M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

RECOGNISINGグラフOF FUNCTIONS

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RECOGNISING GRAPHS

Graphs of functions fall into various categories, but most of those you'll come across will be of a few basic types.

**Straight-line graphs (recalled).**

Constant graphs:

![Graphs of constant functions](image1)

The main diagonals:

![Graphs of diagonal functions](image2)
Other straight line graphs passing through the origin:

- $y = 2x$
- $y = 2x$
- $y = 5x$
- $y = \frac{x}{4}$
- $y = -2x$
- $y = -\frac{x}{3}$
General straight line graphs, not passing through the origin:

\[ y = 3x - 2 \]

\[ y = 1 - x \]

The point where a linear graph cuts the \( y \)-axis is also known as the \( y \)-intercept, or simply the intercept.

The point where the graph cuts the \( x \)-axis is sometimes called the \( x \)-intercept, but is more often called the root (as in the solution of an equation).

The more common form of equation of a straight-line graph encountered at GCSE is the form

\[ y = mx + c \]

known as the gradient-intercept form.

The graph of \( y = 2x - 3 \) has a gradient of 2 and a \( y \)-intercept at \((0, -3)\).

Similarly the graph of \( y = 1 - x \) had a gradient of \(-1\) and its \( y \)-intercept was the point \((0, 1)\).

(Note that \( y = 1 - x \) is the same as \( y = -x + 1 \).)

The gradient of a graph whose equation is given in gradient-intercept form is evidently the multiple of \( x \), and that the graph crosses the \( y \)-axis where \( y \) takes the value of the constant.

- Any graph of the form \( y = mx + c \) has a gradient of \( m \) and a \( y \)-intercept at \((0, c)\).

(The gradient-intercept form cannot be applied to constant graphs of the form \( x = c \), since they are vertical lines and attempting to find a gradient would mean dividing by zero, which is undefined.)
Quadratic graphs.

These graphs are of functions of the form $y = ax^2 + bx + c$ where $a$, $b$ and $c$ are constants, and $a$ is not zero. The highest power of $x$ is 2 (the square of $x$). The basic graph of $y = x^2$ is shown upper left.

These graphs are parabolic or 'bucket-shaped'.

When the $x^2$ term is positive, the graphs point downwards at a trough and the function takes a minimum value. The expansion of $y = (x - 2) (x - 8)$ is $y = x^2 - 10x + 16$.

On the other hand, they point upwards at a crest and have a maximum value when the $x^2$ term is negative. The expansion of $y = (1 - x) (2 + x)$ is $y = 2 - x - x^2$.

The ‘depth’ of a parabolic graph can vary, but this is as dependent on the scaling of the graph axes as on the actual function.
Cubic graphs.

These are a little more complicated than quadratic graphs. Their functions are of the form \( y = ax^3 + bx^2 + cx + d \) where \( a, b, c \) and \( d \) are constants, and \( a \) is not zero. The highest power of \( x \) is 3 (the cube of \( x \)).

The basic graph of \( y = x^3 \) is shown upper left.

These graphs are characterised by a 'double bend'.

If the term in \( x^3 \) is positive, the general slope is upward from lower left, but if the \( x^3 \) term is negative, the general slope is downward from upper left.

The 'bend' can also vary in severity - the graph on lower left has sharper 'bends' than the one on lower right.
Reciprocal graphs.

These graphs are shared by functions of the form $y = \frac{k}{x}$, where $k$ is a non-zero constant. They differ from previous examples in that they seem to be in two unconnected parts: if $k$ is positive, the two sections are in the upper right and lower left, but if $k$ is negative, the sections are in the upper left and lower right.

Note the following features of the standard graph $y = \frac{1}{x}$.

As $x$ becomes large and positive, $y$ stays positive but approaches zero.
As $x$ becomes large and negative, $y$ stays negative but approaches zero.

As positive $x$ approaches zero, $y$ becomes increasingly large and positive. In other words, $y$ tends to infinity ($\infty$).

As negative $x$ approaches zero, $y$ becomes increasingly large and negative, i.e. $y$ tends to minus infinity ($-\infty$).

When $x = 0$, $y$ is undefined - you cannot divide by zero, so it is absurd to say that $\frac{1}{0} = \infty$. 
Exponential Graphs.

These are graphs of \( y = a^x \) where \( a \) is any positive number.

[Graphs showing \( y = 2^x \) and \( y = 0.8^x \)]

Graphs of this form have the following features in common:

Regardless of the value of \( a \), \( y = 1 \) when \( x = 0 \).

For \( x > 1 \):
- As \( x \) increases, \( y \) also increases for all values of \( x \).
- As \( x \) become large and negative, \( y \) tends to zero but never gets there.

For \( 0 < x < 1 \):
- \( y \) would still be 1 when \( x = 0 \).
- As \( x \) decreases, \( y \) also increases for all values of \( x \).
- As \( x \) becomes large and positive, \( y \) tends to zero but never gets there.

(The graph of \( y = a^x \) when \( a = 1 \) is merely the straight line \( y = 1 \).)
Trigonometric Graphs.

The three main trigonometric functions have the following graphs:

The graphs of $\sin x^\circ$ and $\cos x^\circ$ are similar to each other; both functions can only take values in the range -1 to +1, and both repeat themselves every 360°. Indeed, the graph of $\cos x^\circ$ is the same as that of $\sin x^\circ$ translated 90° to the left.

The graph of $\tan x^\circ$ is quite different. It repeats every 180°, and moreover the function is undefined for certain values of $x$, such as 90°, 270°, and all angles consisting of an odd number of right angles. When $x$ approaches 90° from below, $\tan x^\circ$ becomes very large and positive; when $x$ approaches 90° from above, $\tan x^\circ$ becomes very large and negative.
Circular Graphs.

A circle passing through the origin has an equation of \( x^2 + y^2 = r^2 \), where \( r \) is the radius of the circle.

In the example shown on the right, the circle has a radius of 5 units. Remember that the number on the right-hand side of the equation is the square of the radius, and not the radius itself.

Equation of the tangent to a circle.

For any point \((a, b)\) on a circle centred on the origin, and with a radius \( r \), the equation of the tangent to the circle at that point is \( ax + by = r^2 \).

In the above example, the equation of the circle is \( x^2 + y^2 = 25 \), and the selected point on its circumference is \((4, 3)\).

The equation of the tangent is thus \( 4x + 3y = 25 \).

Another method:

Gradient of radius from \((0, 0)\) to \((4, 3)\) = \( \frac{3}{4} \).

Since the tangent is perpendicular to the radius, its gradient is \( -\frac{4}{3} \), as two perpendicular lines have a gradient product of \(-1\).

The equation of the tangent is therefore \( y = -\frac{4}{3}x + c \).

Substituting \( x = 4, y = 3 \), we have \( -\frac{16}{3} + c = 3 \).

Hence \( c = \frac{25}{3} \) and the equation of the tangent is \( y = \frac{25}{3} - \frac{4}{3}x \).

(This equation is the same as \( 4x + 3y = 25 \), but in \( y = mx + c \) form).