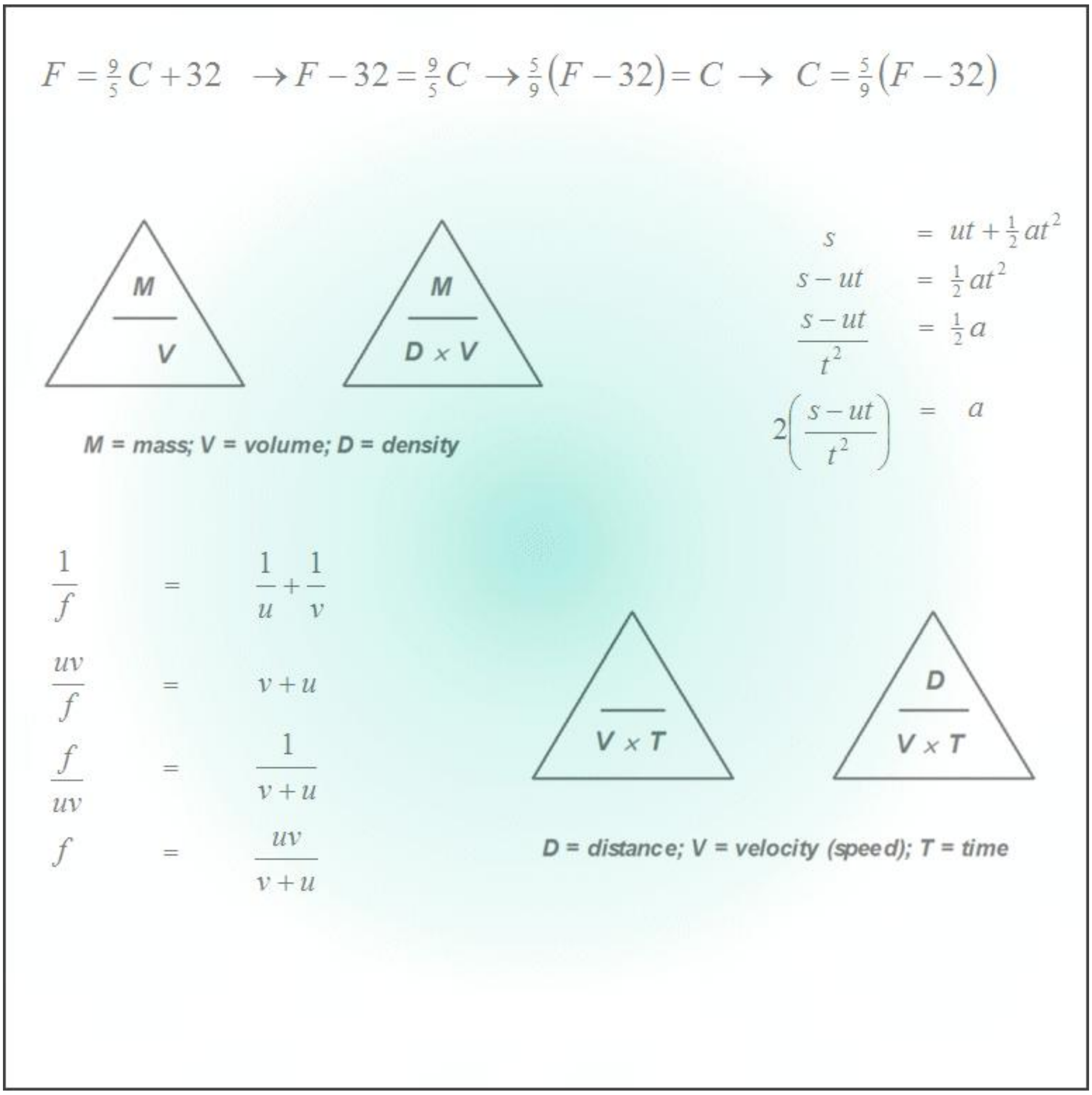
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

FORMULAE



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FORMULAE

A formula is a group of mathematical symbols, usually letters, expressing a relationship between quantities. Typically, a formula consists of a letter on the left-hand side and an expression on the right-hand side, with an equality sign in between.

Take the relationship between mass, volume and density. Since we obtain the mass by multiplying the density by the volume, we can express this relationship as a formula

$M = DV$ where M represents mass, D represents density and V represents volume.

Another example is $A = \pi r^2$ for the area of a circle, where r is the radius and π is a constant number whose value is approximately 3.14 or $\frac{22}{7}$.

Example (1) : In the formula $s = ut + \frac{1}{2}at^2$
find s when $t = 5$, $u = 10$ and $a = 2$.

The solution is

$$s = (10 \times 5) + (\tfrac{1}{2})(2)(5^2) = 75$$

Written this way, the formula is ideal for finding s when u, t and a are known. The answer to the last question was simply obtained by substituting values. The subject of the formula is s, it being the only term on the left-hand side.

Changing the formula subject.

Example (2).
Supposing we wanted to find a, given that s = 114, t = 6 and u = 10. This would require rearranging the formula by changing the subject from s to a.

The rules for changing the subject of a formula are the same as those for equations - just perform the same arithmetic on both sides.

Importantly, the subject must end up entirely on one side of the formula - you cannot have it referred to on both !

s	=	$ut + \frac{1}{2}at^2$	
$s - ut$	=	$\frac{1}{2}at^2$	Subtract ut from both sides
$\frac{s - ut}{t^2}$	=	$\frac{1}{2}a$	Divide both sides by t^2
$2\left(\frac{s - ut}{t^2}\right)$	=	a	Double both sides to remove the fraction

The same formula, but rearranged to make a the subject, is

$$a = 2\left(\frac{s - ut}{t^2}\right)$$

The value of a can then be found by simple substitution

$$a = 2\left(\frac{114 - 60}{36}\right) = 3.$$

This method cannot be used to make t the subject of the formula - this is why.

s	=	$ut + \frac{1}{2}at^2$	
$s - ut$	=	$\frac{1}{2}at^2$	Subtract ut from both sides
$\frac{s - ut}{a}$	=	$\frac{1}{2}t^2$	Divide both sides by a
$2\left(\frac{s - ut}{a}\right)$	=	t^2	Double both sides to remove the fraction

It is impossible to go further, because t is still there on both sides of the formula. No manipulation can isolate t on to one side only. Dividing both sides by t also leads to a dead end.

(There is a way of making t the subject of the formula, but it involves other methods).

Example (3) : Rewrite the formula

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

with *f* as the subject, but without nested fractions.

$\frac{1}{f}$	=	$\frac{1}{u}+\frac{1}{v}$	
uv	=	$f(v+u)$	Multiply both sides by *fuv*
$\frac{uv}{u+v}$	=	f	Divide both sides by (u + v)

The required formula, after rearrangement, is $f = \frac{uv}{u+v}$

Example (4): Rewrite the formula

$$v^2 = u^2 + 2as$$

making *u* the subject, and allowing for negative values of *u*.

$u^2 + 2as$	=	v^2	
u^2	=	$v^2 - 2as$	Subtract 2*as* from both sides.
u	=	$\pm\sqrt{v^2 - 2as}$	Take square root of both sides, **but include positive and negative values if the question requests it.**

The rewritten formula is $u = \pm\sqrt{v^2 - 2as}$

Beware of cases where square roots of both sides need to be taken.

Example (5) (Harder):

Make *x* the subject of the formula $y = \dfrac{x+a}{1-x}$.

The algebra is slightly harder here.

y	=	$\dfrac{x+a}{1-x}$	
$y(1-x)$	=	$x+a$	Multiply both sides by 1-*x*
$y - xy$	=	$x+a$	Expand the LHS
y	=	$x + xy + a$	Bring *x*-terms to RHS
$y - a$	=	$x + xy$	Bring non-*x* terms to LHS
$y - a$	=	$x(1+y)$	Factor *x* from RHS
$\dfrac{y-a}{1+y}$	=	x	Divide both sides by 1+*y*

The required formula, after rearrangement, is $x = \dfrac{y-a}{1+y}$

We had had to collect *x*–terms to one side in order to take *x* out as a factor, and hence make it the formula subject.

Example (6) (Harder):

An experiment was carried out to determine the acceleration due to gravity, *g*. A piece of string 0.5 m long with a bob towards its end was set into motion like a pendulum and the time taken for it to complete 50 oscillations was measured at 71.2 seconds.

Calculate the value of *g* to 3 figures, given that the formula relating a pendulum's period and its length is

$t = 2\pi\sqrt{\frac{l}{g}}$, where *t* is the period (time to complete 1 oscillation in seconds) and *l* is the length of the pendulum in metres.

Here, *l* = 0.5, *t* = 71.3/50 = 1.424, and π = 3.142 (use 4 figures for working)

The formula must be rewritten so that *g* is the subject:

$2\pi\sqrt{\frac{l}{g}}$	=	t	
$\sqrt{\frac{l}{g}}$	=	$\frac{t}{2\pi}$	Divide both sides by 2π.
$\frac{l}{g}$	=	$\frac{t^2}{4\pi^2}$	Square both sides
$\frac{g}{l}$	=	$\frac{4\pi^2}{t^2}$	Take reciprocals of both sides
g	=	$\frac{4\pi^2 l}{t^2}$	Multiply both sides by *l*

Therefore, $g = \frac{4\pi^2 l}{t^2}$, and the values for *l* and *t* can be substituted directly into the formula.

$$g = \frac{4 \times 3.142^2 \times 0.5}{1.424^2} = 9.74$$

to 3 figures.

Finding a formula from requirements.

Example (7): Weather forecasts in the USA use the Fahrenheit scale for temperature, and the rule for converting temperature readings from Fahrenheit to Celsius temperature is

"Subtract 32, then divide by 1.8".

Rewrite this rule as a formula, and use it to convert 77°F to the Celsius scale.

Firstly, we choose letter symbols, say F to represent the Fahrenheit temperature and C to represent Celsius.

Starting with F , we subtract 32 to obtain F – 32. Then we divide by 1.8 to complete the formula

$C = \frac{(F-32)}{1.8}$. Substituting F = 77 gives $C = \frac{(77-32)}{1.8} = \frac{45}{1.8} = 25$.

$\therefore$ **77°F = 25°C.**

Example (8): Deduce a formula which converts a temperature in Celsius to one in Fahrenheit, by changing the formula subject. Use the result to convert 15°C to the Fahrenheit scale.

C	=	$C = \frac{(F-32)}{1.8}$	
$1.8C$	=	$F - 32$	Multiply both sides by 1.8
$1.8C + 32$	=	F	Add 32 to both sides

The formula to convert Celsius to Fahrenheit temperature is therefore $F = 1.8C + 32$.

Substituting C = 15 gives F = (1.8 × 15) + 32 = 27 + 32 = 59.

$\therefore$ **15°C = 59°F.**

Example (9): A bank offers to exchange pounds sterling for euros. If the market euro rate is currently £1 = €1.16, i) write a formula that connects the pounds spent with euros received, and hence find out how many euros can be obtained for £150, and ii) write a formula to convert euros back to sterling at the same rate, and hence convert €203 back to sterling.

Let P denote the number of pounds spent and E the number of euros received.

i) Since £1 = €1.16, the required formula is $E = 1.16P$.
Substituting P = 150, £150 will be converted into 1.16 × 150 euros, or €174.

ii) Since $E = 1.16P$, the formula can be rewritten with P as the subject by dividing both sides by 1.16.

Hence $1.16P = E$, and dividing by 1.16 gives $P = \frac{E}{1.16}$.

Thus, €203 = £ $\frac{203}{1.16}$ = £175.

Example (10).
A car fuel tank has a capacity of 50 litres. Write a formula connecting the litres used, L, and the percentage P of fuel remaining in the tank, assuming no fill-up in between.

The amount of fuel left in the tank is $50 - L$ litres. To express it as a percentage of the total, divide $(50 - L)$ by 50 and multiply it by 100.

The required formula is

$$P = \frac{100(50 - L)}{50} = 2(50 - L)$$

Example (11).
The internal angles of a triangle add up to 180° or two right angles, and those of a quadrilateral add up to 360° or four right angles. For each extra side of the polygon, 2 right angles are added to the angle sum. Write a formula connecting the number of sides, n, of a polygon and the internal angle sum, S.

When n is increased by 1, S increases by 2, suggesting that the formula would include $2n$. When $n = 3$, $2n = 6$ and $S = 2$, and when $n = 4$, $2n = 8$ and $S = 4$.

The required formula is thus $S = 2n - 4$.

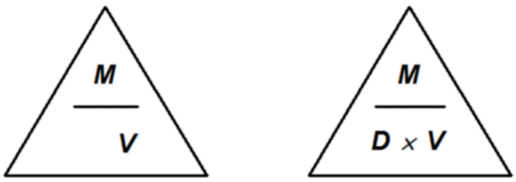
The Formula Triangle.

If a formula has three variables, where one is a product or a quotient of the other two, is is useful to illustrate the relationship by a **formula triangle**.

Example (12): Density is measured by dividing the mass of a substance by its volume, i.e. $D = \frac{M}{V}$.

Draw the formula triangle to show the relationship.

Using the symbols ***D*** for density, ***M*** for mass and ***V*** for volume, we first put the quotient into the empty triangle on the left, and then complete the triangle by putting ***D*** and the × sign on the bottom row.
We can now use the completed formula triangle as follows:



M = mass; V = volume; D = density

If we need to find the mass, we cover up the ***M*** to leave the required formula ***D*** × ***V***.

To find the density, we cover up the ***D*** and the × sign to leave ***M/V***.

To find the volume, we cover up the ***V*** and the × sign to leave ***M/D***.

The next two examples make use of the following densities:

Iron: 7.8 g/cm^3 Lead: 11.4 g/cm^3 Gold: 19.3 g/cm^3

Example (13): Find the mass of an iron cube of side 15 cm. Give your answer in kilograms to the nearest 0.1 kg.

The volume ***V*** of the block is 15 × 15 × 15 = 3375 cm^3 and its density ***D*** is 7.8 g/cm^3 .
Covering up ***M*** on the formula triangle leaves us with ***D*** × ***V*** which works out at

3375 × 7.8 g, or 26,325 g. In kilograms, this is 26.3 kg to the nearest 0.1 kg.

Example (14): A gold ingot was stolen from a display at a museum. Its mass was 41 kg and its dimensions were 30 cm × 12 cm × 10 cm. Was this ingot actually made of gold ?

The volume ***V*** of the ingot was 30 × 12 × 10 = 3600 cm^3 and its mass ***M*** was 41000 g (remember to convert kg to g by multiplying by 1000).

Covering up ***D*** on the formula triangle leaves us with ***M/V*** which works out at
$\frac{41000}{3600}$ or 11.4 g/cm^3 .

The density of the ingot was 11.4 g/cm^3 - the ‘gold’ ingot was actually a dummy made of lead !

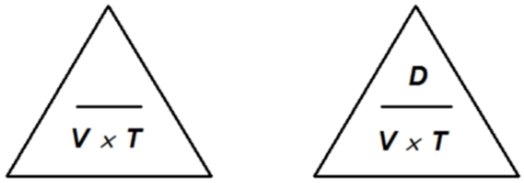
Example (15): Distance is measured by multiplying speed (velocity) by time, i.e ***D*** = ***VT***.
Draw the formula triangle to show the relationship.

Using the symbols ***D*** for distance, ***V*** for speed and ***T*** for time, we first put the product into the empty triangle below left, and then complete the triangle by putting ***D*** and the division bar above it .

To find the distance, we cover up the ***D*** to leave the required formula ***V*** × ***T***.

To find the speed (velocity), we cover up the ***V*** and the × sign to leave ***D/T***.

To find the time, we cover up the ***T*** and the × sign to leave ***D/V***.



D = distance; V = velocity (speed); T = time

Example (16): A train takes 2 hours and 18 minutes to cover the 184 miles between Manchester and London. Find the average speed in miles per hour.

Here we have ***D*** = 184 and ***T*** = 2.3 (remember that 18 minutes = 0.3 hours) and we require ***V***.

We therefore cover up ***V*** to be left with ***D/T*,** which works out at $\frac{184}{2.3}$ or 80 m.p.h.

Example (17): Another train takes 2 hours and 6 minutes to cover the distance between London and York at an average speed of 90 miles per hour. What is the distance between London and York ?

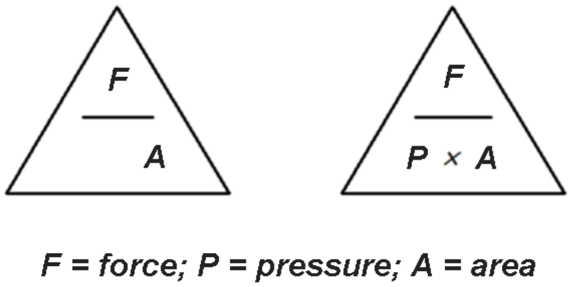
The time ***T* =** 2.1 hours and the speed ***V* =** 90, but we require ***D***.
Covering up ***D*** on the formula triangle leaves us with ***T*** × ***V*** which works out at 2.1 × 90, or 189.

The distance between London and York is therefore 189 miles.

(From 2017)

Example (16): Pressure is the amount of force acting per unit area, and is measured by dividing the force by the area, i.e, ***P*** = ***F/A***.
Draw the formula triangle to show the relationship.

Using the symbols ***F*** for force, ***A*** for area and ***P*** for pressure, , we first put the quotient into the empty triangle (left),and then complete the triangle by putting ***P*** and the × sign on the bottom row.



We can now use the completed formula triangle as follows:

To find the area, we cover up the ***A*** and the × sign to leave ***F/P***.

To find the force, we cover up the ***F*** to leave ***M***× ***D***.

N.B. The unit of force is the **newton (N)**, and in physics, **weight** is a force exerted by an object on the ground due to gravity, which is not the same quantity as **mass**.. The weight is given by the formula $w = mg$ where m is the mass and g is the acceleration due to gravity.

Thus, astronauts might be weightless in the environment in a spacecraft where the gravity is zero, but their mass is still there, the same as on the earth.

Example (17): The cross-section of a vertical solid steel pillar is a rectangle measuring 40 cm × 30 cm. The force exerted by the pillar on the ground is 45,000 newtons. Find the pressure exerted on the ground by the pillar, in newtons per square metre.

We have ***F*** = 45,000 and ***A*** = 0.4 × 0.3 = 0.12 m^2 (remember to convert !), but we need to find ***P***.

We therefore cover up ***P*** to be left with ***F/A*,** which works out at $\frac{45000}{0.12}$,
or **375, 000 N / m^2**.

The unit of pressure equal to 1 newton per square metre is also termed the pascal (Pa), so the answer to the last question could have also been written as **375,000 Pa.**