M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

STANDARD INDEX FORM

$$4.679 \times 10^2 = 467.9$$

$$5.89 \times 10^{-7} = 0.000\ 000\ 589$$

$$(5 \times 10^4) \div (8 \times 10^7) = 0.625 \times 10^{-3} = \mathbf{6.25 \times 10^{-4}}$$

$$6231 = 6.231 \times 10^3$$

$$5.137 \times 10^4 = 51370$$

$$(8 \times 10^7) \times (6 \times 10^{-4}) = 48 \times 10^3 = \mathbf{4.8 \times 10^4}$$

$$(5 \times 10^4)^3 = 125 \times 10^{12} = \mathbf{1.25 \times 10^{14}}$$

$$\mathbf{1.083 \times 10^{12} = 1{,}083{,}000{,}000{,}000}$$

Version: 1.3 Date: 21-04-2013

STANDARD INDEX FORM

This is another name for the 'scientific notation' form as displayed on calculators, when very large or very small numbers are involved.

Examples of such displays (on old calculators) are $2.45^{\text{E+07}}$ and $3.16^{\text{E-08}}$.

Most modern calculators display in the correct form of $2.45\times10^{+07}$ and 3.16×10^{-08}.

These numbers consist of two parts: a number greater than or equal to 1 but less than 10, and an **exponent**. The exponent is the power of 10 by which the number on the left is multiplied.

Thus $2.45^{\text{E+07}}$ actually means 2.45×10^7 and $3.16^{\text{E-08}}$ means 3.16×10^{-8}.

Note that -5.42×10^6 is a negative number, but has a positive exponent, so is greater than 10 in **size.**

A positive exponent denotes a number whose size is greater than or equal to 10; a zero exponent a number greater than or equal to 1, but less than ten, and a negative exponent a number whose size is less than 1.

Note that -5.42×10^6 is a negative number, but has a positive exponent, so is greater than 10 in **size.**

Converting a number from regular notation to standard form.

To convert a large number into standard form, write it out in full, and count the number of places the decimal point must be moved to the **left** until we have one non-zero digit to the left of the decimal point. That number of places becomes the **positive** exponent.

$2614.3 = 2.6143 \times 10^3$ (decimal point moved 3 places to the left).
$597000 = 5.97 \times 10^5$ (assumed final decimal point moved 5 places to the left).

Example (1): The distance from the Earth to the Sun is given as 149,600,000 km. Express this distance in standard form in kilometres.

By assuming a decimal point at the end of the number 149,600,000 we find that we need to move it 8 places to the left until we have 1.496 in the number part.

We moved the decimal point 8 places, so the exponent is 8 and hence the standard form of 149600000 is $\mathbf{1.496 \times 10^8}$.

To convert a small number into standard form, write it out in full, and count the number of places the decimal point must be moved to the **right** until we have one non-zero digit to the left of the decimal point. That number of places is the **negative** exponent.

$0.0394 = 3.94 \times 10^{-2}$ (decimal point moved 2 places to the right).
$0.000000816 = 8.16 \times 10^{-7}$ (decimal point moved 7 places to the right).

Example (2): The mass of the Earth is 0.00000309 times that of the Sun. Express this number in standard form.

We need to move the decimal point six places to the right until we have 3.09 in the number part, and thus the standard form of 0.00000309 is $\mathbf{3.09 \times 10^{-6}}$.

Converting a number from standard form to regular notation.

If the exponent is positive (i.e. we have a large number), write out the number portion and move the decimal point to the **right** by the number of places given in the exponent, inserting extra zeros at the end of the number when necessary.

$4.679 \times 10^2 = 467.9$ (decimal point moved 2 places right)
$6.231 \times 10^3 = 6231$ (decimal point moved 3 places right, and "dropped")
$5.137 \times 10^4 = 51370$ (decimal point moved 4 places right, with an extra 0 inserted)

Example (3): The volume of the Earth is 1.083×10^{12} cubic kilometres.
Express this value as an ordinary number.

After the decimal point has been moved 3 places right, the number part becomes 1083. There are still 9 decimal places of the 12 to go, so we have to tag 9 extra zeros to the end of the number 1083.

$\therefore$ **$1.083 \times 10^{12} = 1{,}083{,}000{,}000{,}000$**

It is worth noting that, for a large number, the number of digits (before any decimal point, if there is one), is one more than the positive exponent of that same number when written in standard form.

Thus 1.083×10^{12} or 1,083,000,000,000 has 13 digits in it , and $4.679 \times 10^2 = 467.9$ has three (before the decimal point).

If the exponent is negative (i.e. we have a small number), write out the number portion and move the decimal point to the **left** by the number of places given in the exponent, inserting extra zeros immediately after the decimal point when necessary.

$4.17 \times 10^{-1} = 0.417$ (decimal point moved 1 place left)
$7.38 \times 10^{-2} = 0.0738$ (decimal point moved 2 places left, with an extra 0 inserted)
$2.156 \times 10^{-4} = 0.0002156$ (decimal point moved 4 places left, with three extra zeros inserted)

Example (4): The wavelength of the light from a sodium street lamp is 5.89×10^{-7} metres.
Express this value as an ordinary number.

After the decimal point has been moved one place left, the number part becomes 0.589. There are still 6 decimal places of the 7 to go, so we have to tag 6 extra zeros between the decimal point and the digit 5.

$\therefore$ **$5.89 \times 10^{-7} = 0.000\ 000\ 589$**

It is also worth noting that, for a small number, the number of zeros between the decimal point and the first non-zero digit is one less than the negative exponent of that same number in standard form.

Thus $5.89 \times 10^{-7} = 0.000\ 000\ 589$ has 6 zeros between the decimal point and the first nonzero digit (the 5), and $2.156 \times 10^{-4} = 0.0002156$ has 3 zeros after the decimal point but before the digit 2.

Standard Form Arithmetic.

Standard form lends itself well to multiplication, division, and power problems.

Because a number in standard form includes an exponent, i.e. a power of 10, we can use the laws of indices by treating the 'number' parts and 'exponent' parts separately.

Examples (5): Evaluate the following, using standard form throughout:

i) $(4 \times 10^3) \times (2 \times 10^5)$; ii) $(6 \times 10^7) \div (3 \times 10^2)$; iii) $(2 \times 10^3)^2$

In Part i) we can multiply the number parts to give $4 \times 2 = 8$ and the exponents to give $10^3 \times 10^5 = 10^8$. We then combine the intermediate results to give 8×10^8.
(Recall the index law: addition of powers corresponds to multiplication of numbers.)

In Part ii) we divide the number parts to give $6 \div 3 = 2$ and the exponents to give $10^7 \div 10^2 = 10^5$. Combining the intermediate results gives 2×10^5.
(Recall the index law: subtraction of powers corresponds to division of numbers.)

In Part iii) we square the number parts to give $2^2 = 4$ and the exponents to give $(10^3)^2 = 10^6$. Combining the intermediate results gives 4×10^6.
(Recall: doubling the powers corresponds to squaring numbers).

The examples above were chosen for simplicity, but sometimes a little adjustment is necessary.

Example (6): Evaluate the following, using standard form throughout:

i) $(8 \times 10^7) \times (6 \times 10^{-4})$; ii) $(5 \times 10^4) \div (8 \times 10^7)$; iii) $(5 \times 10^4)^3$

In Part i) we multiply the number parts to give $8 \times 6 = 48$ and the exponents to give $10^7 \times 10^{-4} = 10^3$. Combining the intermediate results, we have 48×10^3 , but this result is not *quite* in standard form because 48 is not between 1 and 10.

We therefore re-express the result as $48 \times 10^3 = 4.8 \times 10 \times 10^3 = \mathbf{4.8 \times 10^4}$, which this time is in correct standard form.

In Part ii) we divide the number parts to give $5 \div 8 = 0.625$ and the exponents to give $10^4 \div 10^7 = 10^{-3}$. Combining the intermediate results, we have 0.625×10^{-3}, but again this is not standard form because 0.625 is not between 1 and 10.

We therefore re-express the result as $0.625 \times 10^{-3} = 6.25 \times 10^{-1} \times 10^{-3} = \mathbf{6.25 \times 10^{-4}}$, which is correct standard form.

In Part iii) we cube the number parts to give $5^3 = 5$ and the exponents to give $(10^4)^3 = 10^{12}$. Combining the intermediate results gives 125×10^{12}, or $1.25 \times 10^2 \times 10^{12} = \mathbf{1.25 \times 10^{14}}$.

Examples (7): Evaluate the following in standard form:

i) $\sqrt{9 \times 10^8}$; ii) $\sqrt{6.4 \times 10^{11}}$; iii) $\dfrac{1}{5 \times 10^6}$

In i) both the number and exponent are perfect squares, so we can simply use the surd laws and the law of fractional powers:

$\sqrt{9 \times 10^8}$ = $\sqrt{9} \times \sqrt{10^8}$ = $\mathbf{3 \times 10^4}$. (Halve the indices to take square roots.)

In ii) the exponent is odd, so we must put the expression into a form where the exponent is even;

$\sqrt{6.4 \times 10^{11}}$ = $\sqrt{64 \times 10^{10}}$ = $\mathbf{8 \times 10^5}$.

In iii) we redefine the fraction as $\dfrac{1}{5 \times 10^6}$ = $\dfrac{1}{5} \times \dfrac{1}{10^6}$ and use the law of negative powers to obtain $\dfrac{1}{5 \times 10^6}$ = 0.2×10^{-6} , adjusting to $\mathbf{2 \times 10^{-7}}$.

Note: $\dfrac{1}{10^6} = 10^{-6}$.

Addition and subtraction in standard form can be slightly more awkward.

Examples (8): Evaluate the following, giving the answers in standard form :

i) $(8 \times 10^3) - (3 \times 10^3)$; ii) $(7 \times 10^{-4}) + (9 \times 10^{-4})$;
iii) $(6 \times 10^4) + (9 \times 10^2)$; iv) $(7 \times 10^{-2}) - (2 \times 10^{-3})$;
v) $(7 \times 10^{23}) + (5 \times 10^{21})$

When two numbers in standard form have the same exponent, then the number parts can simply be added (or subtracted), whilst leaving the exponent unchanged.

In Parts i) and ii) the exponent parts match, and therefore we can work out
$(8 \times 10^3) - (3 \times 10^3) = \mathbf{5 \times 10^3}$ and $(7 \times 10^{-4}) + (9 \times 10^{-4}) = 16 \times 10^{-4} = \mathbf{1.6 \times 10^{-3}}$.

When the exponents are different, as in parts iii) and iv), it is best in most cases to work in regular notation and then convert to standard form at the end of the sum.

In Part iii) one of the numbers to be added is a multiple of 10^4 but the other is a multiple of 10^2 . We therefore convert both to regular notation, perform the sum, and finally convert back to standard form.

$(6 \times 10^4) + (9 \times 10^2) = 60{,}000 + 900 = 60{,}900 = \mathbf{6.09 \times 10^4}$.

In Part iv) one of the numbers has an exponent of 10^{-2} but the other an exponent of 10^{-3} .

Convert both numbers to ordinary notation, subtract and then convert back to standard form:

$(7 \times 10^{-2}) - (2 \times 10^{-3}) = 0.07 - 0.002 = 0.068 = \mathbf{6.8 \times 10^{-2}}$.

In v) the numbers are too large to convert into ordinary notation, so we adjust one of the numbers in such a way as to make the exponents the same. Thus the number 5×10^{21} can be expressed in "not quite standard form" as 0.05×10^{23}.

We can now perform the addition sum as $(7 \times 10^{23}) + (0.05 \times 10^{23}) = \mathbf{7.05 \times 10^{23}}$.

This is better than treating the sum as $(700 \times 10^{21}) + (5 \times 10^{21}) = 705 \times 10^{21} = 7.05 \times 10^{23}$
as this involves an extra step to convert the result into standard form .