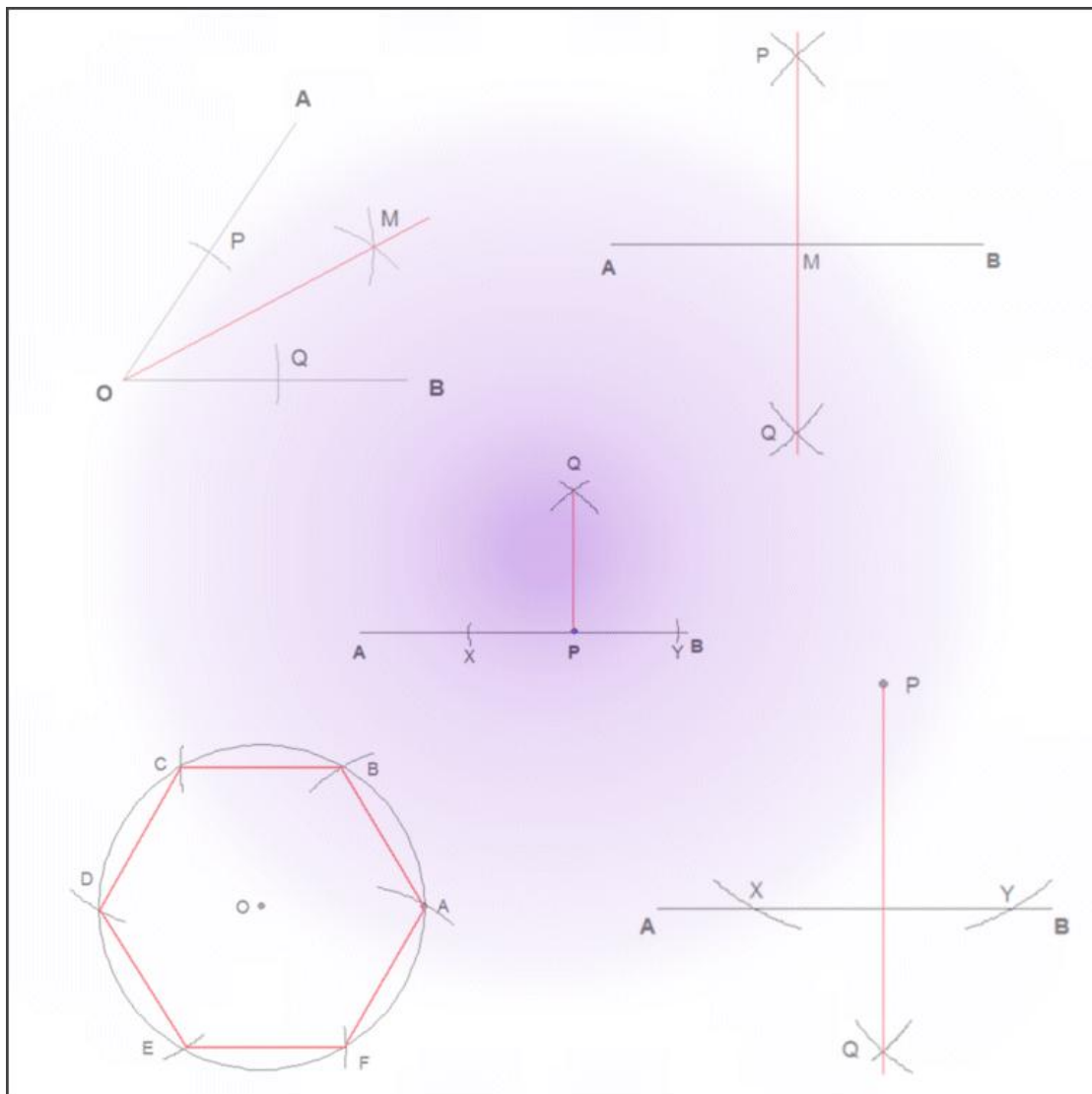


M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

MEASURES AND CONSTRUCTIONS



MEASURES AND CONSTRUCTIONS

Ruler and compass constructions.

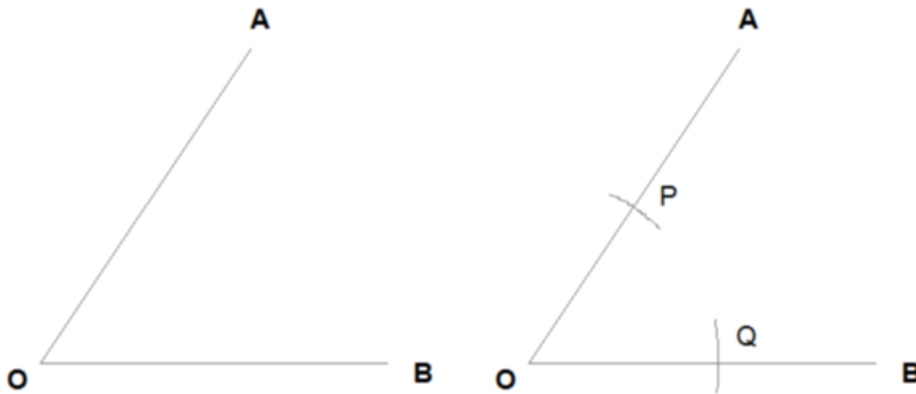
Many geometrical constructions can be carried out using a ruler and a compass, of which the following are the most important and likely to come up in exams.

Bisecting an angle.

(“Bisect” means to divide into two equal parts).

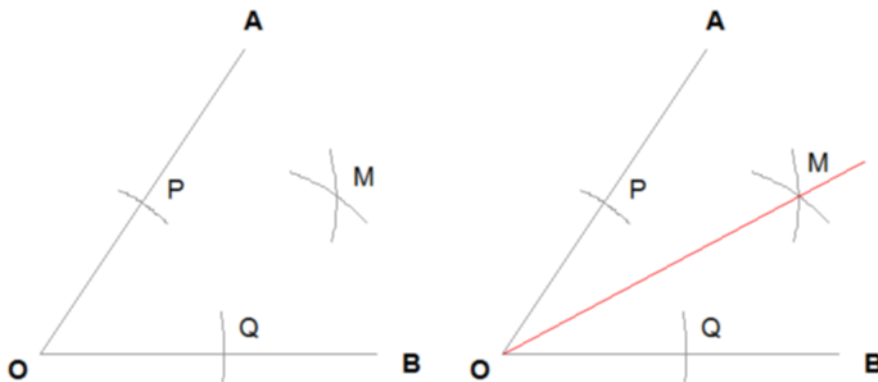
Take the angle AOB, below left.

First, centre a compass at O and draw two arcs to cut OA at P and OB at Q, below right.



Next, centre the compass at P *without altering the opening* and draw another arc inside the angle. Repeat, centring the compass at Q, until there are two arcs crossing at point M, below left.

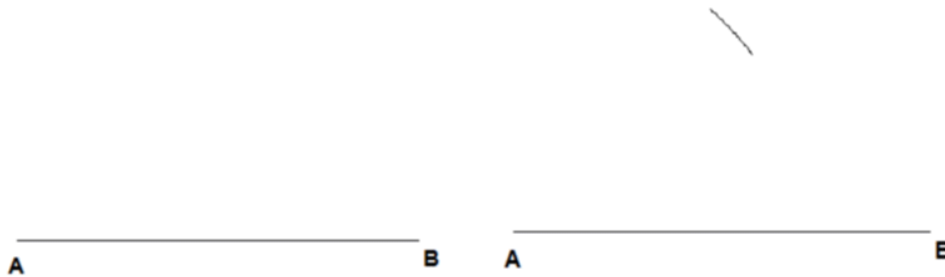
The line OM (below right) is the **bisector** of the angle AOB; angles AOM and BOM are equal.



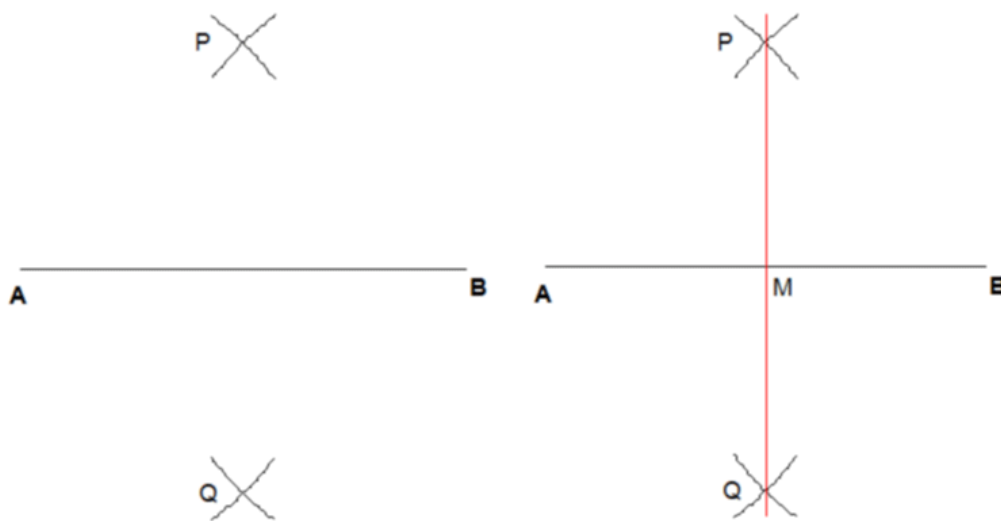
Bisecting a line.

Take a line AB, below left.

Open a compass to about three-quarters of the length of AB, centre it on A, and draw two arcs above and below the centre of the line (below right).



Next, without altering the compass opening, draw another two arcs centred on B, (lower left) and connect the resulting points P and Q .



The line PQ is the **perpendicular bisector** of the line AB; lengths AM and MB are equal.
(Not only does PQ bisect AB, but it also does so at right angles.)

Constructing a perpendicular (an accurate 90° angle) from a point on a given line.

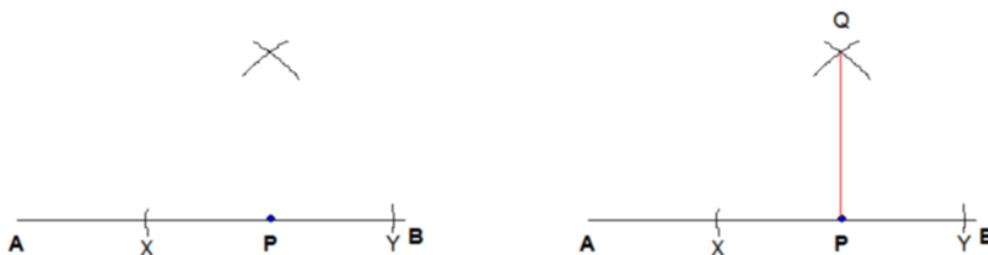
Here we wish to draw a line perpendicular to the given line AB, passing through the point P. The first step is to draw two compass arcs centred on P, cutting the line AB (below right). If the point P is at or near the end of AB, we must extend AB and continue as above.



Label the two points of intersection X and Y.

Open the compass to about three-quarters the distance X-Y and draw an arc centred on X, passing above point P. Without altering the compass opening, draw a similar arc centred on Y.

The two arcs will meet at Q (below right), and joining points P and Q completes the construction.



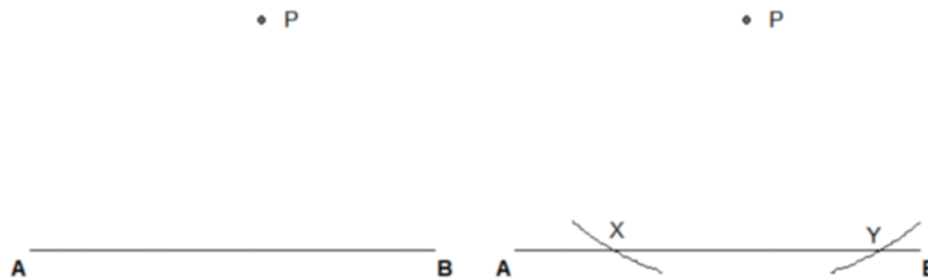
The perpendicular distance PQ is also the shortest distance between point Q and the line AB.

In general, the minimum distance from a point P to a straight line AB is the length of the perpendicular from P to AB.

Constructing a perpendicular from a point to a given line.

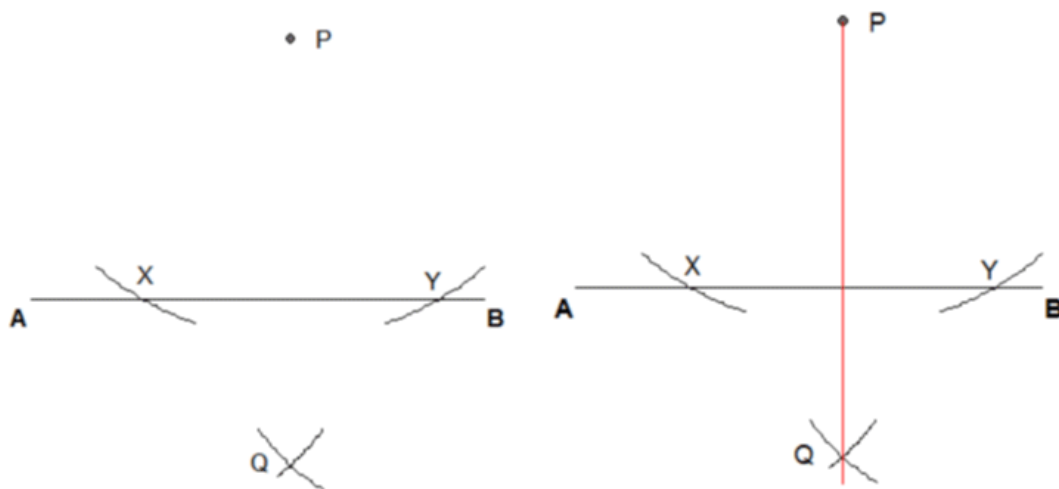
This construction is slightly different from the last one, as we are given a point *away* from the line, and need to construct the perpendicular *to* the line.

Firstly, open out the compass sufficiently to draw two arcs from P, cutting the line AB in two places. Label these points X and Y (lower right)



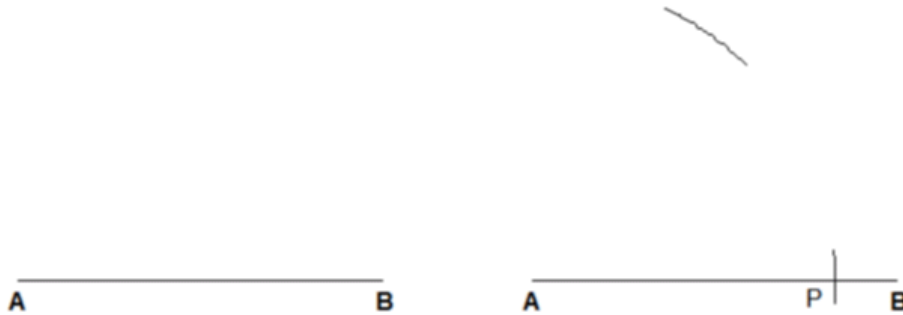
Draw two arcs (of equal radius !) from points X and Y, to intersect at Q, on the side of the line AB opposite to the point P.

Finally, join points P and Q to complete the perpendicular (lower right).



Constructing an accurate 60° angle.

Begin with a baseline AB and draw two arcs from point A; one to cut AB at point P and the other one to pass above the midpoint of AB.

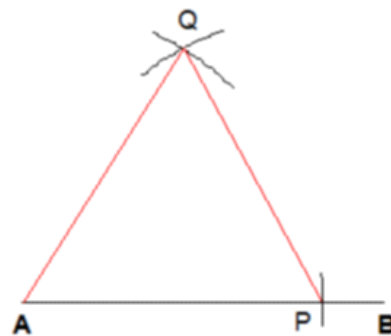


Next, without altering the compass opening, draw an arc centred at P, and intersecting with the arc above AB. Label this point of intersection Q.

Finally connect point Q to point A. The angle QAP is 60° .



Incidentally, joining points P and Q produces another 60° angle at APQ, and therefore the triangle AQP is equilateral.

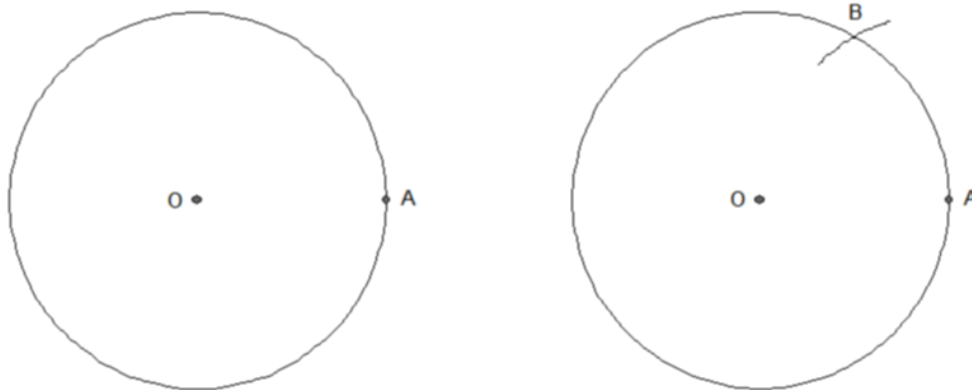


Diversion – constructing a regular hexagon inscribed in a circle.

The construction of an equilateral triangle in the last example can be adapted to construct a regular hexagon inside a circle – very neat !

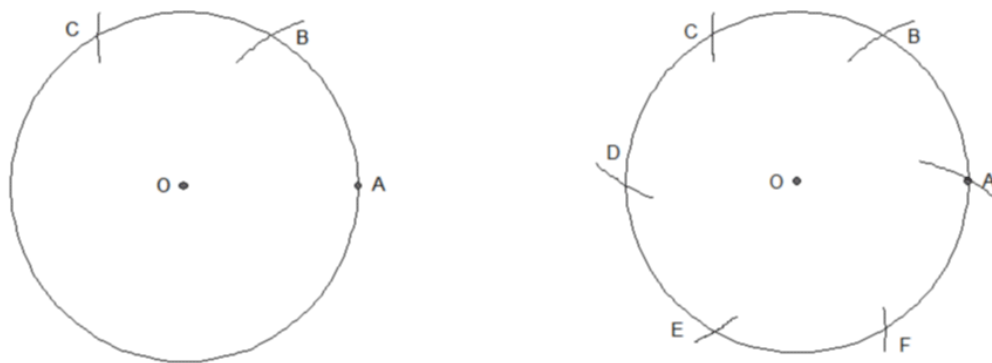
Begin with a circle centred at point O and mark a point A on its circumference (left).
Ensure that the compass radius remains the same throughout the following steps:

Centre the compass at A and draw an arc cutting the circle at point B (right).



Next, centre the compass at B and draw another arc to cut the circle at point C (left).

Continue the process until six arcs have been drawn, with the sixth one meeting the circle at point A. (right).



Finally, join points A to F to form a regular hexagon.

We can check the accuracy of the construction by realising that lines AOD, BOE and COF are diameters of the circle.

