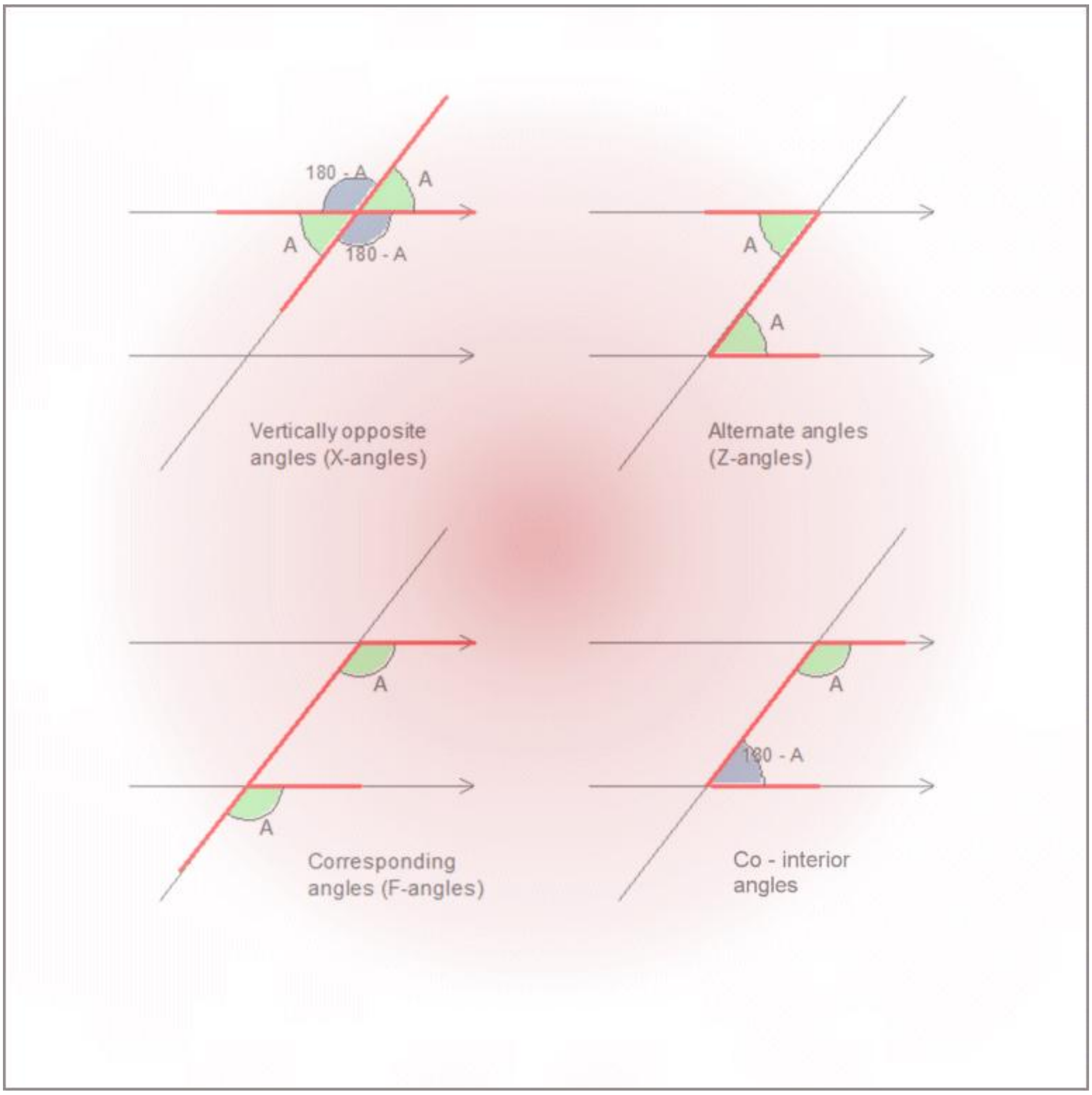
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

SOLVING ANGLE PROBLEMS



Version: 2.2 Date: 21-11-2016

SOLVING ANGLE PROBLEMS.

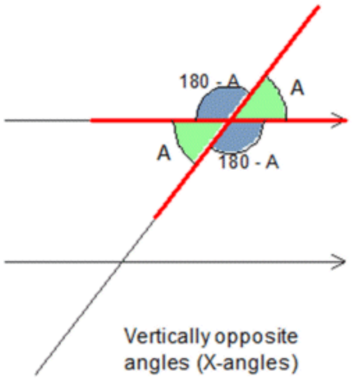
Remember that the angle in a straight line is 180° and that angles around a point sum to 360°.

When two non-parallel lines intersect, we have **vertically opposite** angles.

These are also called X-angles because they occur in the letter X.

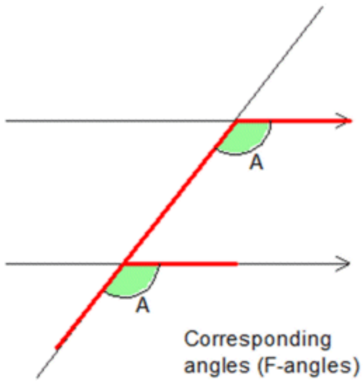
In the diagram, the acute angles marked *A* are vertically opposite, and also equal.

Because there are 180° in a straight line, we have another pair of vertically opposite angles, whose size is (180 – *A*)°.



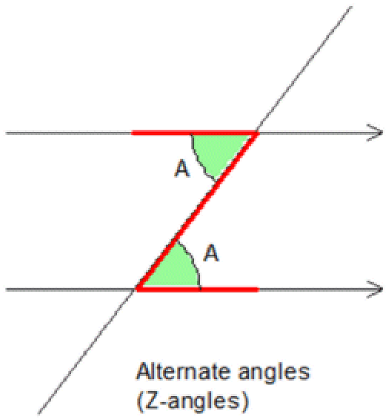
Whenever a third line crosses a pair of parallel lines, we have **corresponding angles,** sometimes called F-angles because they occur in the letter F (which could also be reversed !)

Corresponding angles (the angles *A* form an example here) are equal.



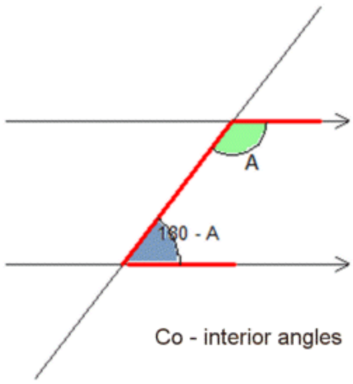
Another relationship is shown by **alternate angles**, also known as Z-angles because they occur in the letter Z (which could be reversed !)

The angles marked *A* form an example of equal and alternate angles.



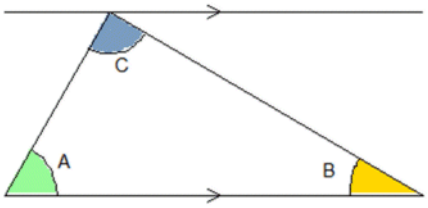
Finally, we have the C- or U-shaped arrangement shown right, or **co-interior angles**.

Co-interior pairs of angles always sum to 180°.

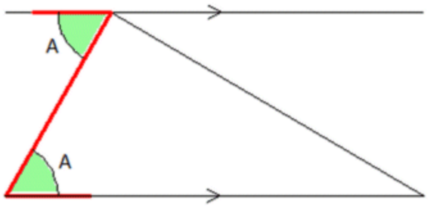


These arrangements are the key to solving many angle problems in plane geometry, especially those involving parallel lines.

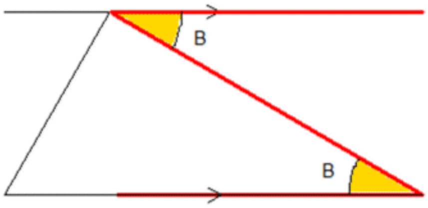
Example (1):. Prove that the angles of a triangle sum up to 180°.



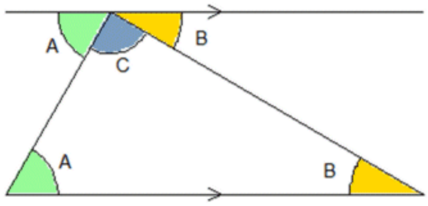
Draw line parallel to base of triangle



Marked angles are equal (alternate)

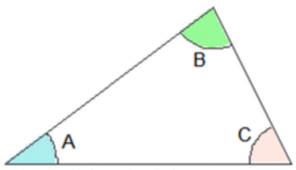


Alternate angles again

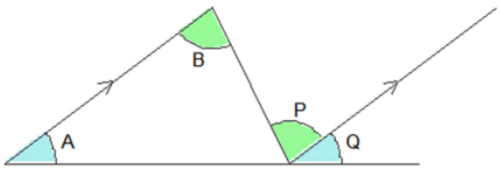


A + B + C = 180° (degrees in a straight line)

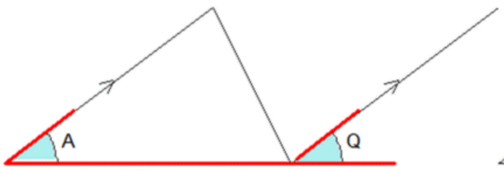
Example(2): Prove that the exterior angle of a triangle is equal to the sum of the interior angles.



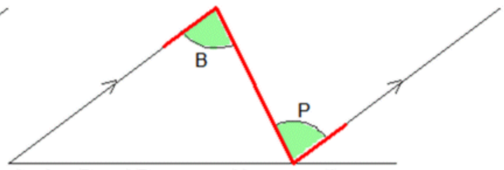
Take a triangle ABC



Draw a line parallel to one side of the triangle and extend the base - two new angles P and Q are formed.



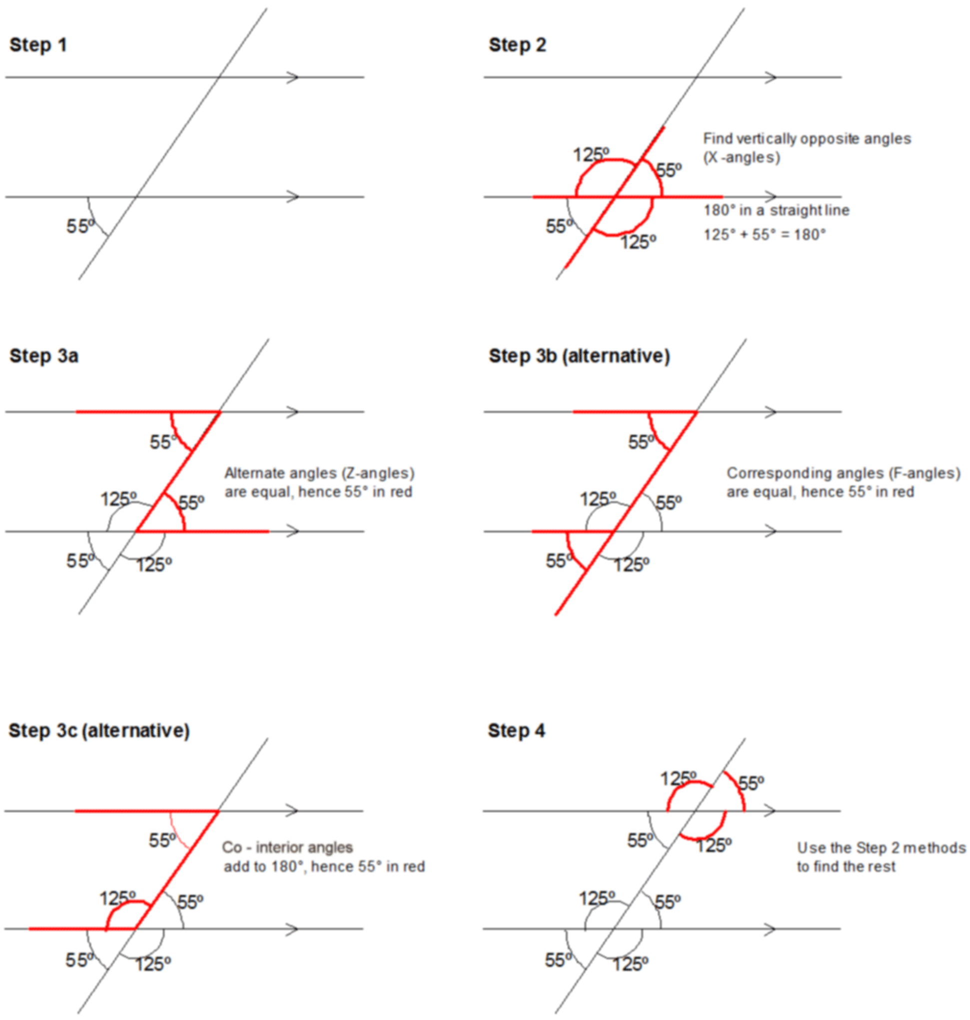
Angles A and Q are equal because they are corresponding angles.



Angles B and P are equal because they are alternate angles.

Angle P + angle Q = Angle A + angle B, therefore the exterior angle of the triangle is equal to the sum of its interior angles.

Example (3): Find all the unknown angles in the "Step 1" diagram. The arrowed lines are parallel. This diagram is not drawn accurately.



The original diagram in Step 1 only gives us an angle of 55° around one intersection.

In Step 2, we use the fact that vertically opposite angles (X-angles) are equal, and also the fact that there are 180° in a straight line, to determine all the angles around that point. Note: 125° + 55° = 180°.

In Step 3, we can choose various strategies to determine the angles around the other intersection. We can use the equality of alternate angles as in 3a, the equality of corresponding angles (F-angles) as in 3b, or co-interior angles summing to 180° as in 3c.

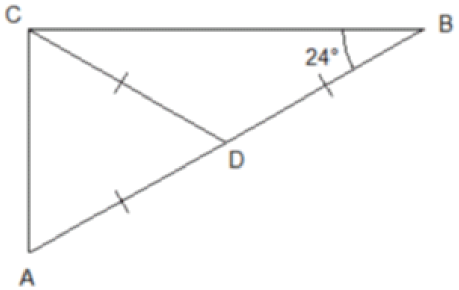
In Step 4, Having found one angle in Step 3, we can use vertically opposite angles and 180° in a straight line to find the other three. Note as well how all four angle around the upper intersection correspond to all four angles around the lower one.

Example (4):

In the triangle on the right (not drawn accurately) , $\angle$ *CBD* = 24°.

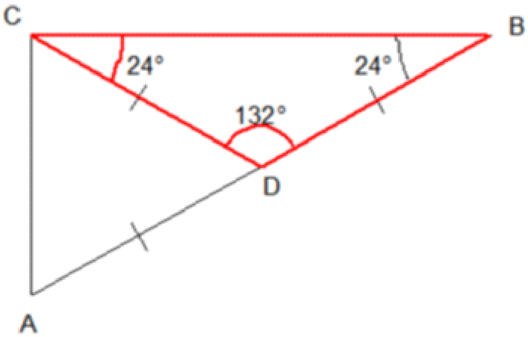
Lengths *AD*, *BD* and *CD* are also all equal.

Find $\angle$ *ACD*, and hence show that $\angle$ *ACB* is a right angle.



Since *CD* = *DB*, Δ*CDB* must be isosceles, and so $\angle$ *DCB* = 24°.

Therefore $\angle$ *CDB* = 180° – (2 × 24)° = 132°

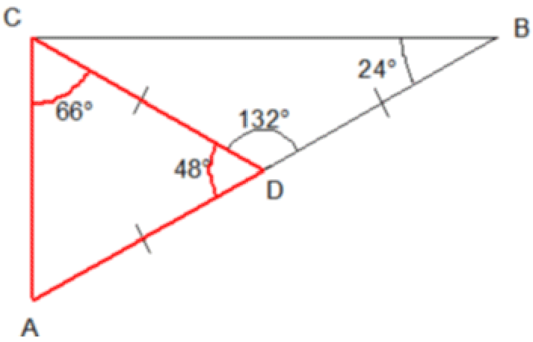


Having found $\angle$ *CDB*, we know that $\angle$ *ADC* = (180 – 132)° = 48°.
(180° in a straight line !)

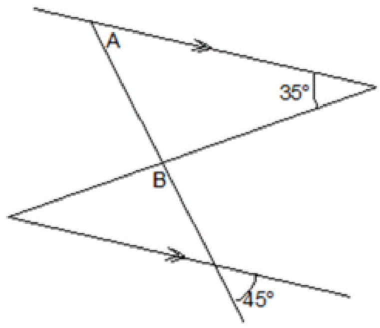
Since *CD* = *AD*, Δ*ACD* must be also be isosceles, and so $\angle$ *CAD* and $\angle$ *ACD* must also be equal.

Thus $\angle$ *ACD* = ½(180-48)° = 66°.

Finally, $\angle$ *ACB* = $\angle$ *ACD* + $\angle$ *DCB*
= 66° + 24° = 90°.

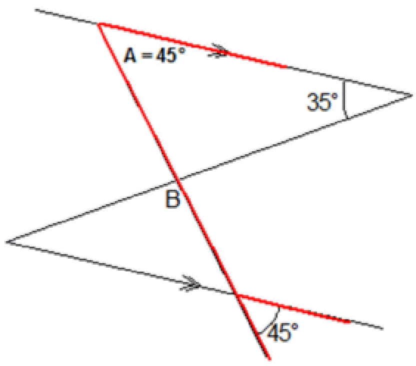


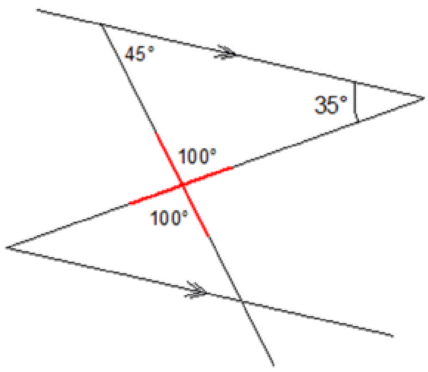
Example (5a): Find angles A and B in the diagram below, which has not been drawn accurately.



We can find angle A by noticing that it and the 45° angle are corresponding angles.

Thus A = 45° (corresponding angles are equal). (see lower left)

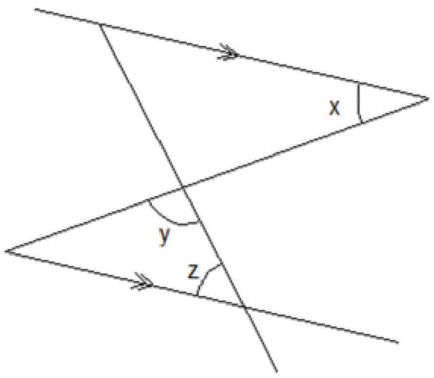


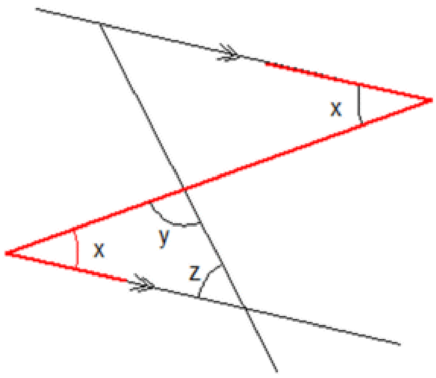


To find angle B, we can subtract 45° + 35° from 180° to find the last angle of the upper triangle, i.e. 100°.

Angle B is vertically opposite, so it is also equal to 100°. (see lower right).

Example (5b): Prove that $x + y + z = 180°$ in the diagram below.



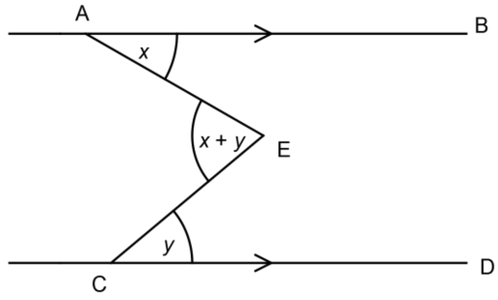


The proof is easily found using alternate angles (right).

We now have the lower triangle complete, so $x + y + z = 180°$ (angle sum of triangle).

Example (5c): Lines *AB* and *CD* are parallel, and point *E* lies between the lines.
Let angle *BAE* = *x* and angle *DCE* = *y*.

Prove that angle *AEC* = *x* + *y*.

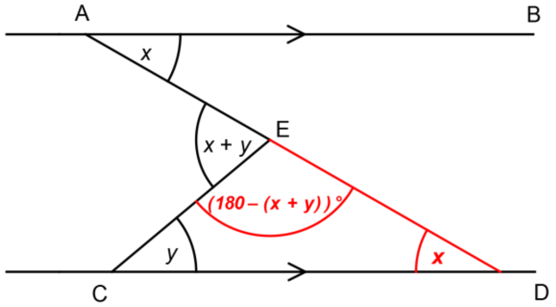


Proof using angle sum of triangle.

Extend line *AE* until it meets line *CD* at point *D*.

By alternate angles, ∠ *CDE* = *x* .
Hence ∠ *CED* = (180- (*x* + *y*))° and
∠ *AEC* = (180-(180-(*x* + *y*)))°
= *x* + *y* as requested.

Also. the exterior angle *AEC* is the sum of the interior angles *ECD* and *EDC.*



Proof using alternate angles only.

Draw a line parallel to *AB* and *CD*, passing through point *E*, and thus dividing angle *AEC* into two parts, angles *AEP* and *CEP*.

By alternate angles, ∠ *AEP* = ∠ *BAE* = *x*.
Also, ∠ *CEP* = ∠ *DCE* = *y*.

Hence ∠ *AEC* = ∠ *AEP* + ∠ *CEP* = *x* + *y*.
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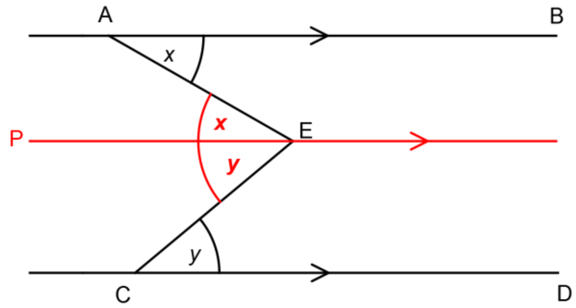
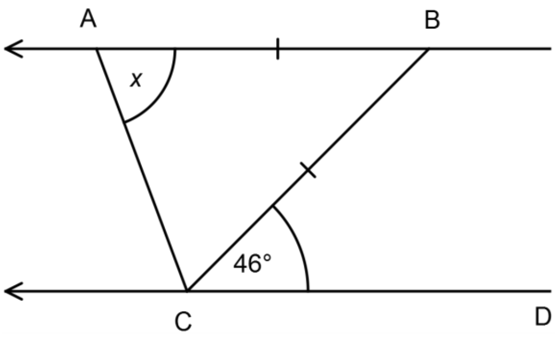
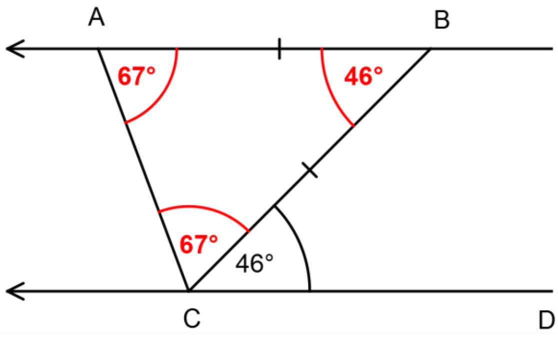


Example (6): Find the size of angle *CAB*, labelled *x* in the diagram (not drawn accurately).





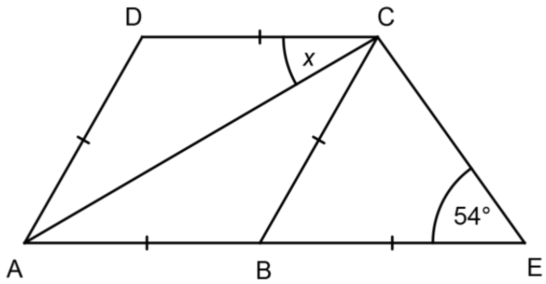
Angles *BCD* and *ABC* form a pair of alternate angles, so $\angle ABC = \angle BCD = 46°$.
Also, *AB* = *BC*, so Δ *ABC* is isosceles, and $\angle ACB$ and $\angle CAB$ must also be equal.

Thus $\angle CAB = \frac{1}{2}(180-46)° = 67°$.

The following examples assume knowledge of the properties of quadrilaterals.

Example (7): Figure *ABCD* is a rhombus, the triangle *BEC* is isosceles, and *ABC* is a straight line.

Work out the size of angle *DCA*, labelled *x*.
(Diagram not drawn accurately)

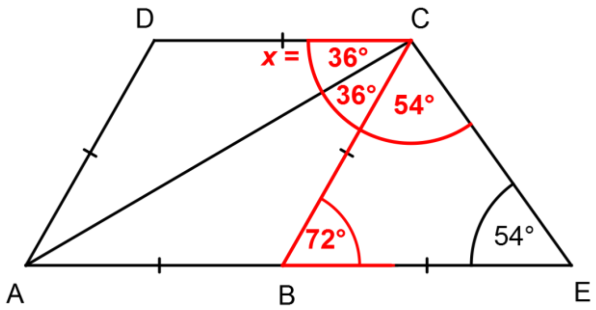


Because the triangle *BEC* is isosceles with *BC* and *BE* as the two equal sides, ∠ *BCE* = 54° and the third angle, ∠ *CBE* = 72°.

Next, we deduce that ∠ *DCB* = ∠ *CBE* = 72°, using alternate angles.

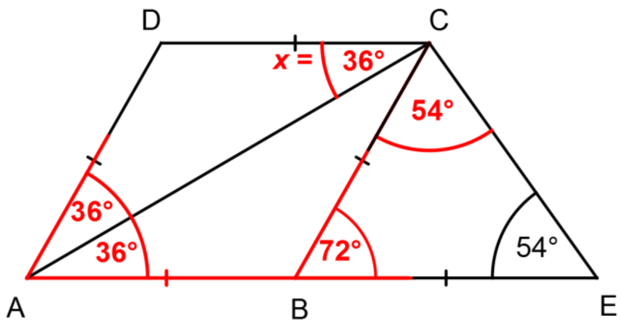
The diagonals of a rhombus are also its lines of symmetry, therefore they bisect its angles.

Hence angle *DCA*, labelled *x*, is half of 72° or **36°**.



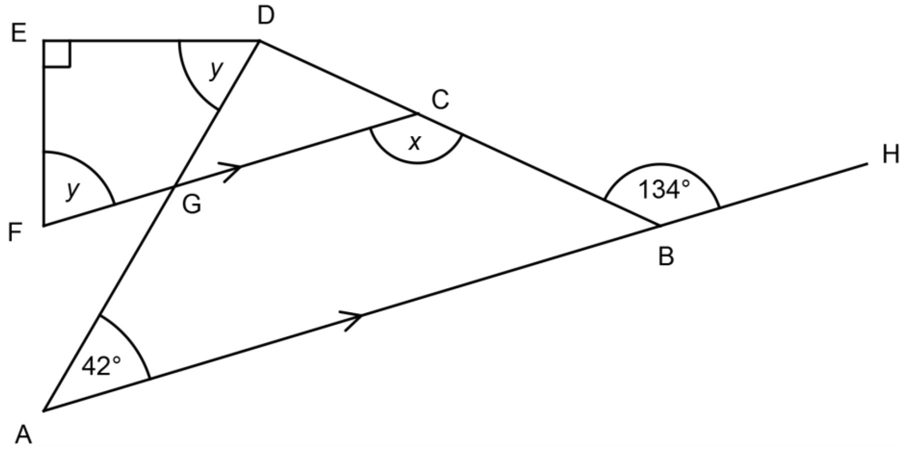
Alternatively, we could have deduced that ∠ *DAB* = ∠ *CBE* = 72°, using corresponding angles and then continued with ∠ *DCB* = ∠ *DAB* = 72° because opposite angles of a rhombus are equal.

Many angle problems can be solved in such a variety of ways, all equally valid. Just don't waste time on unnecessary detours !



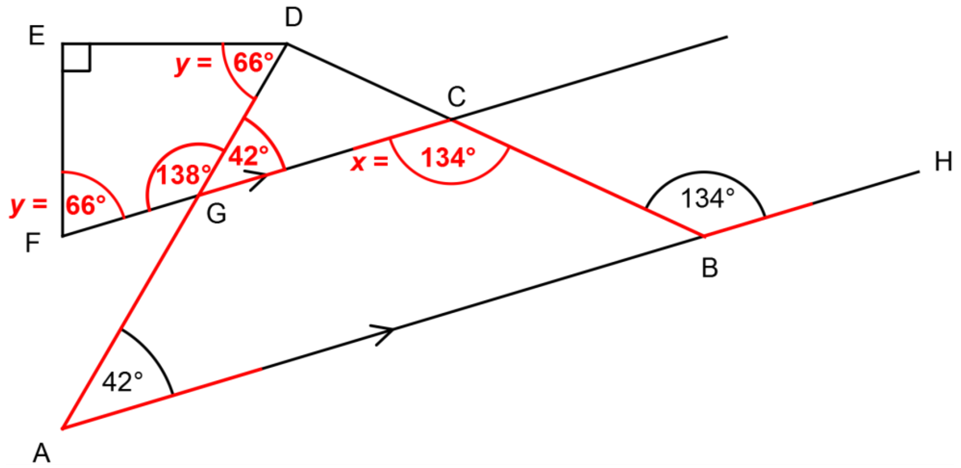
Example (8): Lines *ABH* and *FGC* are parallel and angle *DEF* is a right angle.
The angles *GFE* and *GDE*, labelled *y*, are equal. (Diagram not drawn accurately).

i) State the size of angle *x*, giving a reason for your answer.
ii) Work out the size of angle *y*, showing clear working and reasons.



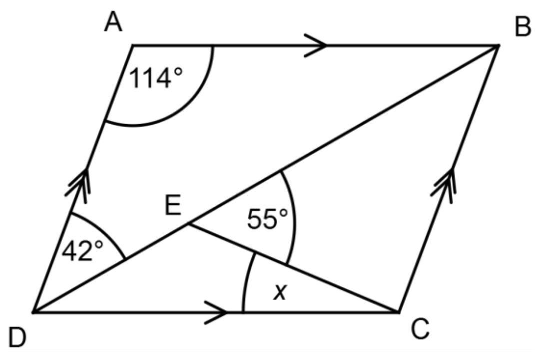
i) Angle *DCB* (marked *x*) = **134°** because it and angle *CBH* are alternate angles and therefore equal.

ii) By corresponding angles, ∠ *DGC* = ∠ *DAB* = 42°, so ∠ *FGD* = (180 – 42)° = 138°.
We now have two angles of the quadrilateral *DEFG*, so the two angles labelled *y* must sum to (360 – (138 + 90))° or 132°. Hence each angle labelled *y* is half of 132°, or **66°**.



Example (9): Figure *ABCD* is a parallelogram, where ∠ *ADB* = 42°, ∠ *CEB* = 55° and ∠ *DAB* = 114°.

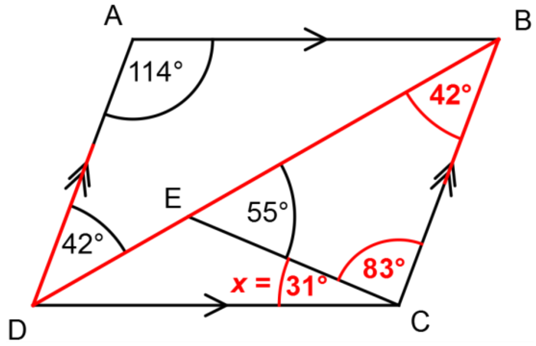
Calculate the size of angle ∠ *DCE*, labelled *x* in the diagram (not accurately drawn).



Because *ABCD* is a parallelogram, sides *AD* and *BC* are parallel, so angles *ADB* and *DBC* form a pair of alternate angles, each of size 42°.

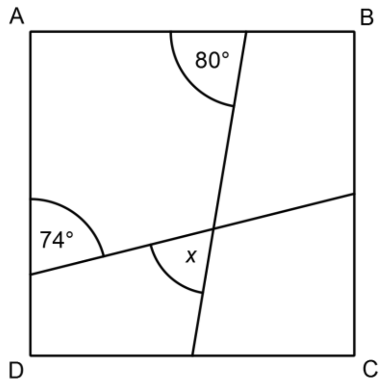
We now have two angles of triangle *EBC*, so the missing angle *ECB* = (180 – (55 + 42))° = 83°.
(Interior angle-sum of a triangle = 180°).

Because opposite angles of a parallelogram are equal, ∠ *DCB* = ∠ *DAB* = 114°.
Hence the missing angle *DCE* (labelled *x*) = ∠ *DCB* - ∠ *ECB* = 114° - 83° = **31°**.



Example (10): *ABCD* is a square.

Work out the size of angle *x*, showing all working.
(Not drawn accurately)



We cannot use corresponding angles to say that ∠ *HJG* = *x* = 80° because the line *HF* is not parallel to line *AB.*

Neither can we use alternate angles to say that ∠ *HJG* = *x* = 74° since lines *EG* and *AD* are not parallel.

Because *ABCD* is a square, ∠ *DAB* = 90°. Also, the angles of a quadrilateral add to 360°, so ∠ *HJE* = (360 – (90 + 80 + 74))° or 116 °.

Finally, ∠ *HJE* = *x* = (180-116)° = **64°** because there are 180° in a straight line.