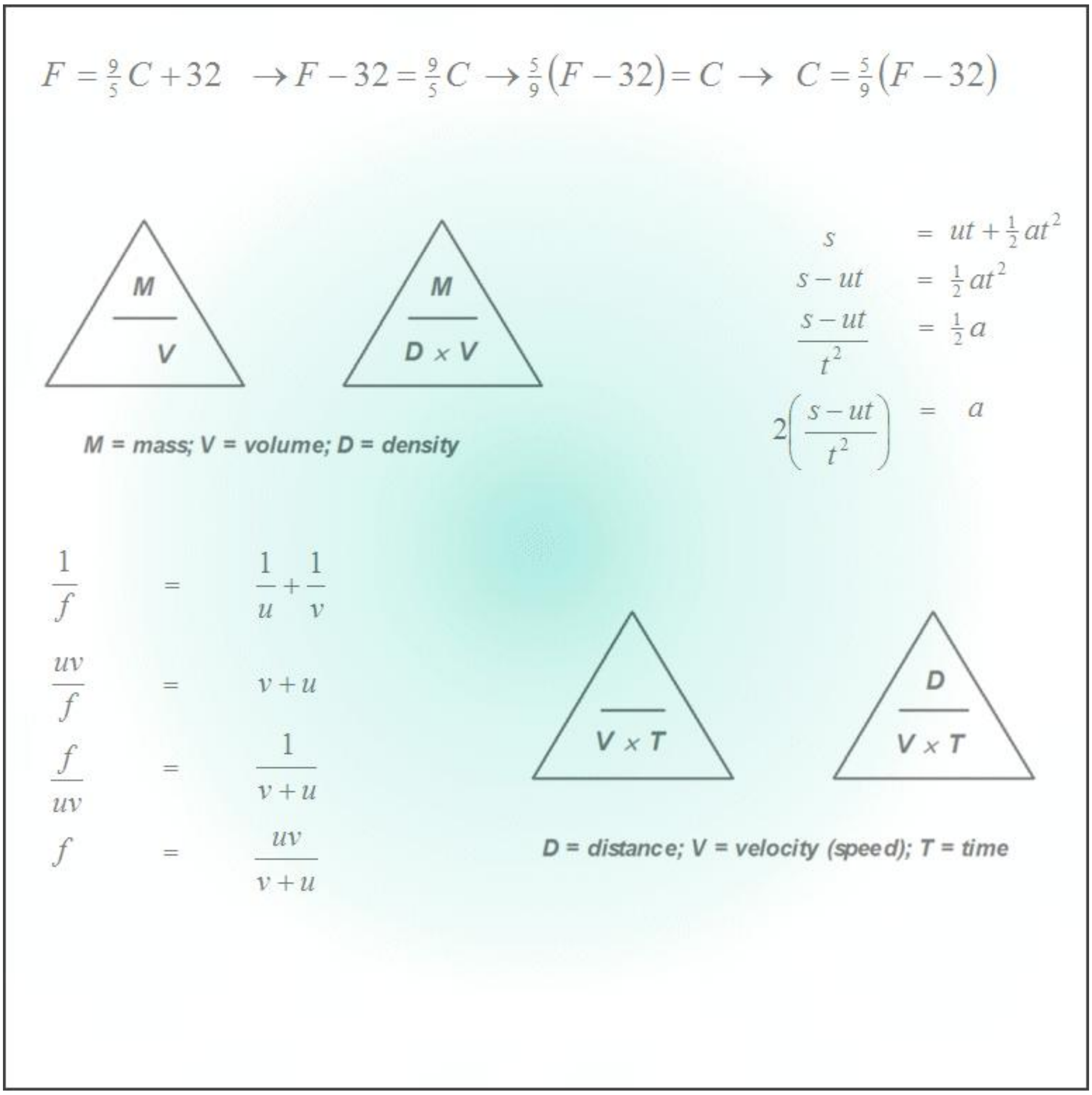
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

FORMULAE



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FORMULAE

A formula is a group of mathematical symbols, usually letters, expressing a relationship between quantities. Typically, a formula consists of a letter on the left-hand side and an expression on the right-hand side, with an equality sign in between.

Take the relationship between mass, volume and density. Since we obtain the mass by multiplying the density by the volume, we can express this relationship as a formula

$M = DV$ where M represents mass, D represents density and V represents volume.

Example (1) : The formula for the area A of a trapezium is $A = \frac{1}{2}(a+b)h$
where a and b are the parallel sides and h is the vertical height of the trapezium.

Find the area A of the trapezium where a = 7 cm, b = 13 cm and h = 6cm.

Substituting for a , b and h we have $A = \frac{1}{2}(7+13)\times 6 = 60\,\text{cm}^2$.

Written this way, the formula is ideal for finding A when a, b and h are known. The answer to the last question was simply obtained by substituting values. The subject of the formula is A, it being the only term on the left-hand side.

Changing the formula subject.

Example (1a) : A trapezium has an area of 54 cm^2, a vertical height of 9 cm, and one of its parallel sides 7 cm long. Find the length of the other parallel side.

We are thus required to use the formula to find b where A = 54, h = 9 and a = 7.
We could substitute for A, a and h in the original formula $A = \frac{1}{2}(a+b)h$ to obtain the equation
$\frac{1}{2}(7 + b) \times 9 = 54$, rearrange the terms and solve for b.

However, this chapter is about formulae, so we shall take the approach of rearranging the formula by changing the subject from A to b.

The rules for changing the subject of a formula are the same as those for equations - just perform the same arithmetic on both sides.

Importantly, the subject must end up entirely on one side of the formula - you cannot have it referred to on both !

A	=	$\frac{1}{2}(a+b)h$	
$2A$	=	$(a+b)h$	Double both sides to get rid of the ½
$\frac{2A}{h}$	=	$a+b$	Divide both sides by h
$\frac{2A}{h} - a$	=	b	Subtract a from both sides

The formula $b = \frac{2A}{h} - a$ now has b as its subject after exchanging left and right-hand sides.
It is now just a matter of substitution to find b where A = 54, h = 9 and a = 7.

Hence the length of the other parallel side of the trapezium, b, is $\frac{2\times 54}{9} - 7 = 5$ cm.

Example (2) : In the formula $s = ut + \tfrac{1}{2}at^2$
find *s* when *t* = 5, *u* = 10 and *a* = 2.

The solution is $s = (10 \times 5) + (\tfrac{1}{2})(2)(5^2) = 75$

Example (2a).
Supposing we wanted to find *a*, given that *s* = 114, *t* = 6 and *u* = 10. This would require rearranging the formula by changing the subject from *s* to *a.*

s	=	$ut + \tfrac{1}{2}at^2$	
$s - ut$	=	$\tfrac{1}{2}at^2$	Subtract *ut* from both sides
$2(s - ut)$	=	at^2	Double both sides to remove the fraction
$\dfrac{2(s - ut)}{t^2}$	=	a	Divide both sides by t^2

The same formula, but rearranged to make *a* the subject, is $a = \dfrac{2(s - ut)}{t^2}$.

The value of *a* can then be found by simple substitution : $a = \dfrac{2(114 - 60)}{36} = 3$

This method cannot be used to make *t* the subject of the formula - this is why.

s	=	$ut + \tfrac{1}{2}at^2$	
$s - ut$	=	$\tfrac{1}{2}at^2$	Subtract *ut* from both sides
$2(s - ut)$	=	at^2	Double both sides to remove the fraction
$\dfrac{2(s - ut)}{a}$	=	t^2	Divide both sides by *a*

It is impossible to go further, because *t* is still there on both sides of the formula. No manipulation can isolate *t* on to one side only. Dividing both sides by *t* also leads to a dead end.

(There is a way of making *t* the subject of the formula, but it involves other methods).

Example (3) : Rewrite the formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

with *f* as the subject, but without nested fractions.

One method of rearranging the formula is :

$\frac{1}{f}$	=	$\frac{1}{u}+\frac{1}{v}$	
$\frac{uv}{f}$	=	$\frac{uv}{u}+\frac{uv}{v}=v+u$	Multiply both sides by *uv* to perform the fraction addition
$\frac{f}{uv}$	=	$\frac{1}{u+v}$	Take reciprocals of both sides
f	=	$\frac{uv}{u+v}$	Multiply both sides by *uv*

The required formula, after rearrangement, is $f=\frac{uv}{u+v}$

Example (4): Rewrite the formula

$$v^2 = u^2 + 2as$$

making u the subject, and allowing for negative values of u.

$u^2 + 2as$	=	v^2	
u^2	=	$v^2 - 2as$	Subtract $2as$ from both sides.
u	=	$\pm\sqrt{v^2 - 2as}$	Take square root of both sides, **but include positive and negative values if the question requests it.**

The rewritten formula is $u = \pm\sqrt{v^2 - 2as}$

Beware of cases where square roots of both sides need to be taken.

Example (5) (Harder):

Make x the subject of the formula $y = \dfrac{x+a}{1-x}$.

The algebra is slightly harder here.

y	=	$\dfrac{x+a}{1-x}$	
$y(1-x)$	=	$x+a$	Multiply both sides by 1-x
$y - xy$	=	$x+a$	Expand the LHS
y	=	$x + xy + a$	Bring x-terms to RHS
$y - a$	=	$x + xy$	Bring non-x terms to LHS
$y - a$	=	$x(1+y)$	Factor x from RHS
$\dfrac{y-a}{1+y}$	=	x	Divide both sides by 1+y

The required formula, after rearrangement, is $x = \dfrac{y-a}{1+y}$

We had had to collect x–terms to one side in order to take x out as a factor, and hence make it the formula subject.

Example (6) (Harder):

An experiment was carried out to determine the acceleration due to gravity, *g*. A piece of string 0.5 m long with a bob towards its end was set into motion like a pendulum and the time taken for it to complete 50 oscillations was measured at 71.2 seconds.

Calculate the value of *g* to 3 figures, given that the formula relating a pendulum's period and its length is

$t = 2\pi\sqrt{\frac{l}{g}}$, where *t* is the period (time to complete 1 oscillation in seconds) and *l* is the length of the pendulum in metres.

Here, $l = 0.5$, $t = 71.3/50 = 1.424$, and $\pi = 3.142$ (use 4 figures for working)

The formula must be rewritten so that *g* is the subject:

$2\pi\sqrt{\frac{l}{g}}$	=	t	
$\sqrt{\frac{l}{g}}$	=	$\frac{t}{2\pi}$	Divide both sides by 2π.
$\frac{l}{g}$	=	$\frac{t^2}{4\pi^2}$	Square both sides
$\frac{g}{l}$	=	$\frac{4\pi^2}{t^2}$	Take reciprocals of both sides
g	=	$\frac{4\pi^2 l}{t^2}$	Multiply both sides by *l*

Therefore, $g = \frac{4\pi^2 l}{t^2}$, and the values for *l* and *t* can be substituted directly into the formula.

$$g = \frac{4 \times 3.142^2 \times 0.5}{1.424^2} = 9.74$$

to 3 figures.

Finding a formula from requirements.

Example (7): Weather forecasts in the USA use the Fahrenheit scale for temperature, and the rule for converting temperature readings from Fahrenheit to Celsius temperature is

"Subtract 32, then divide by 1.8".

Rewrite this rule as a formula, and use it to convert 77°F to the Celsius scale.

Firstly, we choose letter symbols, say F to represent the Fahrenheit temperature and C to represent Celsius.

Starting with F, we subtract 32 to obtain F – 32. Then we divide by 1.8 to complete the formula

$C = \frac{(F-32)}{1.8}$. Substituting F = 77 gives $C = \frac{(77-32)}{1.8} = \frac{45}{1.8} = 25$.

∴ 77°F = 25°C.

Example (8): Deduce a formula which converts a temperature in Celsius to one in Fahrenheit, by changing the formula subject. Use the result to convert 15°C to the Fahrenheit scale.

C	=	$C = \frac{(F-32)}{1.8}$	
$1.8C$	=	F - 32	Multiply both sides by 1.8
$1.8C + 32$	=	**F**	Add 32 to both sides

The formula to convert Celsius to Fahrenheit temperature is therefore $F = 1.8C + 32$.

Substituting C = 15 gives F = (1.8 × 15) + 32 = 27 + 32 = 59.

∴ 15°C = 59°F.

Example (9): A bank offers to exchange pounds sterling for euros. If the market euro rate is currently £1 = €1.16, i) write a formula that connects the pounds spent with euros received, and hence find out how many euros can be obtained for £150, and ii) write a formula to convert euros back to sterling at the same rate, and hence convert €203 back to sterling.

Let P denote the number of pounds spent and E the number of euros received.

i) Since £1 = €1.16, the required formula is $E = 1.16P$.
Substituting P = 150, £150 will be converted into 1.16 × 150 euros, or €174.

ii) Since $E = 1.16P$, the formula can be rewritten with P as the subject by dividing both sides by 1.16.

Hence $1.16P = E$, and dividing by 1.16 gives $P = \frac{E}{1.16}$.

Thus, €203 = £$\frac{203}{1.16}$ = £175.

Example (10).
A car fuel tank has a capacity of 50 litres. Write a formula connecting the litres used, L, and the percentage P of fuel remaining in the tank, assuming no fill-up in between.

The amount of fuel left in the tank is $50 - L$ litres. To express it as a percentage of the total, divide $(50 - L)$ by 50 and multiply it by 100.

The required formula is

$$P = \frac{100(50 - L)}{50} = 2(50 - L)$$

Example (11).
The internal angles of a triangle add up to 180°, and those of a quadrilateral add up to 360°. For each extra side of the polygon, an extra 180° is added to the angle sum. Write a formula connecting the number of sides, n, of a polygon and the internal angle sum, S.

When n is increased by 1, S increases by 180°, suggesting that the formula would include $180n$. When $n = 3$, $S = 180 \times 1$, and when $n = 4$, $S = 360 = 180 \times 2$. The multiple of 180° is evidently two less than the number of sides of the polygon.

The required formula is thus $S = 180(n - 2)°$.

The Formula Triangle.

If a formula has three variables, where one is a product or a quotient of the other two, is is useful to illustrate the relationship by a **formula triangle**.

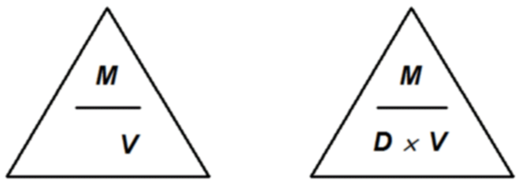
Some of the more important relationships are shown as follows; physics students might be familiar of other examples.

<u>Density.</u>

Example (12): Density is measured by dividing the mass of a substance by its volume, i.e. $D = \frac{M}{V}$.

Draw the formula triangle to show the relationship.

Using the symbols ***D*** for density, ***M*** for mass and ***V*** for volume, we first put the quotient into the empty triangle on the left, and then complete the triangle by putting ***D*** and the × sign on the bottom row.
We can now use the completed formula triangle as follows:



M = mass; V = volume; D = density

If we need to find the mass, we cover up the ***M*** to leave the required formula ***D*** × ***V***.

To find the density, we cover up the ***D*** and the × sign to leave ***M/V***.

To find the volume, we cover up the ***V*** and the × sign to leave ***M/D***.

The next two examples make use of the following densities:

Iron: 7.8 g/cm^3 Lead: 11.4 g/cm^3 Gold: 19.3 g/cm^3

Example (13): Find the mass of an iron cube of side 15 cm. Give your answer in kilograms to the nearest 0.1 kg.

The volume ***V*** of the block is 15 × 15 × 15 = 3375 cm^3 and its density ***D*** is 7.8 g/cm^3 .
Covering up ***M***on the formula triangle leaves us with ***D*** × ***V*** which works out at

3375 × 7.8 g, or 26,325 g. In kilograms, this is 26.3 kg to the nearest 0.1 kg.

Example (14): A gold ingot was stolen from a display at a museum. Its mass was 41 kg and its dimensions were 30 cm × 12 cm × 10 cm. Strangely enough, the police were not too concerned about the theft when they were told the facts about the ingot. Could you suggest why ?

The volume ***V*** of the ingot was 30 × 12 × 10 = 3600 cm^3 and its mass ***M*** was 41000 g (remember to convert kg to g by multiplying by 1000).

Covering up ***D*** on the formula triangle leaves us with ***M/V*** which works out at
$\frac{41000}{3600}$ or 11.4 g/cm^3 .

The density of the ingot was 11.4 g/cm^3 - the ‘gold’ ingot was actually a dummy made of lead !

Density of Mixtures.

Example (15): A garage owner produces a motor oil blend by mixing 17 litres of light oil of density 0.85 g/cm^3 with 8 litres of heavy oil of density 0.9 g/cm^3. Calculate the density of the blended oil.

We multiply the volume of each individual oil by its density to find the mass :

17 litres of light oil of density 0.85 g/cm^3 has a total mass of 17×0.85 kg = 14.45 kg.
8 litres of heavy oil of density 0.9 g/cm^3 has a total mass of 8×0.9 kg = 7.20 kg.

By addition, 25 litres of the blend has a mass of 21.65 kg, and so the density is $\frac{21.65}{25}$ or 0.866 kg / l .

As 1 litre = 1000 cm^3 and 1 kg = 1000 g, this density can be stated as **0.866 g/cm^3**.

Example (16) : A candlemaker mixes 17kg of light wax of density 0.85 g/cm^3 and 8 kg of heavy wax of density 0.9 g/cm^3. Calculate the density of the mixed wax.

The quantities and values in this example are the same as those in the previous one, but there is an important difference, because here we work with *masses* rather than *volumes.*

17 kg of light wax of density 0.85 g/cm^3 has a volume of $\frac{17000}{0.85} = 20{,}000$ cm^3.

Likewise 8 kg of heavy wax of density 0.9 g/cm^3 has a volume of $\frac{8000}{0.9} = 8{,}889$ cm^3.

By addition, 25 kg of the mixed wax has a volume of 28,889 cm^3.

Hence the density of the wax mixture is $\frac{25000}{28889} = 0.865$ g/cm^3.

Note the slight difference between the calculated density between this example and the previous one.

Example (17) (Algebra tie-in): Tom is an amateur winemaker, but the juice from his own grapes has a density of 1.07 g/cm^3 which is too low to make a decent wine. He wants to increase the density to 1.09 g/ cm^3, and to do this, he adds concentrated grape juice of density 1.34 g/cm^3.

Given that Tom has 5 litres of grape juice to begin with, how much concentrate does he need to increase the density of the juice from 1.07 to 1.09 g/cm^3 ?

Tom's original 5 litres of grape juice has a mass of 5×1.07 kg, or 5.35 kg.
He needs x litres of concentrate of density 1.34 g/cm^3 , and that adds $1.34x$ kg extra mass to his juice.
After adding the concentrate, he will have a total volume of $(5 + x)$ litres of juice, and the mass of the juice will be $(5.35 + 1.34x)$ kg.

Since density is mass divided by volume, and Tom requires a density of 1.09 g/cm^3 for his juice, we form the equation $\frac{5.35+1.34x}{5+x} = 1.09$.

This rearranges to $5.35 + 1.34x = 1.09(5 + x) \rightarrow 5.35 + 1.34x = 5.45+ 1.09x$
$\rightarrow 1.34x = 0.1 + 1.09x \rightarrow 0.25x = 0.1 \rightarrow x = 0.4$.

Tom therefore needs **0.4 litres**, or **400 ml**, of concentrate to raise the density of the juice from 1.07 to 1.09 g/cm^3.

Distance, Speed and Time.

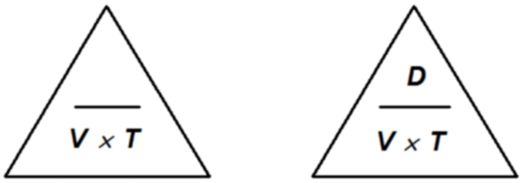
Distance is measured by multiplying speed (velocity) by time, i.e $D = VT$.

Using the symbols ***D*** for distance, ***V*** for speed and ***T*** for time, we first put the product into the empty triangle below left, and then complete the triangle by putting ***D*** and the division bar above it .

To find the distance, we cover up the ***D*** to leave the required formula $V \times T$.

To find the speed (velocity), we cover up the ***V*** and the × sign to leave ***D/T***.

To find the time, we cover up the ***T*** and the × sign to leave ***D/V***.



D = distance; V = velocity (speed); T = time

Example (18): A train takes 2 hours and 18 minutes to cover the 184 miles between Manchester and London. Find the average speed in miles per hour.

Here we have $D = 184$ and $T = 2.3$ (remember that 18 minutes = 0.3 hours) and we require ***V***.

We therefore cover up ***V*** to be left with ***D/T*,** which works out at $\frac{184}{2.3}$ or 80 m.p.h.

Example (19): Another train takes 2 hours and 6 minutes to cover the distance between London and York at an average speed of 90 miles per hour. What is the distance between London and York ?

The time $T = 2.1$ hours and the speed $V = 90$, but we require ***D***.
Covering up ***D*** on the formula triangle leaves us with $T \times V$ which works out at 2.1×90, or 189.

The distance between London and York is therefore 189 miles.

Difference between velocity and speed.

Although velocity and speed are often taken to be the same, there is a subtle difference between the two, and it is particularly important in physics.

Speed has **magnitude** but no **direction**, but velocity has both.

A southbound train from Manchester to London might have the same average *speed* of 80 miles per hour as a northbound one from London to Manchester, but when it comes to velocities, we would have to choose one of the directions as being positive and the other one as negative.

If we took the southbound direction as positive, then the southbound train could be said to have a velocity of 80 mph, but the northbound one a velocity of -80 mph.

Similarly a car going round a roundabout might have a constant *speed*, but because of its changing direction, its *velocity* would not be constant.

<u>Further Speed and Time Problems.</u>

Example (20): Jeremy drove off from Newcastle at 10:15 to attend a conference in Liverpool at 14:00. He by-passed Leeds, 105 miles past Newcastle, at 11:55, but the second part of his journey to Liverpool was much slower, averaging only 45 mph, while still taking the same time as the Newcastle-Leeds section.

i) How many minutes did Jeremy have to spare when he arrived at Liverpool ?
ii) Calculate Jeremy's average speed in miles per hour between Newcastle and Leeds.
iii) Calculate the distance between Leeds and Liverpool.
iv) Hence calculate Jeremy's average speed between Newcastle and Liverpool.

i) The time interval from 10:15 to 11:55 is 1 hour and 40 minutes, or 100 minutes, for the Newcastle-Leeds section, and it took Jeremy the same time to continue on to Liverpool, so he arrived there at 13:35, leaving him with 25 minutes to spare before the start of the conference.

ii) Jeremy's average speed between Newcastle and Leeds is obtained by dividing the distance of 105 miles by the time of 100 minutes, but that would give the average speed in miles *per minute* rather than in miles *per hour*, so we have to multiply by 60.
Hence Jeremy's average speed between Newcastle and Leeds was $\frac{105}{100} \times 60 = 63$ m.p.h.

iii) Distance is speed multiplied by time, so we multiply the speed of 45 m.p.h. by the time, which is 100 minutes, or 1 hour and 40 minutes, or $1\frac{2}{3}$ hours. (Because the speed is stated in m.p.h. we use hours rather than minutes as our time units).

Hence the distance from Leeds to Liverpool is $45 \times 1\frac{2}{3} = 75$ miles.

iv) The total distance from Newcastle to Liverpool is 180 miles, and Jeremy's total driving time was 200 minutes.
Calculating as in ii), the average speed between Newcastle and Liverpool was $\frac{180}{200} \times 60 = 54$ m.p.h.

We could have tabled the initial facts and completed as we went along:

After part i): Travel times calculated

	Distance	Time	Average Speed
Newcastle-Leeds	105 miles	**1 h 40 m**	
Leeds-Liverpool		**1 h 40 m**	45 mph
Whole journey		**3 h 20 m**	

After parts ii) and iii): Average speed for first stage, and distance for second stage calculated

	Distance	Time	Average Speed
Newcastle-Leeds	105 miles	1 h 40 m	**63 mph**
Leeds-Liverpool	**75 miles**	1 h 40 m	45 mph
Whole journey		3 h 20 m	

After part iv): Distances totalled and average speed for whole journey calculated

	Distance	Time	Average Speed
Newcastle-Leeds	105 miles	1 h 40 m	63 mph
Leeds-Liverpool	75 miles	1 h 40 m	45 mph
Whole journey	**180 miles**	3 h 20 m	**54 mph**

Average Speed.

Example (20a): Jeremy discussed the journey with his colleague James, remarking on his average speeds.

Jeremy: "James, my average speeds for each part of my trip were 63 m.p.h. and 45 m.p.h, and my average speed for the whole journey was 54 m.p.h..That means you can find the average speed for a whole trip by adding the speeds for the two separate stages and halving. How hard can it be ?"

James: "It's not as easy as that. It only works if the times taken for each section of the journey happen to be the same."

Show that James's conclusion is correct by representing the time for each part by t, the speed of the first part as u, and the speed of the second part as v.

Distance is speed × time, so the first part of the distance is ut , and the second part is vt.
The total time is $2t$ and the total distance is $ut + vt$ or $(u + v)\,t$.

Hence the average speed in this special case is $\frac{(u+v)t}{2t} = \frac{(u+v)}{2}$, i.e. the mean of u and v.

Example (20b): Some time later, James was driving down the motorway with Jeremy as passenger, when both came across a two-mile stretch of road works with an average speed camera set to a strict maximum speed limit of 40 mph. James therefore proceeded to drive the first mile at 60 mph, but slowed down to 30 mph for his second mile.

Jeremy: "You idiot, James. We were talking average speeds only the other day, and you did half the distance at 60 and half at 30, so that's an average speed of 45. We'll get a speeding fine for this."

James: "If you remember what I told you, your average speed would have been 45 if you'd spent the same *time*, not the same *distance*, doing 60 for one part and 30 for the other. I think we'll be all right here, after all that."

Show, with full working, why James was right not to be worried about receiving a speeding fine.

A speed of 60 miles per hour is the same as one mile per minute, so it took James one minute to cover his first mile. He drove the second mile at 30 miles per hour, which is half a mile per minute, or a mile in two minutes.

James had driven 2 miles in 3 minutes, or $\frac{2}{3} \times 60 = 40$ m.p.h.

We could also have reasoned : 2 miles in 3 minutes = 20 miles in 30 minutes ,
or 40 miles in 60 minutes – i.e. 40 m.p.h.

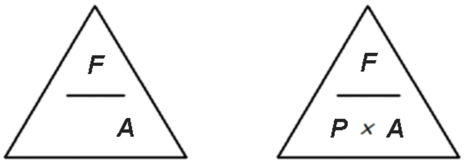
James was therefore correct in saying he would escape a speeding fine.

Pressure.

Pressure is the amount of force acting per unit area, and is measured by dividing the force by the area, i.e, $\boldsymbol{P} = \boldsymbol{F/A}$.

The completed formula triangle is shown on the right.

To find the area, we cover up the $\boldsymbol{A}$ and the $\times$ sign to leave $\boldsymbol{F/P}$; to find the force, we cover up the $\boldsymbol{F}$ to leave $\boldsymbol{P} \times \boldsymbol{A}$.



F = force; P = pressure; A = area

N.B. The unit of force is the **newton (N)**, and in physics, **weight** is a force exerted by an object on the ground due to gravity, which is not the same quantity as **mass**.. The weight is given by the formula $w = mg$ where m is the mass and g is the acceleration due to gravity.

Thus, astronauts might be weightless in the environment in a spacecraft where the gravity is zero, but their mass is still there, the same as on the earth.

Example (21): The cross-section of a vertical solid steel pillar is a rectangle measuring 40 cm × 30 cm. The force exerted by the pillar on the ground is 54,000 newtons. Find the pressure exerted on the ground by the pillar, in newtons per square metre.

We have $\boldsymbol{F} = 54{,}000$ and $\boldsymbol{A} = 0.4 \times 0.3 = 0.12$ m^2 (remember to convert !), but we need to find $\boldsymbol{P}$.

We therefore cover up $\boldsymbol{P}$ to be left with $\boldsymbol{F/A}$**,** which works out at $\frac{54000}{0.12}$, or **450, 000 N / m^2**.

The unit of pressure equal to 1 newton per square metre is also termed the pascal (Pa), so the answer to the last question could have also been written as **450,000 Pa.**

Example (21a): (Ratio / similarity tie-in)

The vertical solid steel pillar in the previous example is used to support the roof of a building, and the architect wants to redesign the pillar such that the pressure exerted on the ground by the pillar is reduced to 200,000 N / m^2., whilst the force exerted by the pillar on the ground is still 54,000 newtons. He wants the base tapering outwards so that it is a similar rectangle to the 40 × 30 cm main cross-section of the pillar.

Find the dimensions of the base of the pillar.

We have $\boldsymbol{F} = 54{,}000$ and $\boldsymbol{P} = 200{,}000$, but we need to find $\boldsymbol{A}$.

We therefore cover up $\boldsymbol{A}$ to be left with $\boldsymbol{F/P}$**,** which works out at $\frac{54000}{200000}$, or **0.27 m^2**.

The cross-sectional area of the main pillar is 0.12 m^2, but that of the tapered base is 0.27 m^2.
The area ratio between the tapered base and the main pillar is 0.27 : 0.12, or 9 : 4.

Taking square roots, the corresponding length ratio is 3 : 2.

The linear dimensions of the pillar's base are therefore $\frac{3}{2}$ times those of the main pillar.

Hence the pillar base is a rectangle of $(\frac{3}{2} \times 40) \times (\frac{3}{2} \times 30)$, or **60 × 45 cm.**