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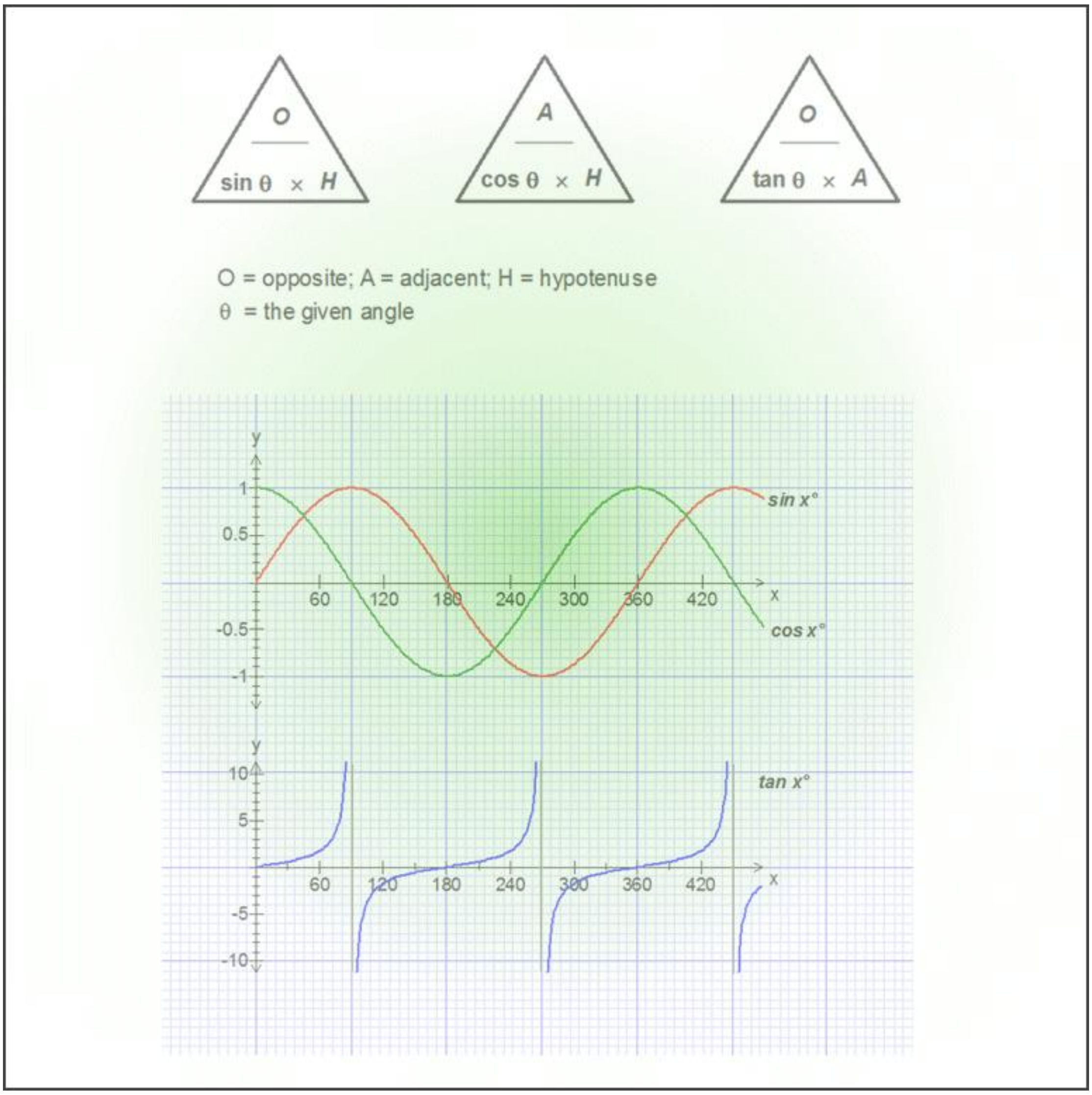
# M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

# TRIGONOMETRIC RATIOS – SINE, COSINE AND TANGENT

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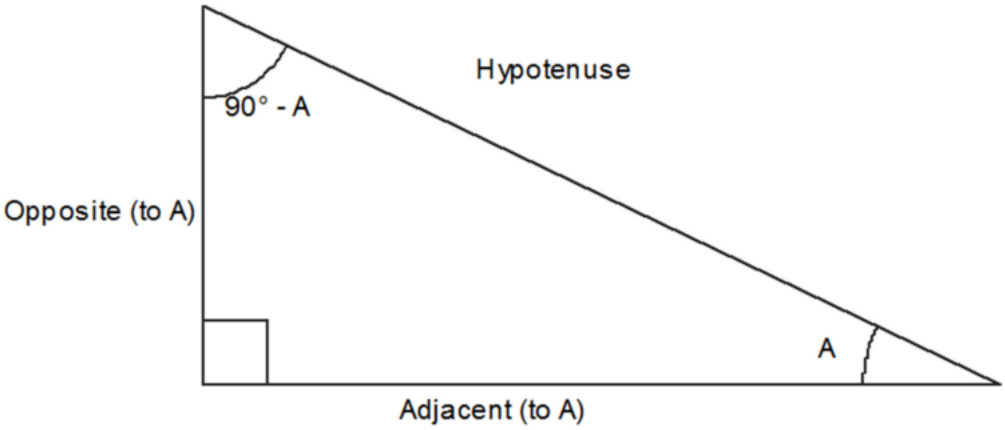
## TRIGONOMETRIC RATIOS.

The general right-angled triangle looks like this:




The hypotenuse is the longest side of the triangle, and is the side opposite the right angle.

Using angle A as the reference, the other two sides of the triangle are the 'opposite' which is opposite A, and the 'adjacent' which runs between A and the right angle.

The three sides are all related by the following ratios; the **sine**, **cosine** and **tangent** of A. They are abbreviated to **sin**, **cos** and **tan**.

The sine of angle A (sin A) is the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$

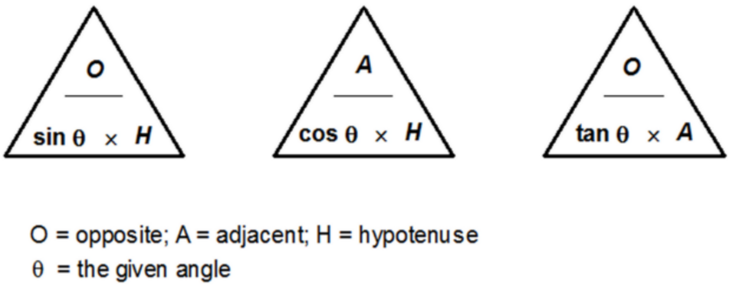
The cosine of angle A (cos A) is the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$

The tangent of angle A (tan A) is the ratio $\frac{\text{opposite}}{\text{adjacent}}$

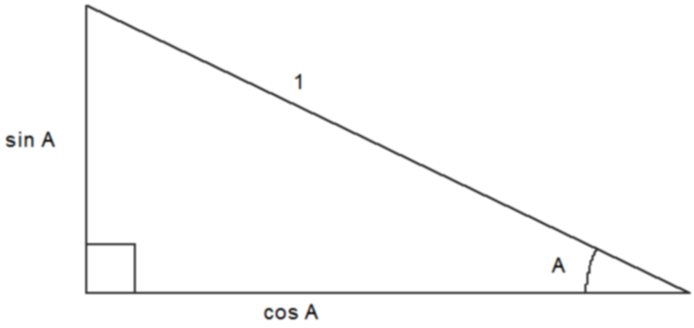
Also, $\frac{\text{opposite}}{\text{hypotenuse}} \div \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{opposite}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\text{opposite}}{\text{adjacent}}$

therefore tan A = $\frac{\sin A}{\cos A}$

Recall the SOHCAHTOA formula triangles:



**An important identity.**



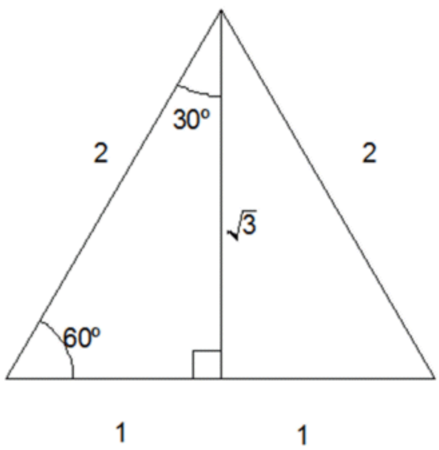

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares on the other two sides, therefore

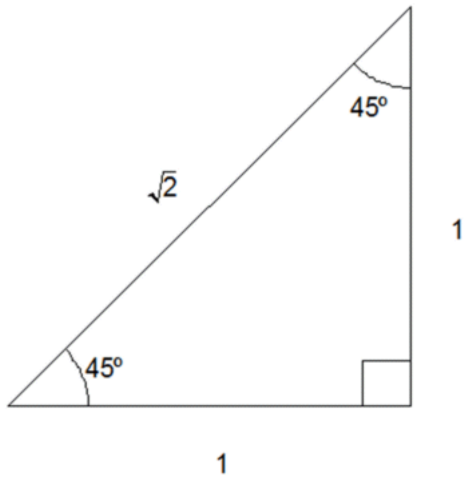
$\mathbf{\cos^2 A + \sin^2 A = 1.}$ (This holds true for all angles **A**).

**Trigonometric ratios of special angles.**

The trigonometric ratios of certain angles can be deduced by using Pythagoras' theorem.
Notice how the equilateral triangle has been split into two congruent right-angled triangles, each with acute angles of 60° and 30°.

You will find both of these right-angled triangles in a standard geometry set. They are also known as set-squares.




$\sin 30° = \cos 60° = \frac{1}{2}$

$\sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan 60° = \sqrt{3}$

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

$\sin 90° = \cos 0° = 1$

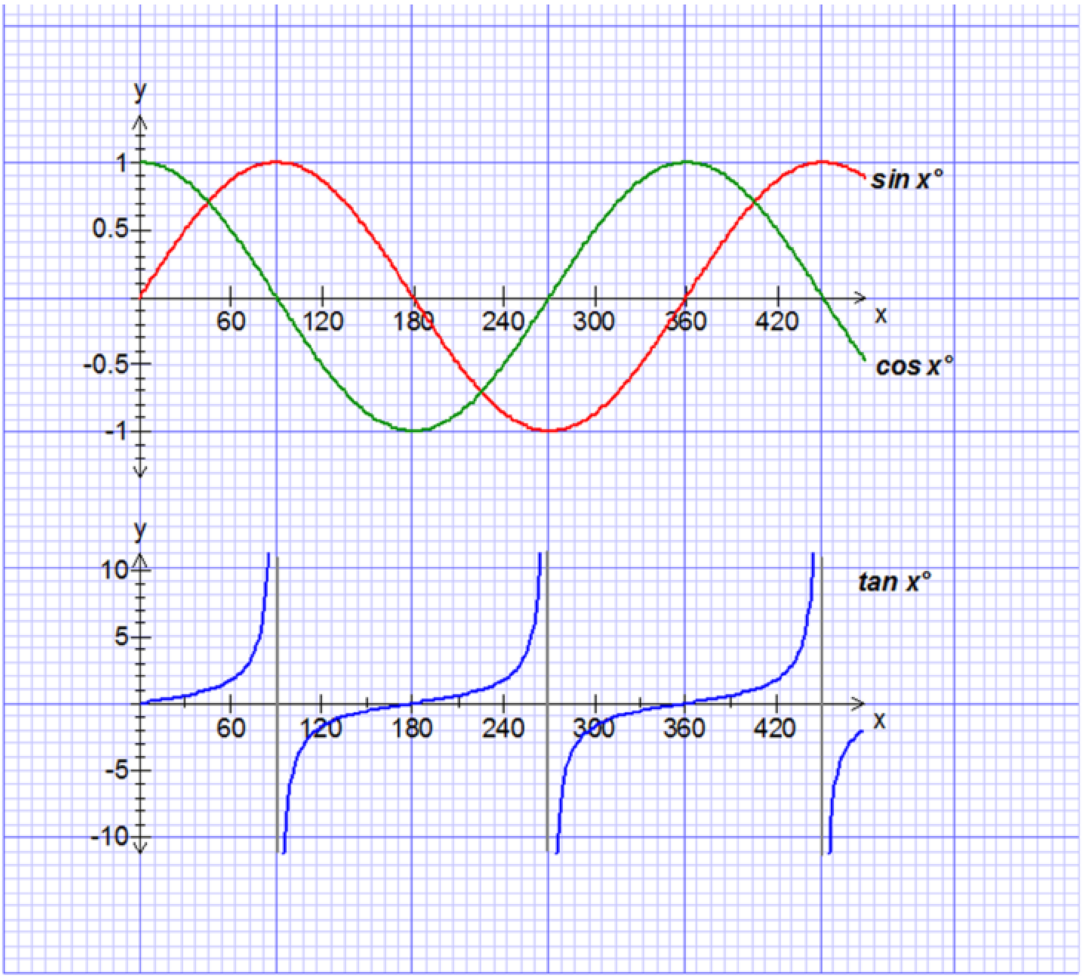
$\cos 90° = \sin 0° = 0$

$\tan 0° = 0$

Generally, we shall only be dealing with the trig ratios of acute angles at GCSE level, but we must be able to recognise the shapes and symmetries of their graphs for all angles.

**Trigonometric Graphs.**

The three main trigonometric functions have the following graphs**:**

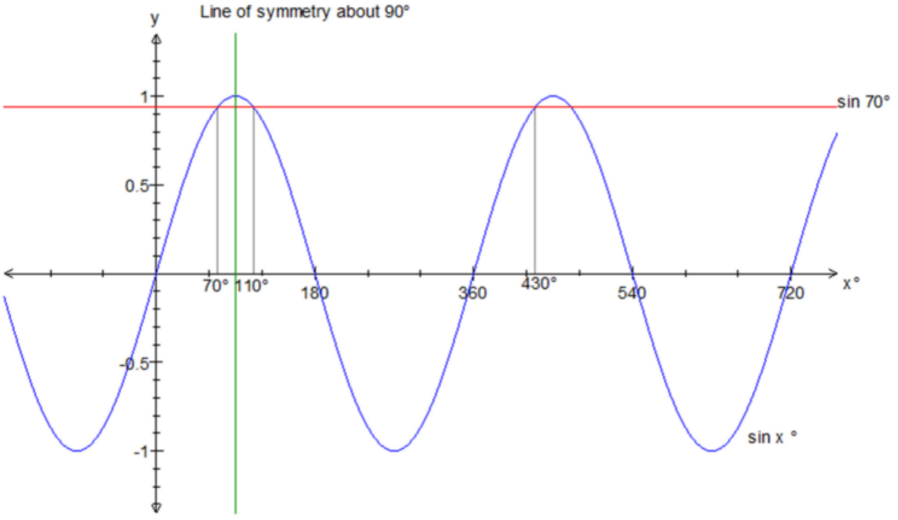



The graphs of sin *x*° and cos *x*° are similar to each other; in fact they are shown together for comparison. Both functions can only take values in the range -1 to +1, and both repeat themselves every 360°. Indeed, the graph of cos *x*° is the same as that of sin *x*° shifted 90° to the left.

The graph of tan *x*° is quite different. It repeats every 180°, and moreover the function is undefined for certain values of *x*, such as 90°, 270°, and all angles consisting of an odd number of right angles. When *x* approaches 90° from below, tan *x*° becomes very large and positive; when *x* approaches 90° from above, tan *x*° becomes very large and negative. The tangent graph therefore has discontinuities at 90°, 270°, and all angles 90° + 180*n*° where *n* is an integer.

Looking at the graphs, it can be seen that infinitely many angles can share the same values for their sine, cosine or tangent.

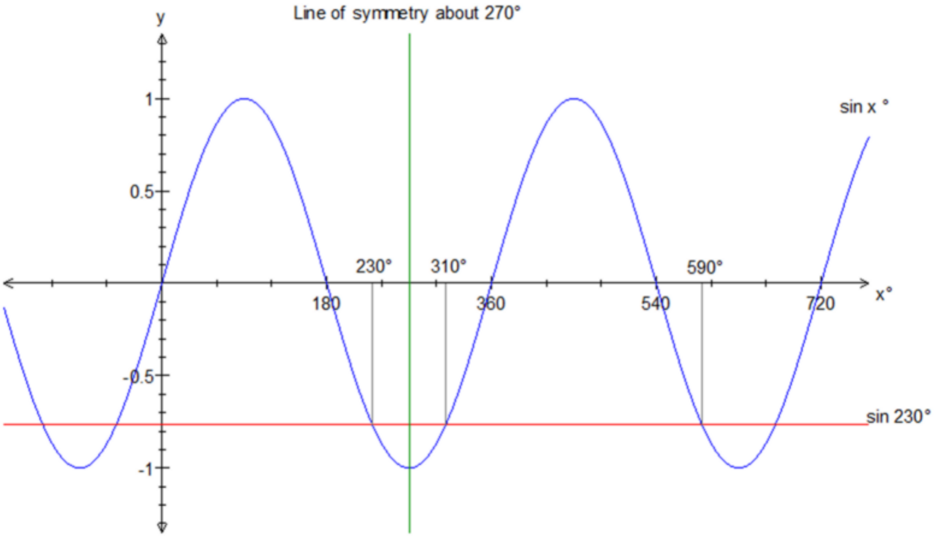
**Example (1):** Find two other angles $x$° where sin $x$° = sin 70°; ii) sin 230°.




The graph of the sine function has a line of symmetry at $x$ = 90° and a repeating period of 360°.
Therefore, *one* other angle having the same sine as 70° is (360 + 70)° or 430°.
We could have added 360° again to obtain 790°, or subtracted 360° to obtain -290°.

We also use the symmetry of the graph of sin $x$ to deduce that sin 110° is the same as sin 70°. This is because 110° is the same distance from 90° as 70°, but on opposite sides of the line of symmetry.
Again, we could keep adding or subtracting multiples of 360° from the 110°, to get 470° and so forth.

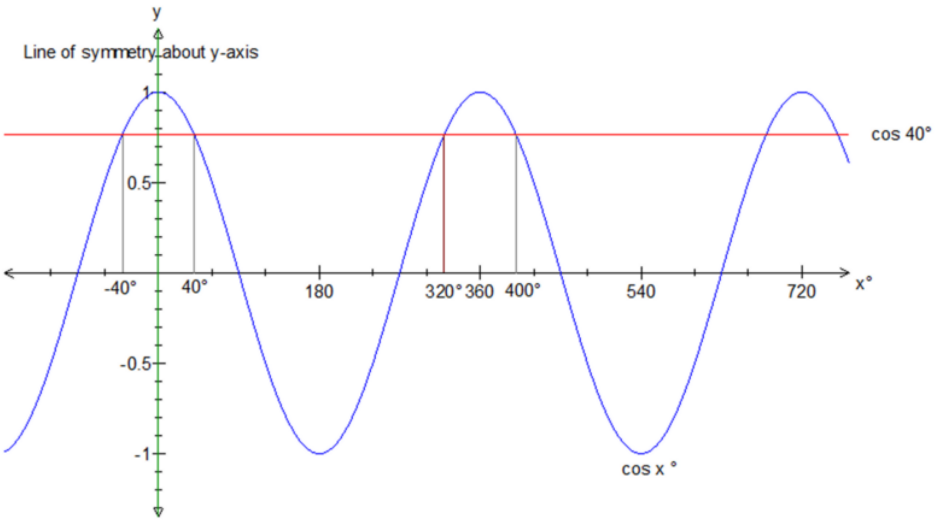
∴ two other angles $x$° where sin $x$° = sin 70° are 110° and 430°.




The graph of the sine function also has a line of symmetry at $x$ = 270°.
As in i), *one* other angle having the same sine as 230° is (360 + 230)° or 590°.

Using the symmetry of the graph of sin $x$ , we can deduce that sin 310° is the same as sin 230°, because both angles are equidistant from 270°.
∴ two other angles $x$° where sin $x$° = sin 230° are 310° and 590°.

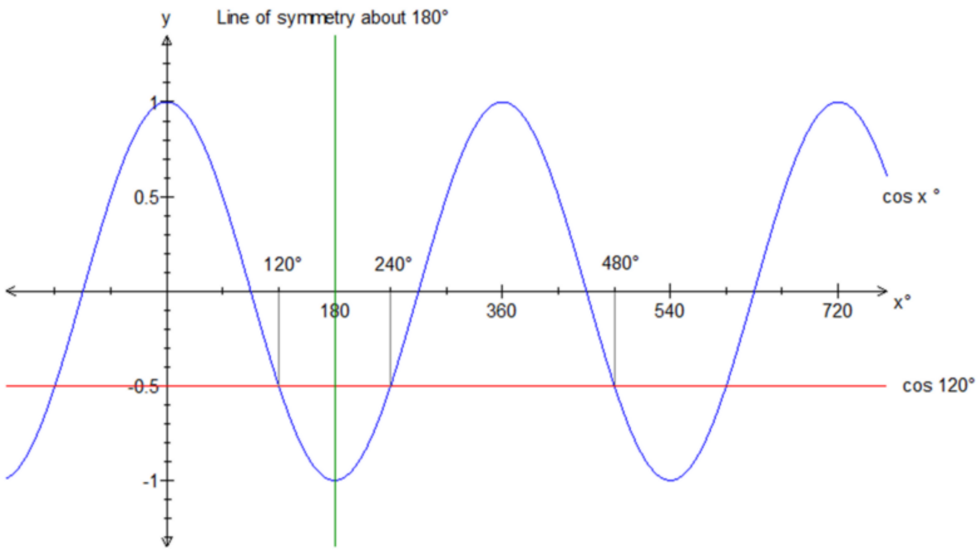
**Example (2):** Find two other positive angles $x$° where cos $x$° = i) cos 40°; cos 120°.




The graph of the cosine function has a line of symmetry about the $y$-axis and a period of 360°.
Therefore, *one* other angle having the same cosine as 70° is (360 + 40)° or 400°.
We could have added 360° again to obtain 760°, or subtracted 360° to obtain -320°.

Also, by symmetry, cos( -40°) is the same as cos 40°, because 40° and -40° are equidistant from the $y$-axis. The question asked for a positive angle, so we must add 360° to -40° to get 320°.

∴ two other positive angles $x$° where cos $x$° = cos 40° are 400° and 320°. (Note that $x$ = 360° is also a line of symmetry.)




Here we note the line of symmetry at $x$ = 180° as well as the period of 360°..
Therefore, *one* other angle having the same cosine as 120° is (360 + 120)° or 480°.
We could have added 360° again to obtain 760°, or subtracted 360° to obtain -320°.

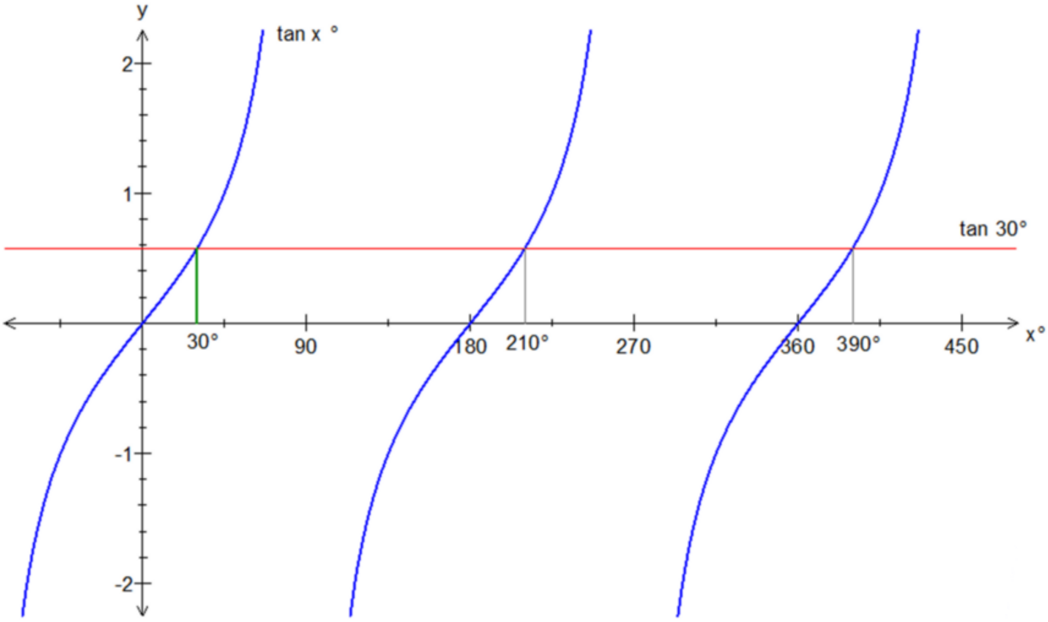
By symmetry, cos 240° = cos 120°, because 120° and 240° are equidistant from the line $x$ = 180°.
∴ two other positive angles $x$° where cos $x$° = cos 120° are 240° and 480°.

The symmetrical pairing of solutions is common to both the sine and cosine graphs, for all angles on either side of the graphs' lines of symmetry.

Thus sin 115° = sin 65° (both equidistant from line of symmetry at 90°), and cos 155° = cos 215° (both equidistant from line of symmetry at 180°).

**Example (3):** Find two other positive angles *x*° where tan *x*° = tan 30°.

The graph of tan *x*° is different from the other two inasmuch as it has a period of 180° and not 360°, as well as having no lines of symmetry.




We merely need to add or subtract multiples of 180° to find additional angles whose tangents are equal to tan 30°. Two others are (180 + 30)° or 210°, and (360 + 30)°, or 390°.

∴ two other positive angles *x*° where tan *x*° = tan 30° are 210° and 390°.