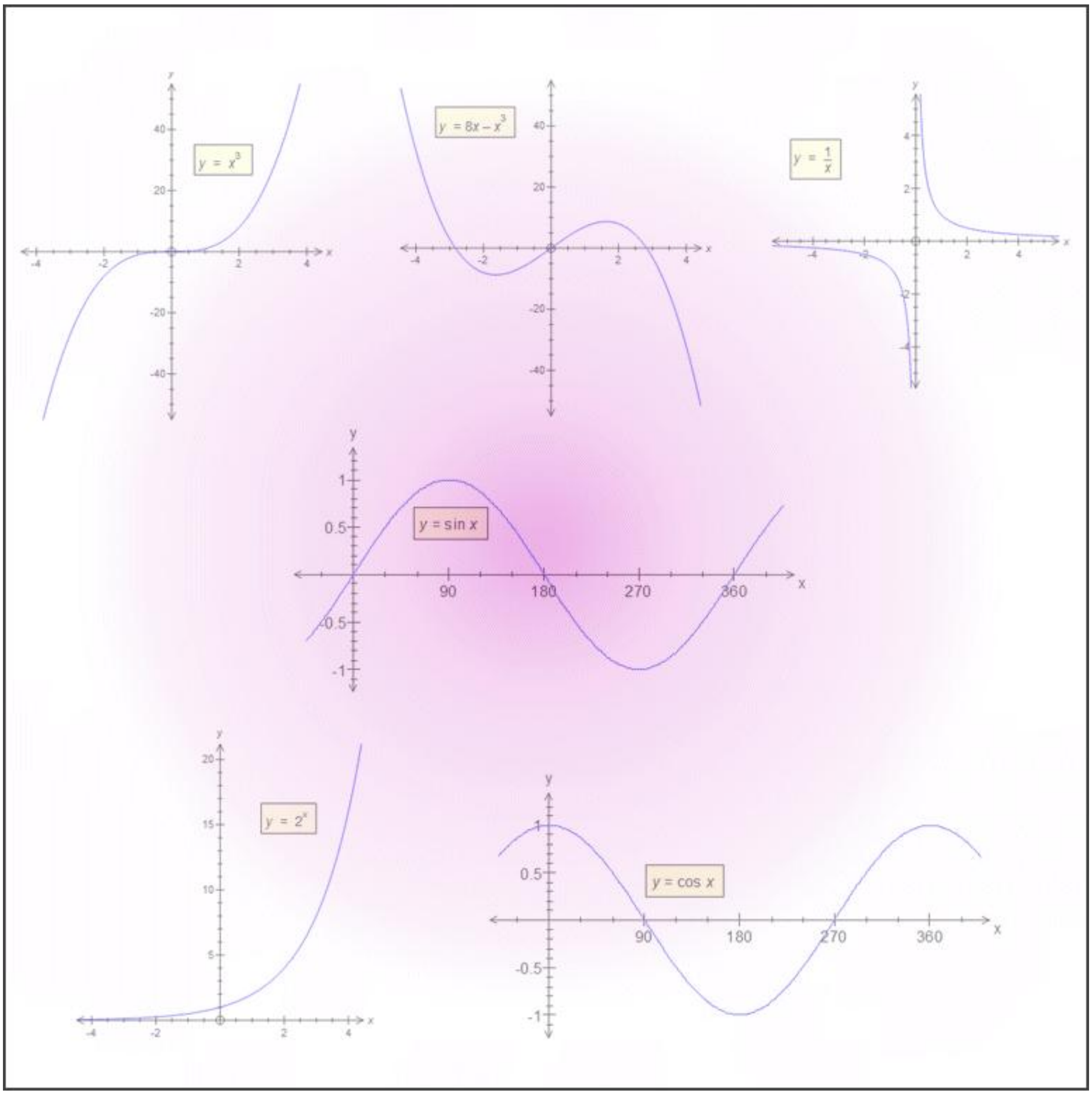
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

RECOGNISING GRAPHS OF FUNCTIONS



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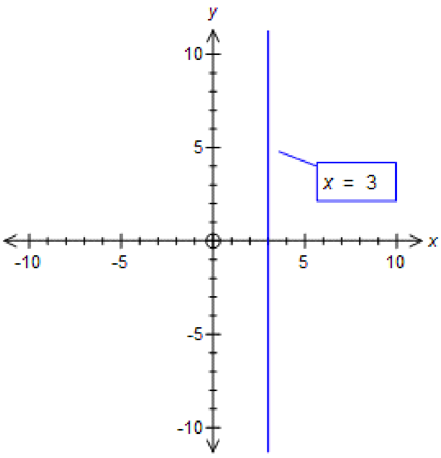
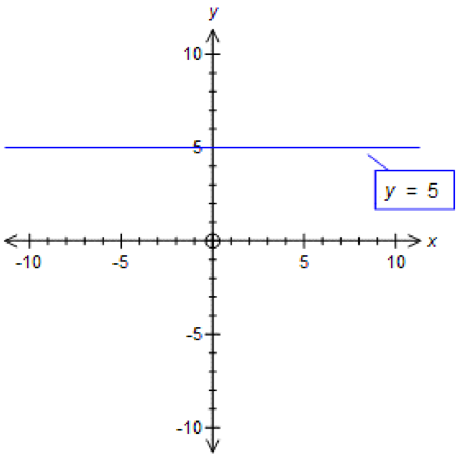
RECOGNISING GRAPHS

Graphs of functions fall into various categories, but most of those you'll come across will be of a few basic types.

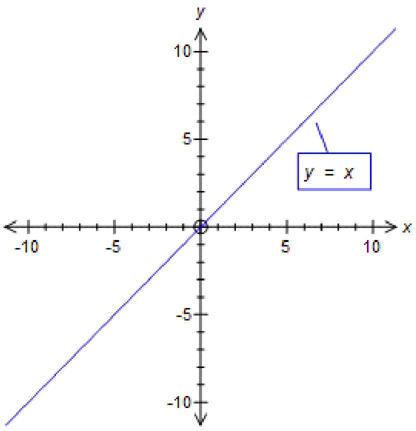
Straight-line graphs (recalled).

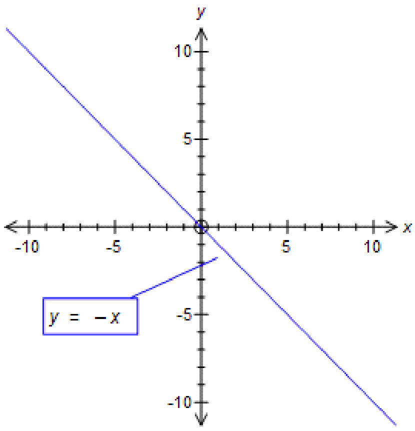
Constant graphs:



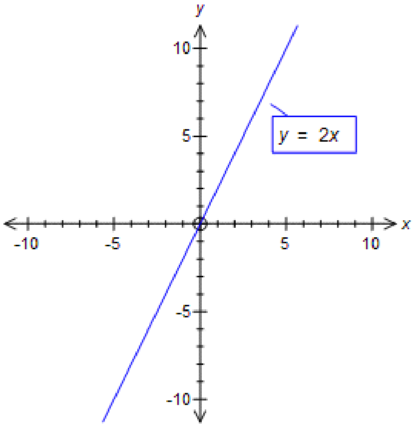


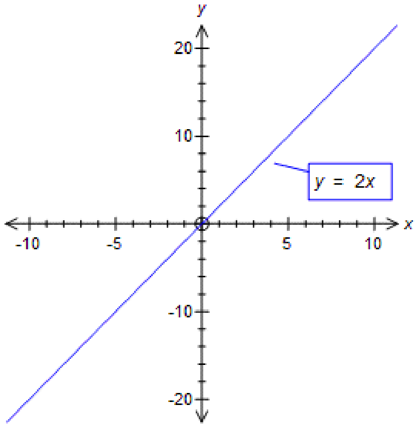
The main diagonals:

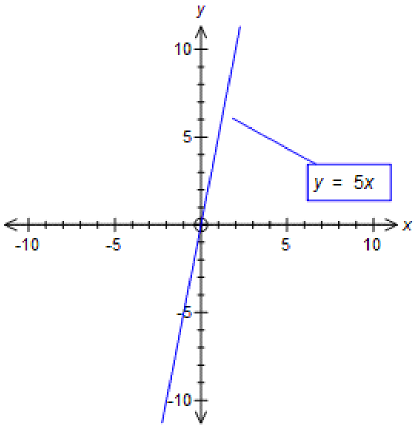


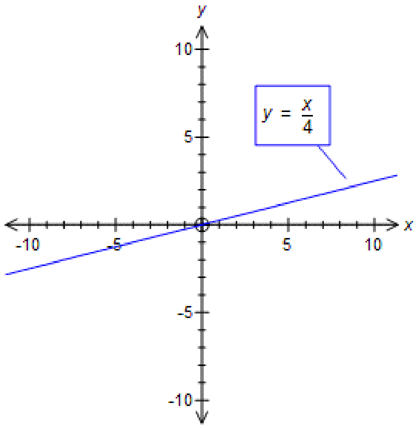


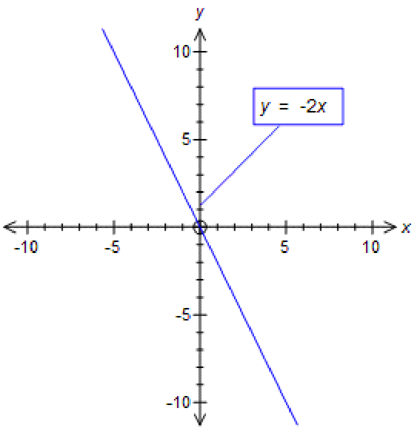
Other straight line graphs passing through the origin:

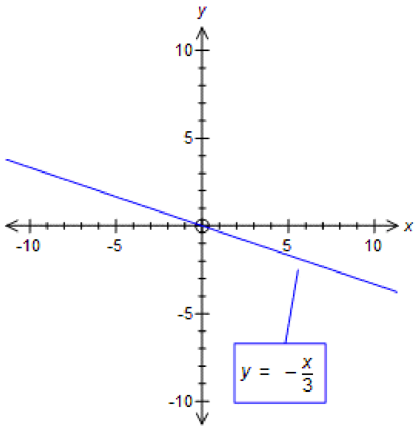




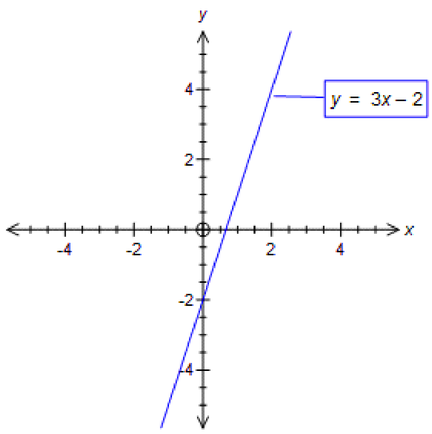


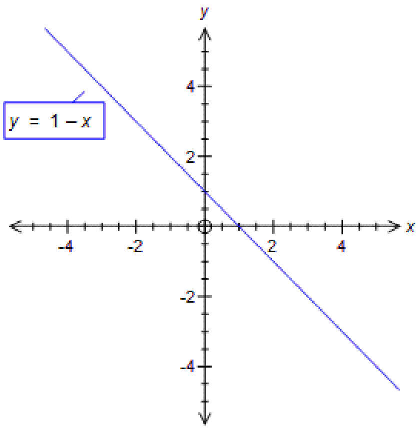






General straight line graphs, not passing through the origin:





The point where a linear graph cuts the *y*-axis is also known as the *y*-intercept, or simply the intercept.

The point where the graph cuts the *x*-axis is sometimes called the *x*-intercept, but is more often called the root (as in the solution of an equation).

The more common form of equation of a straight-line graph encountered at GCSE is the form

$\boldsymbol{y = mx + c}$, known as the **gradient-intercept** form.

The graph of $y = 2x - 3$ has a gradient of 2 and a *y*-intercept at (0, -3).
Similarly the graph of $y = 1 - x$ had a gradient of –1 and its *y*-intercept was the point (0, 1).
(Note that $y = 1 - x$ is the same as $y = -x + 1$).

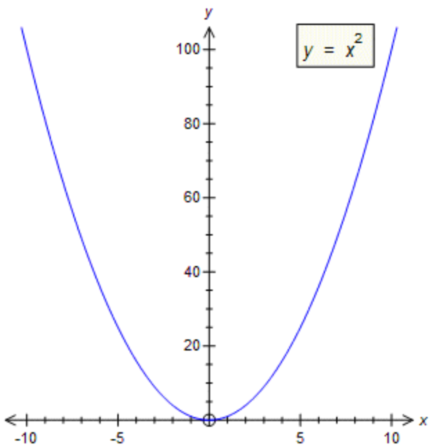
The gradient of a graph whose equation is given in gradient-intercept form is evidently the multiple of *x*, and that the graph crosses the *y*-axis where *y* takes the value of the constant.

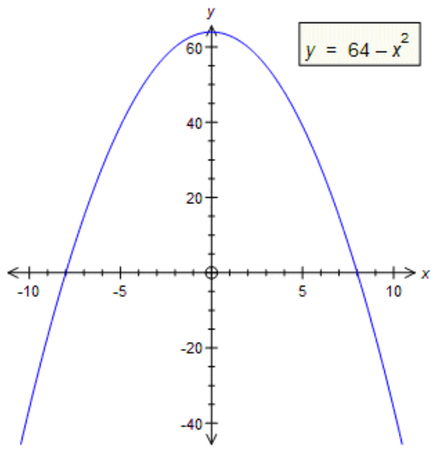
- Any graph of the form $\boldsymbol{y = mx + c}$ has a gradient of *m* and a *y*-intercept at (0, *c*).

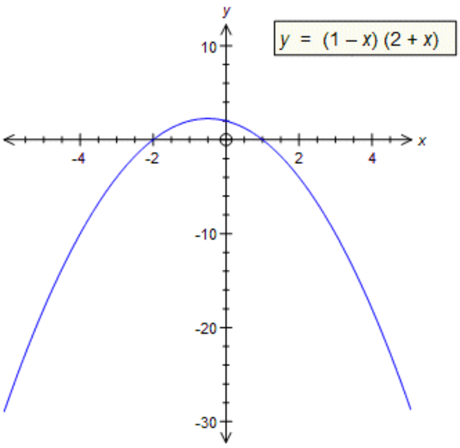
(The gradient-intercept form cannot be applied to constant graphs of the form $x = c$, since they are vertical lines and attempting to find a gradient would mean dividing by zero, which is undefined.)

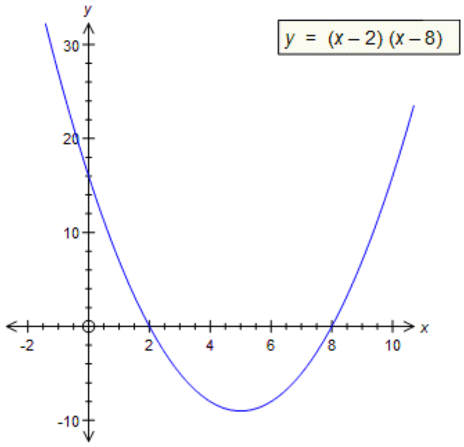
Quadratic graphs.

These graphs are of functions of the form $y = ax^2 + bx + c$ where a, b and c are constants, and a is not zero. The highest power of x is 2 (the square of x). The basic graph of $y = x^2$ is shown upper left.









These graphs are parabolic or 'bucket-shaped'.

When the x^2 term is positive, the graphs point downwards at a trough and the function takes a minimum value. The expansion of $y = (x - 2)(x - 8)$ is $y = x^2 - 10x + 16$.

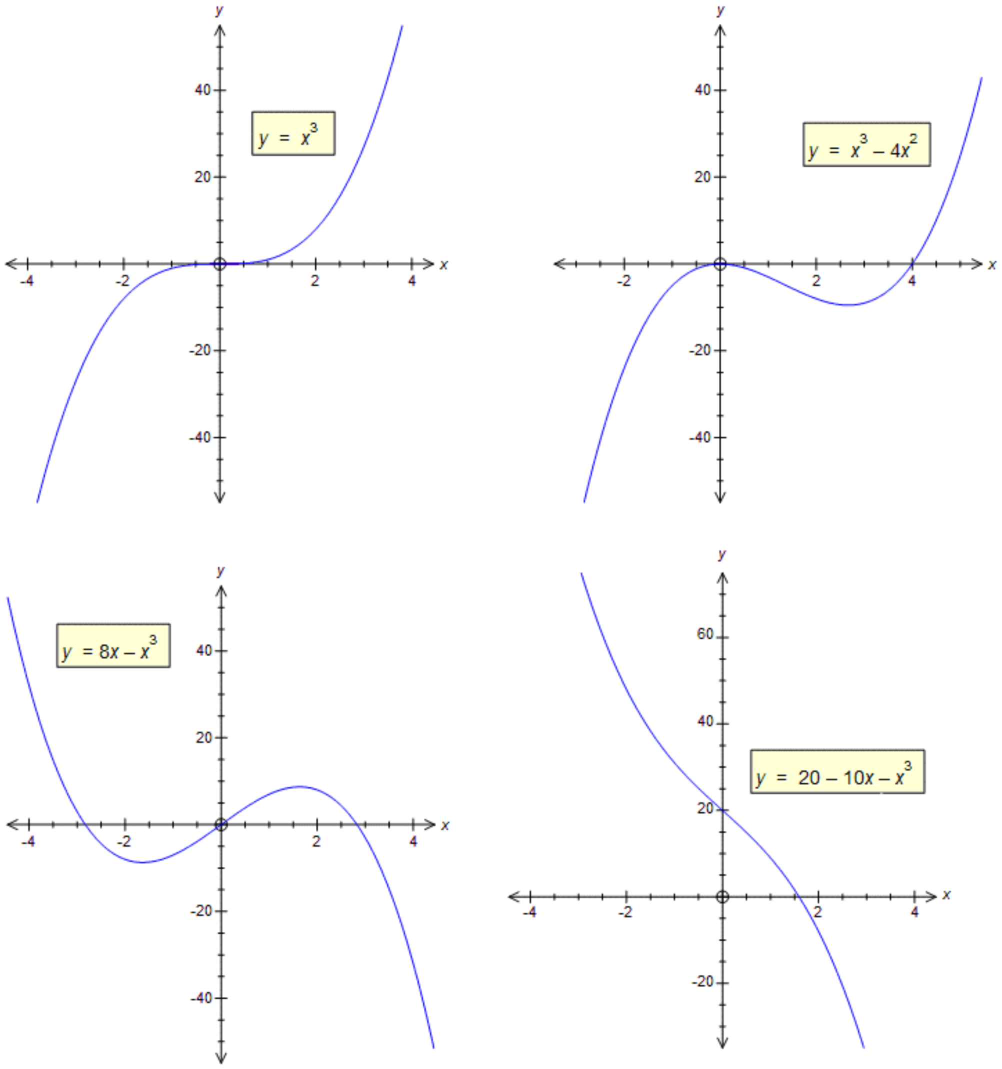
On the other hand, they point upwards at a crest and have a maximum value when the x^2 term is negative. The expansion of $y = (1 - x)(2 + x)$ is $y = 2 - x - x^2$.

The 'depth' of a parabolic graph can vary, but this is as dependent on the scaling of the graph axes as on the actual function.

Cubic graphs.

These are a little more complicated than quadratic graphs.
Their functions are of the form $y = ax^3 + bx^2 + cx + d$ where a, b, c and d are constants, and a is not zero. The highest power of x is 3 (the cube of x).

The basic graph of $y = x^3$ is shown upper left.

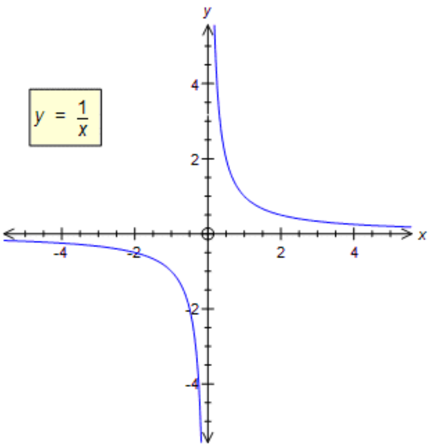


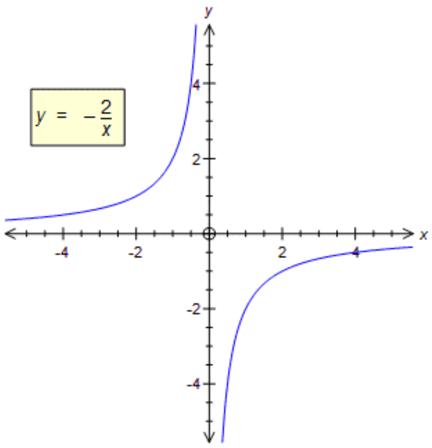
These graphs are characterised by a 'double bend'.

If the term in x^3 is positive, the general slope is upward from lower left, but if the x^3 term is negative, the general slope is downward from upper left.

The 'bend' can also vary in severity - the graph on lower left has sharper 'bends' than the one on lower right.

Reciprocal graphs.





These graphs are shared by functions of the form $y = \frac{k}{x}$, where k is a non-zero constant. They differ from previous examples in that they seem to be in two unconnected parts: if k is positive, the two sections are in the upper right and lower left, but if k is negative, the sections are in the upper left and lower right.

Note the following features of the standard graph $y = \frac{1}{x}$.

As x becomes large and positive, y stays positive but approaches zero.
As x becomes large and negative, y stays negative but approaches zero.

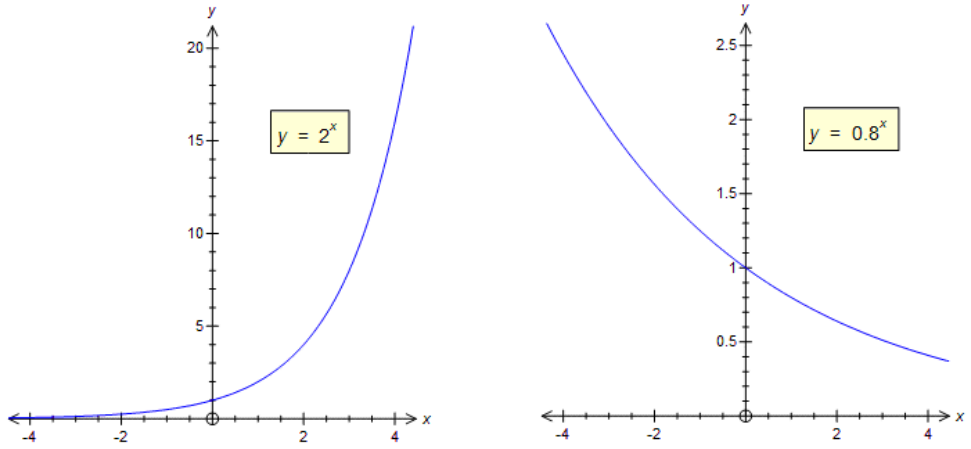
As positive x approaches zero, y becomes increasingly large and positive.
In other words, y **tends to infinity** (∞).

As negative x approaches zero, y becomes increasingly large and negative,
i.e. y tends to minus infinity ($-\infty$).

When $x = 0$, y is undefined - you cannot divide by zero, so it is absurd to say that $\frac{1}{0} = \infty$.

Exponential Graphs.

These are graphs of $y = a^x$ where a is any *positive* number.



Graphs of this form have the following features in common:

Regardless of the value of a, $y = 1$ when $x = 0$.

For $x > 1$:

As x increases, y also increases for all values of x.
As x become large and negative, y tends to zero but never gets there.

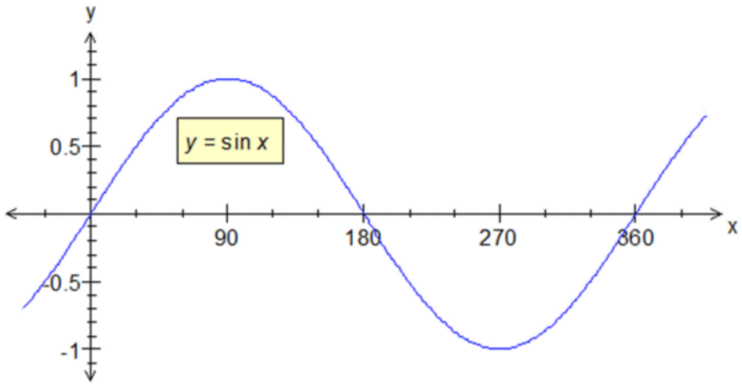
For $0 < x < 1$:

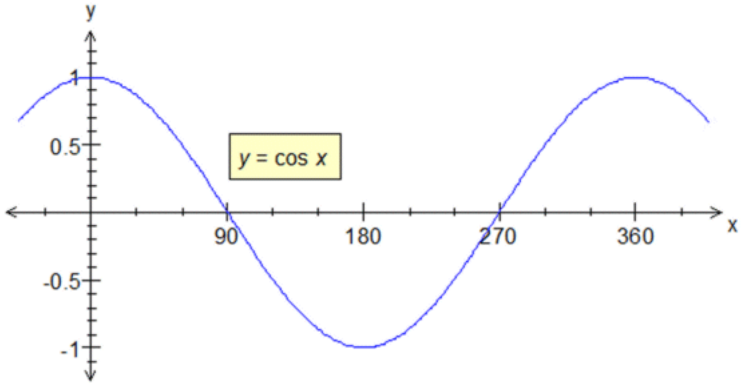
y would still be 1 when $x = 0$.
As x decreases, y also increases for all values of x.
As x becomes large and positive, y tends to zero but never gets there.

(The graph of $y = a^x$ when $a = 1$ is merely the straight line $y = 1$.)

Trigonometric Graphs.

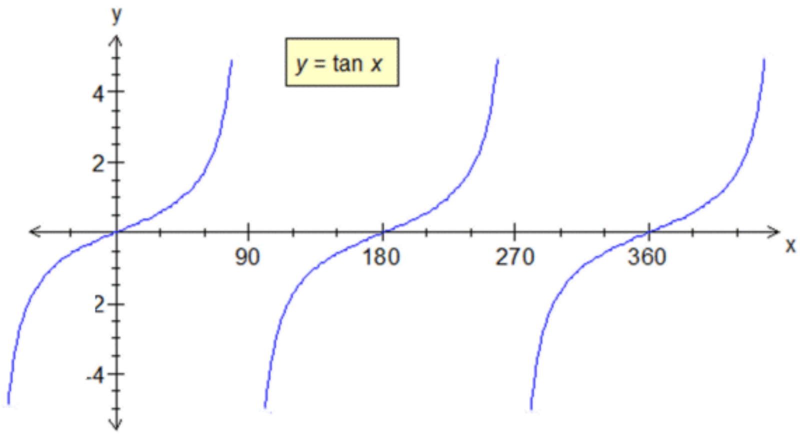
The three main trigonometric functions have the following graphs**:**





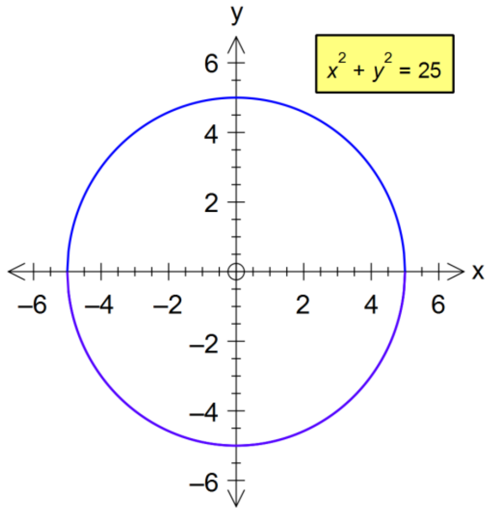
The graphs of sin x° and cos x° are similar to each other; both functions can only take values in the range -1 to +1, and both repeat themselves every 360°. Indeed, the graph of cos x° is the same as that of sin x° translated 90° to the left.

The graph of tan x° is quite different. It repeats every 180°, and moreover the function is undefined for certain values of x, such as 90°, 270°, and all angles consisting of an odd number of right angles. When x approaches 90° from below, tan x° becomes very large and positive; when x approaches 90° from above, tan x° becomes very large and negative.



Circular Graphs.

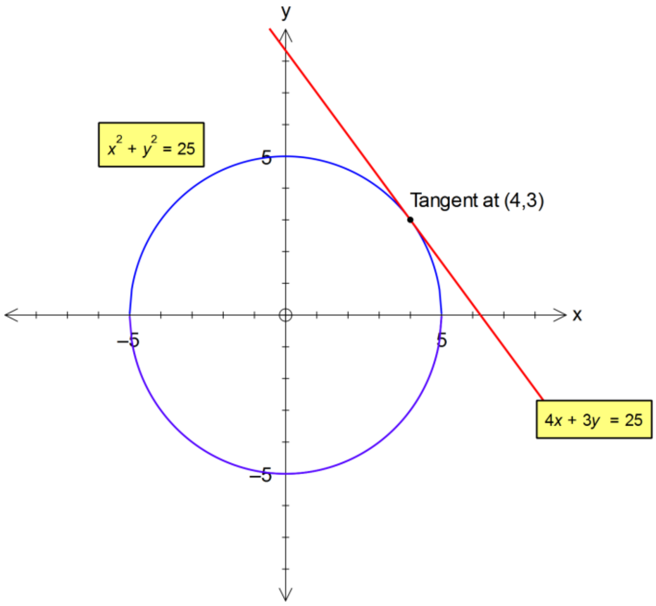
A circle passing through the origin has an equation of $x^2 + y^2 = r^2$, where r is the radius of the circle.



In the example shown on the right, the circle has a radius of 5 units. Remember that the number on the right-hand side of the equation is the *square* of the radius, and not the radius itself.

Equation of the tangent to a circle.

For any point (a,b) on a circle centred on the origin, and with a radius r, the equation of the tangent to the circle at that point is $\boldsymbol{ax + by = r^2}$.



In the above example, the equation of the circle is $x^2 + y^2 = 25$, and the selected point on its circumference is (4, 3).

The equation of the tangent is thus $\mathbf{4x + 3y = 25}$.

Another method:

Gradient of radius from (0, 0) to (4, 3) = $\frac{3}{4}$.
Since the tangent is perpendicular to the radius, its gradient is $-\frac{4}{3}$, as two perpendicular lines have a gradient product of -1.

The equation of the tangent is therefore $y = -\frac{4}{3}x + c$.

Substituting $x = 4$, $y = 3$, we have $-\frac{16}{3} + c = 3$.

Hence $c = \frac{25}{3}$ and the equation of the tangent is $\boldsymbol{y = \frac{25}{3} - \frac{4}{3}x}$.

(This equation is the same as $4x + 3y = 25$, but in $y = mx + c$ form).