction)
The last example could also have been written as a **percentage**, as 80%.

The % symbol is short for "÷ 100" or "$\frac{}{100}$".
Here are some fractions converted into percentages:
$\frac{1}{2}$ = 50%; $\frac{1}{4}$ = 25%; $\frac{1}{5}$ = 20%; $\frac{3}{4}$ = 75%; $\frac{7}{10}$ = 70%; $\frac{47}{100}$ = 47%; $\frac{3}{100}$ = 3%.

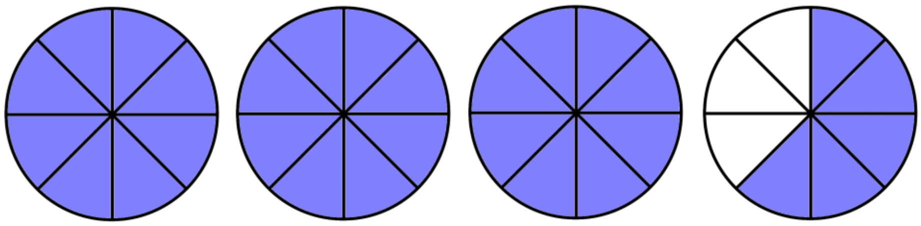
Mixed numbers and improper fractions.

A mixed number consists of a whole number and a **proper** fractional part, for example $1\frac{4}{7}$, $-2\frac{3}{5}$.
In all cases, it is customary to express a fraction in its simplest form, so that the top and bottom have no common factors; the fraction is then said to be in its **lowest terms**.

Thus $3\frac{5}{8}$ and $\frac{22}{7}$ are already in their lowest terms, but $\frac{8}{10}$ is not, because we can 'cancel out' a common factor of 2 from top and bottom to give $\frac{4}{5}$ which *is* in its lowest terms.

A mixed number can be re-expressed as an improper fraction by multiplying the whole number part by the bottom line of the fractional part, and then adding the top line of the fractional part.

Thus $3\frac{5}{8}$ can be converted to $\frac{(3\times 8)+5}{8} = \frac{29}{8}$.



Alternatively, we can see there are 8 eighths in a whole, so there are 24 eighths in the 3 wholes, plus the 5 eighths in the $\frac{5}{8}$ part, giving 29 eighths in all.

Conversely, an improper fraction can be turned into a mixed number by division. The whole number part will be the quotient, whilst the fractional part will be the remainder over the bottom line.

Thus, dividing 47 by 5 gives a quotient of 9 and a remainder of 2, so $\frac{47}{5} = 9\frac{2}{5}$.
Another example: dividing 40 by 6 gives a quotient of 6 and a remainder of 4, so $\frac{40}{6} = 6\frac{4}{6} = 6\frac{2}{3}$.
(We have reduced the result to its lowest terms).

Overview of fraction arithmetic.

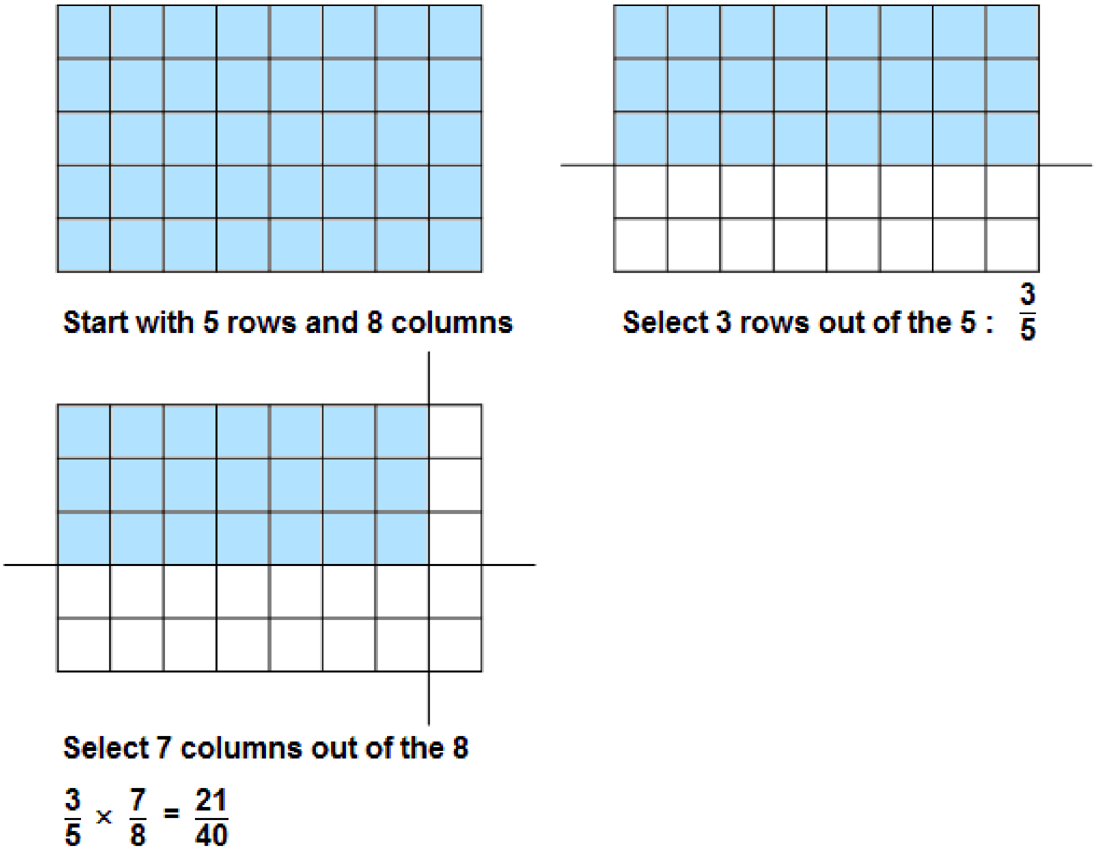
Multiplication:

The diagram below illustrates how to multiply two fractions, namely to find the value of $\frac{3}{5}\times\frac{7}{8}$.

We begin with a grid of 5 rows and 8 columns, containing 40 cells in total. Next, we select 3 rows out of the 5, thus leaving $\frac{3}{5}$ of the total area, or 24 cells, shaded and the remainder blanked out.

Then we select 7 columns out of the 8 , leaving $\frac{7}{8}$ of the last shaded area, or 21 cells, still shaded and the remainder blanked out.

The final result is $\frac{3}{5}\times\frac{7}{8}=\frac{3\times 7}{5\times 8}=\frac{21}{40}$. Notice how 21 cells out of the original 40 are still shaded.



Note again the case of equivalent fractions : $\frac{3}{4}\times\frac{2}{2}=\frac{3\times 2}{4\times 2}=\frac{6}{8}$, since $\frac{2}{2}$ is simply 1.

As the worked example above showed, multiplication of fractions is quite straightforward; we just multiply the top lines and bottom lines together and simplify the result.

Example (2): $\frac{2}{5}\times\frac{6}{7}=\frac{2\times6}{5\times7}=\frac{12}{35}$.

Example (3): $\frac{4}{7}\times\frac{5}{8}=\frac{4\times5}{7\times8}=\frac{20}{56}=\frac{5}{14}$.

This time, we had to divide the top and bottom by 4 to get the result in its lowest terms.

We could have alternatively 'cancelled' a common factor of 4 from top and bottom *before* the final section, by spotting it in the product:

$$\frac{4}{7}\times\frac{5}{8}=\frac{\overset{1}{\cancel{4}}\times5}{7\times\underset{2}{\cancel{8}}}=\frac{5}{14}\ .$$

Mixed numbers **must** be put into improper fraction form first :

Example (4): $3\frac{1}{5}\times3\frac{1}{8}=\frac{16}{5}\times\frac{25}{8}=\frac{\overset{2}{\cancel{16}}\times\overset{5}{\cancel{25}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{8}}}=10$

Here, the 'cancelling' process of 8 and 5 from top and bottom has left us with a whole number result.

Never try to reckon as follows: multiply the whole numbers as 3 × 3 = 9 , then multiply the fractions to get $\frac{1}{5}\times\frac{1}{8}=\frac{1}{40}$, and add them to get $9\frac{1}{40}$. This is totally incorrect.

Multiplying a fraction by a whole number.

To multiply a fraction by a whole number, we simply multiply the **top line** of the fraction by it.

Example (5) : $\frac{3}{7}\times8=\frac{3\times8}{7}=\frac{24}{7}=3\frac{3}{7}$.

This is the same as saying $\frac{3}{7}\times\frac{8}{1}=\frac{3\times8}{7\times1}=\frac{24}{7}=3\frac{3}{7}$.

Example (6) : $\frac{5}{6}\times12=\frac{5\times12}{6}=\frac{60}{6}=10$.
(This last example has given a whole number result).

Examples like the last two are also often quoted using the word 'of' instead of the × sign;

e.g. $\frac{3}{7}$ of 8 = $3\frac{3}{7}$, $\frac{5}{6}$ of 12 = 10.

Division:
A division sum involving two fractions can be turned into a multiplication sum by inverting the top and bottom lines of the fraction to *the right* of the division sign, namely the one we are dividing *by*.

Example (7): $\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{3 \times 5}{4 \times 4} = \frac{15}{16}$.

Mixed numbers are handled as in multiplication:

Example (8): $2\frac{3}{4} \div 1\frac{5}{6} = \frac{11}{4} \div \frac{11}{6} = \frac{11}{4} \times \frac{6}{11} = \frac{{}^{1}\cancel{11} \times {}^{3}\cancel{6}}{{}_{2}\cancel{4} \times {}_{1}\cancel{11}} = \frac{3}{2} = 1\frac{1}{2}$

Again, 'cancelling' first 11 and then 2 from the top and bottom has simplified the result.

Dividing a fraction by a whole number.

To divide a fraction by a whole number, we simply multiply the **bottom line** of the fraction by it.

Example (9): $\frac{5}{7} \div 3 = \frac{5}{7 \times 3} = \frac{5}{21}$.

This is the same as saying $\frac{5}{7} \div \frac{3}{1} = \frac{5}{7} \times \frac{1}{3} = \frac{5 \times 1}{7 \times 3} = \frac{5}{21}$.

Example (10): $\frac{3}{8} \div 6 = \frac{{}^{1}\cancel{3}}{8 \times {}_{2}\cancel{6}} = \frac{1}{16}$.

(Here we have cancelled 3 from top and bottom.)

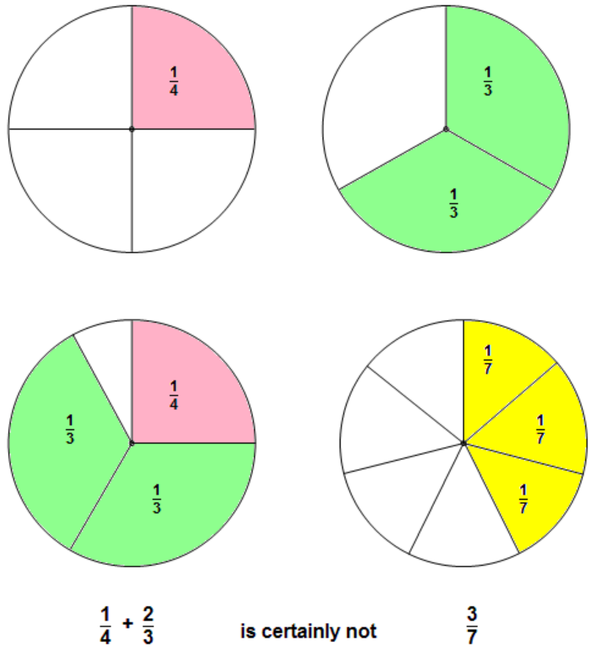
Addition and subtraction of fractions needs more care.

Addition:

Take the sum $\frac{1}{4}+\frac{2}{3}$.

The result is most certainly not
$\frac{1}{4}+\frac{2}{3}=\frac{1+2}{4+3}=\frac{3}{7}$.

One quarter plus two thirds does not make three of anything, and it definitely does not make three sevenths either, as the diagrams show !



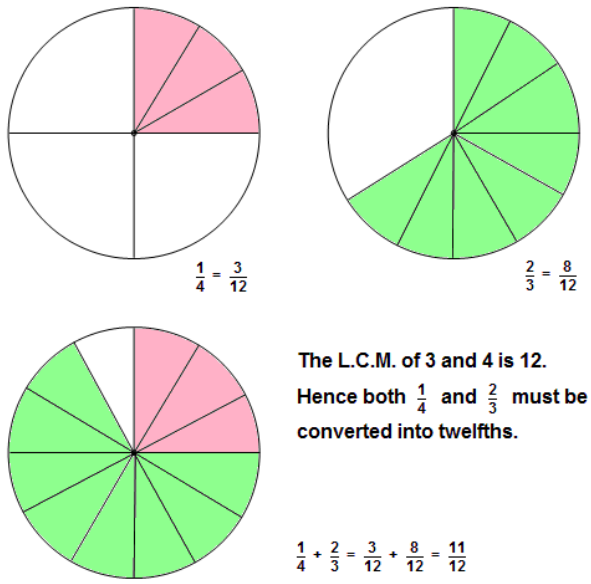
The correct approach is to convert both fractions into equivalent forms with the same denominator, namely the L.C.M. of 3 and 4, or 12. In other words, we work with twelfths.

Here, $\frac{1}{4}\times\frac{3}{3}=\frac{3}{12}$ and $\frac{2}{3}\times\frac{4}{4}=\frac{8}{12}$.

Now that we are dealing with fractions having the same denominator, the addition can now be carried out easily.

$\frac{1}{4}+\frac{2}{3}=\frac{3}{12}+\frac{8}{12}=\frac{3+8}{12}=\frac{11}{12}$.

Note that we still do not add the two denominators, i.e. the 12s, together !



If the fractions have the same denominators (bottom lines), the process is easy enough – we just add the numerators (top lines).

Example (11): $\frac{4}{5}+\frac{3}{5}=\frac{4+3}{5}=\frac{7}{5}=1\frac{2}{5}$

There are two methods in common use for adding mixed numbers.

We can either turn them all into improper fractions followed by turning the result into a mixed number, as in the examples below:

Example (12): $2\frac{3}{8}+4\frac{1}{8}=\frac{19}{8}+\frac{33}{8}=\frac{\cancel{52}^{13}}{\cancel{8}_{2}}=6\frac{1}{2}$ (4 cancelled from top and bottom.)

Example (13) : $5\frac{3}{5}+3\frac{4}{5}=\frac{28}{5}+\frac{19}{5}=\frac{47}{5}=9\frac{2}{5}$

Alternatively (WARNING – can be awkward !) , we can add the whole numbers first, followed by adding the fractions.

Example (12a): $2\frac{3}{8}+4\frac{1}{8}=(2+4)+\left(\frac{3}{8}+\frac{1}{8}\right)=6\frac{\cancel{4}^{1}}{\cancel{8}_{2}}=6\frac{1}{2}$ (4 cancelled from top and bottom.)

Example (13a) : $5\frac{3}{5}+3\frac{4}{5}=(5+3)+\left(\frac{3}{5}+\frac{4}{5}\right)=8+\frac{7}{5}=8+1\frac{2}{5}=9\frac{2}{5}$

Notice how we had to 'carry' 1 in example 13a.

If the fractions have different bottom lines, then we need to find the lowest common multiple of the bottom lines, and turn one or both into equivalent fractions by multiplying top and bottom by the same number.

Example (14): Evaluate $2\frac{1}{4}+5\frac{7}{8}$.

We must find the L.C.M. of 4 and 8 – here it is 8 itself. The first fraction , $\frac{1}{4}$, must therefore be turned into a form with 8 on the bottom line. This can be done by multiplying both top and bottom by 2 to give $\frac{1}{4}\times\frac{2}{2}=\frac{2}{8}$.

The sum thus becomes $2\frac{2}{8}+5\frac{7}{8}=(2+5)+\left(\frac{2+7}{8}\right)=7+\frac{9}{8}=8\frac{1}{8}$.

Note how the second fraction did not need to be converted into an equivalent.

(We have used the method of adding the whole numbers and fractions separately).

Subtraction:

If the fractions have the same denominators (bottom lines), the process is easy enough – we just subtract the numerators (top lines).

Example (15): $\frac{7}{9}-\frac{2}{9}=\frac{7-2}{9}=\frac{5}{9}$

To subtract one mixed number from another, we can turn them all into improper fractions followed by turning the result into a mixed number.

Example (16): $5\frac{5}{7}-1\frac{2}{7}=\frac{40-9}{7}=\frac{31}{7}=4\frac{3}{7}$

Example (17): $8\frac{1}{6}-2\frac{5}{6}=\frac{49-17}{6}=\frac{\cancel{32}^{16}}{\cancel{6}_{3}}=5\frac{1}{3}$ (2 cancelled from top and bottom)

Alternatively (WARNING – can be awkward !), we can subtract the whole numbers, followed by subtracting the fractions, and then **adding** the two results.

Example (16a): $5\frac{5}{7}-1\frac{2}{7}=(5-1)+\left(\frac{5}{7}-\frac{2}{7}\right)=4+\frac{3}{7}=4\frac{3}{7}$

Example (17a): $8\frac{1}{6}-2\frac{5}{6}=(8-2)+\left(\frac{1}{6}-\frac{5}{6}\right)=6+\left(\frac{1-5}{6}\right)=6-\frac{\cancel{4}^{2}}{\cancel{6}_{3}}==5\frac{1}{3}$

The second example appears tricky because we are attempting to subtract a larger fraction $\left(\frac{5}{6}\right)$ from a smaller one $\left(\frac{1}{6}\right)$, leaving us with a fraction to be subtracted, rather than added, in the final stage.

If the fractions have different bottom lines, then we need to find the lowest common multiple of the bottom lines, and turn one or both into equivalent fractions by multiplying top and bottom by the same number, as in addition.

Example (18): evaluate $3\frac{1}{6}-1\frac{3}{4}$.

We first find the L.C.M. of the bottom lines, namely 12, and thus proceed to convert both fractions into twelfths. (We have used the method of turning the mixed numbers into improper fractions first)

$$3\frac{1}{6}-1\frac{3}{4}=\frac{19}{6}-\frac{7}{4}=\left(\frac{19}{6}\times\frac{2}{2}\right)-\left(\frac{7}{4}\times\frac{3}{3}\right)=\frac{38-21}{12}=\frac{17}{12}=1\frac{5}{12}.$$

Converting fractions into decimals.

To convert a fraction into a decimal, divide the top line (numerator) by the bottom line (denominator).

Examples are:

Example (19): $\frac{3}{5} = 0.6$; working : $\begin{array}{r} 0.6 \\ 5\overline{)3.0} \end{array}$ - division terminates.

Example (20): $\frac{1}{6} = 0.16666.....$ working : $\begin{array}{r} 0.1\ 6\ 6\ 6 \\ 6\overline{)1.0^4 0^4 0^4 0} \end{array}$ and so on....

No matter how many extra zeros are tagged on to continue the division, we will keep on getting a quotient of 6 and a remainder of 4, so the decimal will go on for ever without giving an exact answer.

Example (21): $\frac{7}{8} = 0.875$ working: $\begin{array}{r} 0.8\ 7\ 5 \\ 8\overline{)7.0^6 0^4 0} \end{array}$; division terminates.

Example (22): $\frac{3}{7} = 0.428571428571...$ working; $\begin{array}{r} 0.4\ 2\ 8\ 5\ 7\ 1\ 4\ 2\ 8 \\ 7\overline{)3.0^2 0^6 0^4 0^5 0^1 0^3 0^2 0^6 0} \end{array}$

This time, we cycle through all the possible remainders from 1 to 6 until the remainder of 2 makes its second appearance.

As the examples show, not all fractions give an exact result (a **terminating** decimal). Others appear to go on for ever (**recurring** decimals).

If the denominator of the fraction (in lowest terms) yields only powers of 2 and/or 5 when separated into its prime factors, then the resulting decimal will terminate. In all other cases it will recur.

Thus, $\frac{1}{40}$ gives rise to a terminating decimal (0.025) because 40 = $2^3 \times 5$.

Conversely, $\frac{1}{48}$ produces a recurring decimal (0.02083333..) because 48 = $2^4 \times 3$.

A more succinct way of expressing a recurring decimal is to put dots over the start and end digits of the recurring period.

0.333333.... = $0.\dot{3}$

0.166666... = $0.1\dot{6}$ (note that the 1 does not repeat)

0.954545... = $0.9\dot{5}\dot{4}$ (the 9 does not repeat)

0.142857142857... = $0.\dot{1}4285\dot{7}$

Converting terminating decimals into fractions.

To convert a terminating decimal into a fraction, multiply both top and bottom by the required power of 10 to make the top an integer, followed by cancelling out common factors. .

Example (23) : $0.625 = \frac{0.625}{1} = \frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \mathbf{\frac{5}{8}}$ (We have cancelled 5 three times)

Example (24): $0.36 = \frac{0.36}{1} = \frac{36}{100} = \mathbf{\frac{9}{25}}$ (We have cancelled 2 twice)

Converting recurring decimals into fractions.

This process is a bit trickier, but there are two methods of finding the fraction corresponding to a recurring decimal.

Method 1 (Systematic).

Take the original fraction and multiply it by the power of 10 required to align the decimal portions, so that they cancel out when subtracting.

Example (25): convert $0.\dot{6}$ to a fraction.

Multiply the recurring decimal by 10 so that it lines up with the original:	$10F =$ $F =$	6.666666...... 0.666666......
Subtract :	$9F =$	6

$F = \frac{6}{9}$, $\therefore\ 0.\dot{6} = \mathbf{\frac{2}{3}}$.

Example (26): convert $0.\dot{7}\dot{2}$ to a fraction.

Here, the repeating portion is two digits long, so the decimal needs multiplying by 100 to ensure decimal alignment:

Multiply the recurring decimal by 100 so that it lines up with the original:	$100F =$ $F =$	72.7272... 0.7272....
Subtract :	$99F =$	72

$F = \frac{72}{99}$, $\therefore\ 0.\dot{7}\dot{2} = \mathbf{\frac{8}{11}}$.

Example (27): Convert $0.8\dot{3}$ to a fraction.

The 3 is repeated here but not the 8, so

Multiply original by 10 as the repeating portion has one digit:	10***F*** **=**	8.333333......
	F **=**	0.833333...
Subtract :	9***F*** **=**	7.5

F **=** $\frac{7.5}{9} = \frac{15}{18}$ **,** and thus $0.8\dot{3} = \mathbf{\frac{5}{6}}$.

Example (28): Convert $0.0\dot{2}$ to a fraction.

The first zero after the decimal point is non-repeating (careful with that 2), and the repeating period is one digit long;

Multiply original by 10 as the repeating portion has one digit:	10***F*** **=**	0.222222.....
	F **=**	0.022222.....
Subtract:	9***F*** **=**	0.2

F **=** $\frac{0.2}{9} = \frac{2}{90}$ **,** and thus $0.0\dot{2} = \frac{1}{45}$.

Method 2 ("Nines").

A recurring decimal can be converted to a fraction by looking at the repeating period.

From that we can write the corresponding fraction with the repeating digits on the top line, whilst the bottom line will have a string of 9s of the same length as the repeating period.

Example (25a): convert $0.\dot{6}$ to a fraction.

The recurring decimal has just a **single-digit** repeating pattern, namely $\dot{6}$, and so its denominator will have **one** 9 in it.

$0.\dot{6} = \frac{6}{9}$, simplifying to $\mathbf{\frac{2}{3}}$.

Example (26a): convert $0.\dot{7}\dot{2}$ to a fraction.

Here, the repeating pattern of $\dot{7}\dot{2}$ has **two** digits in it, so its denominator will have **two** 9s in it.

$0.\dot{7}\dot{2} = \frac{72}{99}$**,** and finally $\mathbf{\frac{8}{11}}$.

This method will need to be modified slightly if the decimal contains non-repeating digits.

Example (27a): Convert $0.8\dot{3}$ to a fraction.

The 3 is repeated here but not the 8, and so we do not have a fraction of the required form to use the method directly.

We can however use the 'dodge' of multiplying $0.8\dot{3}$ by 10 to obtain $8.\dot{3}$ which is of the right form, i.e. $8\frac{1}{3}$.

Dividing back by 10, we have $8\frac{1}{3} \div 10 = \frac{25}{3} \div 10 = \frac{25}{30} = \frac{5}{6}$.

Example (28a): Convert $0.0\dot{2}$ to a fraction.

The first zero after the decimal point is non-repeating, but we can use the 'dodge' of multiplying $0.0\dot{2}$ by 10 to use the method directly.

Multiplying $0.0\dot{2}$ by 10 gives us $0.\dot{2}$ which is of the required form, namely $\frac{2}{9}$.

Dividing back by 10 we then have $0.0\dot{2} = \frac{2}{90} = \frac{1}{45}$.

Comparing fractions.

Example (29): Arrange the fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{5}{6}$ and $\frac{3}{4}$ in descending order of size.

To compare fractions, we must follow the same methods as in addition and subtraction. The fractions must all be converted into equivalent fractions with the same common denominator.

Here, the required denominator is the L.C.M. of 3, 4, 6 and 8.
Since any multiple of 6 is always a multiple of 3, and any multiple of 8 is also a multiple of 4, the required L.C.M. is that of 6 and 8, namely 24.

The fractions must therefore be converted into 24ths by multiplying top and bottom by the same number, thus $\frac{2}{3} = \frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$.
Multiplying top and bottom of $\frac{5}{8}$ by 3 gives $\frac{15}{24}$; similarly, we can work out $\frac{5}{6} = \frac{20}{24}$
(top and bottom × 4) and $\frac{3}{4} = \frac{18}{24}$.

We can therefore deduce that $\frac{20}{24} > \frac{18}{24} > \frac{16}{24} > \frac{15}{24}$, i.e. $\frac{5}{6} > \frac{3}{4} > \frac{2}{3} > \frac{5}{8}$.

Sometimes a question might have the fractions in varied forms (decimal, common and percentages). In such cases, convert all of them into one form, whichever is the easiest.

Example (30) : Arrange $\frac{3}{4}$, 70%, $\frac{4}{5}$ and 0.72 in descending order of size.
We could convert them all into decimals; $\frac{3}{4}$ = 0.75; 70% = 0.7; $\frac{4}{5}$ = 0.8.
Hence 0.8 > 0.75 > 0.72 > 0.7, i.e. $\frac{4}{5} > \frac{3}{4}$ > 0.72 > 70%.

We end this section with a few examination-style problem-solving questions.

Example (31): Tim is looking after his neighbours' two dogs while their owners are on holiday for 8 days. The terrier requires $\frac{3}{4}$ of a can of food a day and the retriever needs $1\frac{1}{2}$ cans. How many cans of dog food does Tim need to buy ?

We need to add together $\frac{3}{4}$ and $1\frac{1}{2}$, then multiply the result by 8.

$\frac{3}{4}+1\frac{1}{2} = \frac{3}{4}+\frac{3}{2} = \frac{3}{4}+\frac{6}{4} = \frac{9}{4} = 2\frac{1}{4}$, so Tim needs $2\frac{1}{4}$ cans of food for the dogs each day.

He needs to feed the dogs for 8 days, so he needs $2\frac{1}{4}\times 8 = \frac{9}{4}\times 8 = \frac{9\times 8}{4} = \frac{72}{4} = 18$ cans of food in total.

Alternatively, we could have multiplied the individual fractions by 8 and added the result.

$\frac{3}{4}\times 8 = \frac{3\times 8}{4} = \frac{24}{4} = 6$; $1\frac{1}{2}\times 8 = \frac{3}{2}\times 8 = \frac{3\times 8}{2} = \frac{24}{2} = 12$; $6+12=18$ cans of food needed.

Example (32): At a college, $\frac{2}{5}$ of the students are female, and $\frac{3}{8}$ of the females are taking courses in maths. What fraction of **all** students are females taking courses in maths?

Given that there are 280 students enrolled in total, how many are female and taking courses in mathematics ?

This time we need to multiply $\frac{2}{5}$ (females as fraction of all students) by $\frac{3}{8}$ (females taking maths as fraction of all females).

$\frac{2}{5}\times\frac{3}{8} = \frac{\overset{1}{\cancel{2}}\times 3}{5\times \underset{4}{\cancel{8}}} = \frac{3}{20}$, which is the fraction of females taking maths courses, relative to all of the enrolled students at the college.

The actual number of females taking maths courses is thus $\frac{3}{20}\times 280 = \frac{3\times 280}{20} = \frac{840}{20} = 42$.

Example (33): The distribution of seats in a theatre is as follows :

Stalls, $\frac{5}{12}$ of the total
Circle, $\frac{3}{8}$ of the total
Balcony, all the remaining 150 seats

Calculate the numbers of seats in the stalls and the circle respectively.

The key here is to work out what fraction of all the seats are balcony seats. As there are only two other seat types, namely stalls and circle, we add their fractions together and subtract from 1:

$$\frac{5}{12}+\frac{3}{8} = \frac{10}{24}+\frac{9}{24} = \frac{19}{24}; \quad 1-\frac{19}{24}=\frac{24}{24}-\frac{19}{24}=\frac{5}{24}$$

(the L.C.M. of 8 and 12 is 24)

We now know that the 150 balcony seats represent $\frac{5}{24}$ of the total.
Dividing by 5, we reason that 30 seats represent $\frac{1}{24}$ of the total.
The seats in the stalls represent $\frac{5}{12}$ of the total, or $\frac{10}{24}$, so they number $10\times 30 = 300$.
Similarly the circle seats are $\frac{3}{8}$, or $\frac{9}{24}$, of the total, so we have $9\times 30 = 270$ of them in the theatre.

Example (34): Gift-wrapping ribbon is produced in spools $37\frac{1}{2}$ metres in length. It is then cut by machine into $2\frac{1}{2}$ metre lengths for packing and sending to the shops. Calculate the number of lengths that could be cut from one such spool, and also show that there is no wastage in the cutting process.

This time we have a mixed number division problem, where we must divide $37\frac{1}{2}$ by $2\frac{1}{2}$.

$$37\frac{1}{2}\div 2\frac{1}{2} = \frac{75}{2}\div\frac{5}{2} = \frac{75}{2}\times\frac{2}{5} = \frac{\overset{15}{\cancel{75}}\times\overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{5}}} = 15.$$

It is possible to cut 15 lengths of $2\frac{1}{2}$ metres from the spool, and since the result of the division is a whole number, there is no wastage.