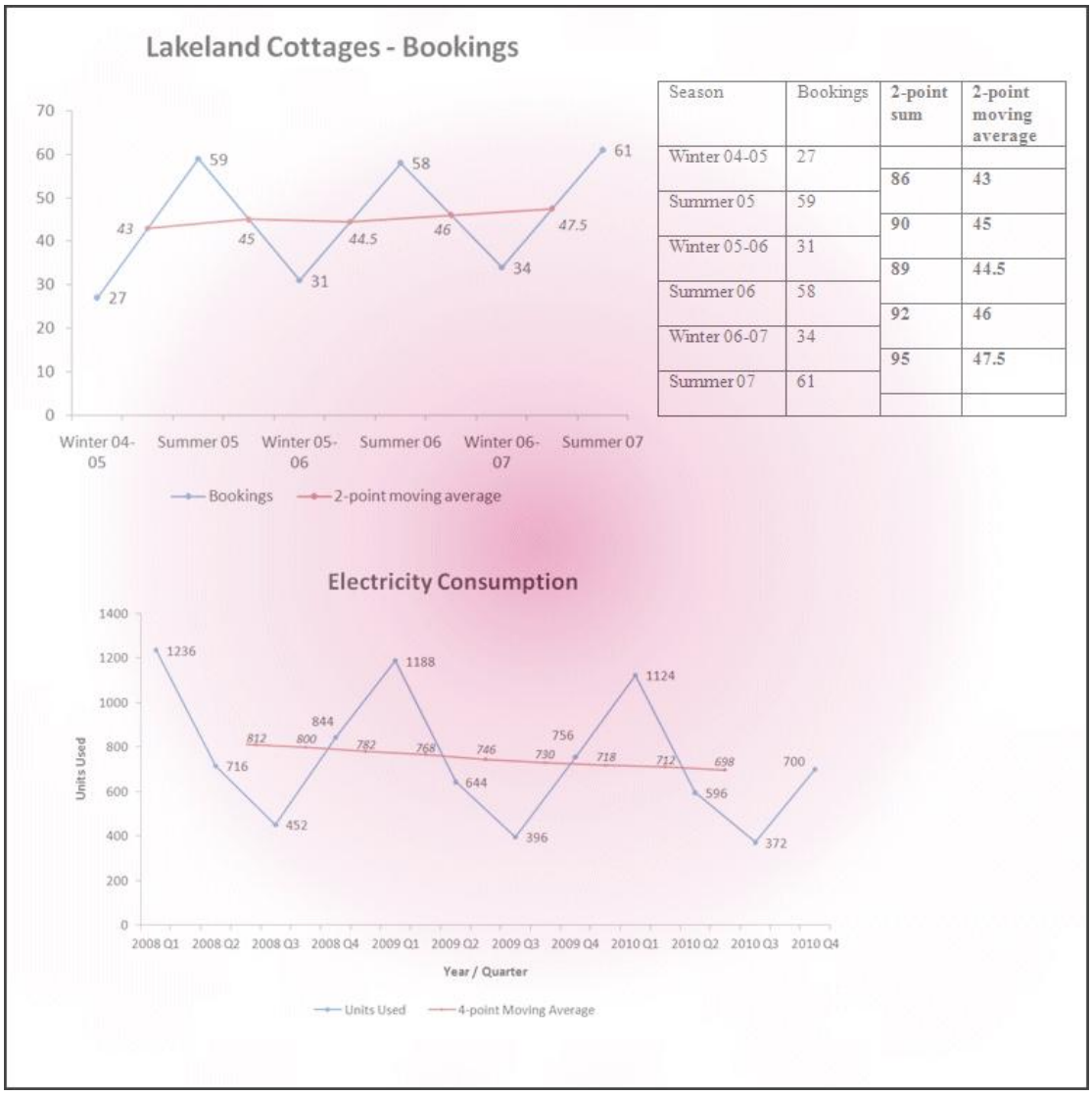
M.K. HOME TUITION

Mathematics Revision Guides
Level: GCSE Higher Tier

TIME SERIES



Season	Bookings	2-point sum	2-point moving average
Winter 04-05	27		
		86	43
Summer 05	59		
		90	45
Winter 05-06	31		
		89	44.5
Summer 06	58		
		92	46
Winter 06-07	34		
		95	47.5
Summer 07	61		

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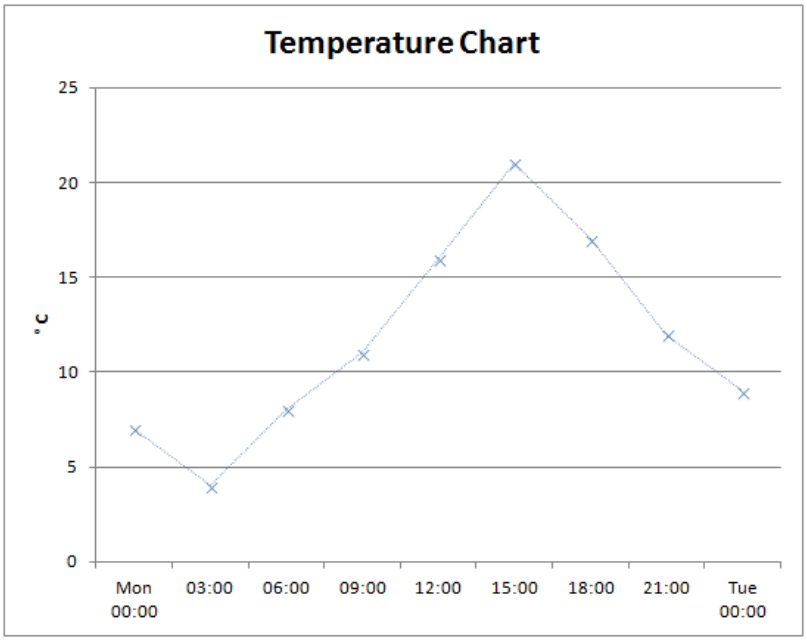
Time Series.

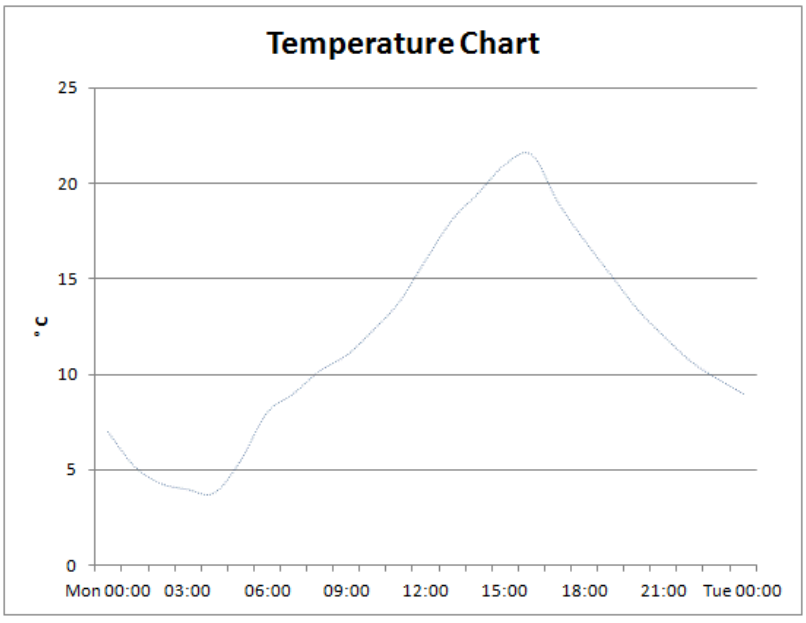
A time series is a collection of data where the horizontal axis is connected with **time**, such as monthly shop sales figures, or weather readings over a 24-hour time period.

Example (1): Here are some temperature readings taken at discrete 3-hour intervals over 24 hours. Plot the results on a frequency polygon.

Time	Mon 00:00	03:00	06:00	09:00	12:00	15:00	18:00	21:00	Tue 00:00
Temp (°C)	7	4	8	11	16	21	17	12	9



The temperature readings were selected at 3-hour intervals, and so the readings had been joined by straight lines. A weather station however monitors the temperature at far more frequent intervals, and by plotting the temperatures over smaller more intervals, the temperature chart describes an ever-smoother curve, as shown below.



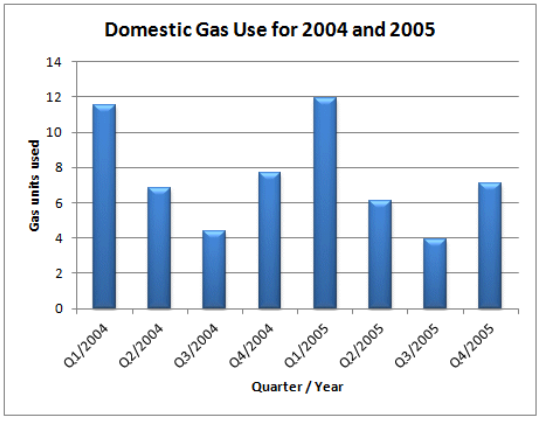
Time series also lend themselves well to comparative graphs, such as year-on-year comparisons.

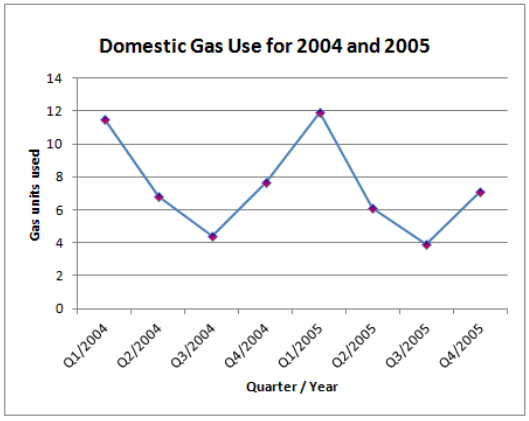
Example (2): A household is monitoring its quarterly gas consumption over two years.

Year	2004				2005			
Qtr.	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Units	11.5	6.8	4.4	7.7	11.9	6.1	3.9	7.1

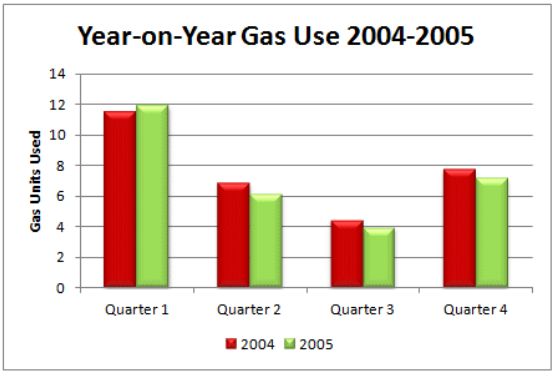
Here, the quarter of the year is the data label, and the units of gas used are the frequency (even though it is no longer a whole number). This also happens to be a time-related series.
Again, the most suitable forms of representation are the bar and line charts.

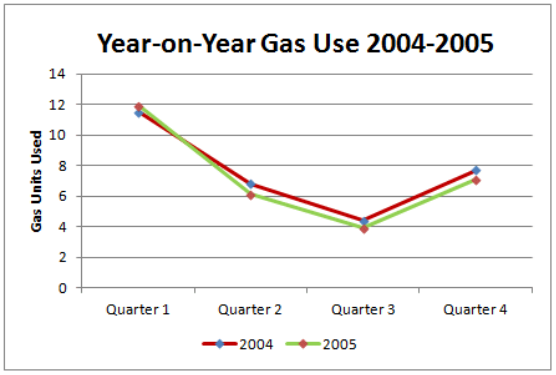
We can display the gas consumption as a straight time series as below:





It might however be more useful to display year-on-year charts, as they can reveal trends:





Based on the 2004 consumption of gas, the household was able to reduce gas usage in the second, third and fourth quarters in 2005, after a slight increase in the first quarter.

Time series are also used for monitoring seasonal trends and 'smoothing out' differences.

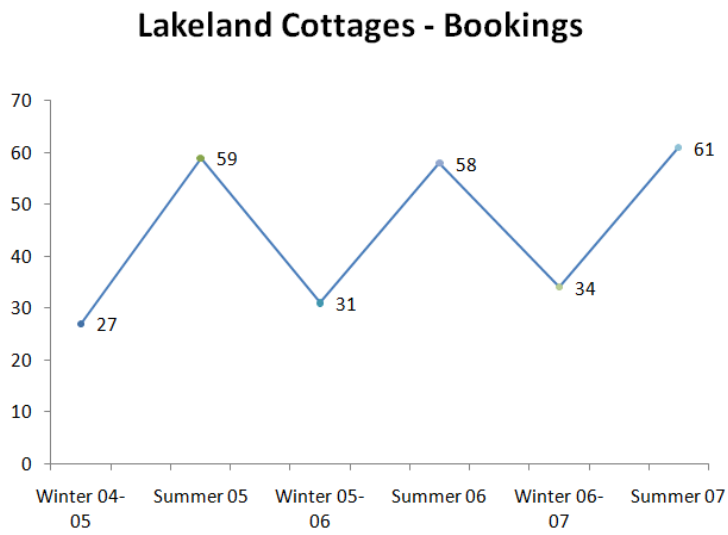
Example (3): Lakeland Cottages is a small hospitality business renting out accommodation for holidaymakers in the Lake District. There are two seasons, 'winter' from October to March, and 'summer' from April to September.

The number of bookings over the last three years is summarised thus:

Season	Winter 04-05	Summer 05	Winter 05-06	Summer 06	Winter 06-07	Summer 07
No. of Bookings	27	59	31	58	34	61

If we were to plot the time series as a line graph, we would have the switchback-like result below.

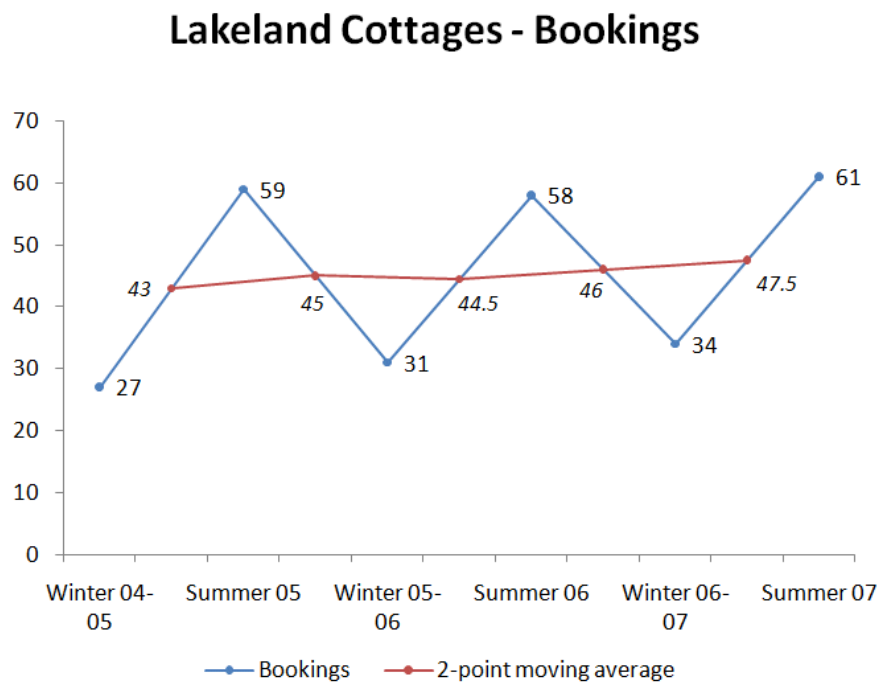
We can see a vague upwards trend, but to confirm this, we need to 'smooth out' the seasonal irregularities.



We can see that the data is collected **twice** in a **year**, so we need to combine the pairs of winter and summer bookings and halve them to produce a **two-point moving average.**

Season	Bookings	**2-point sum**	**2-point moving average**
Winter 04-05	27		
		86	**43**
Summer 05	59		
		90	**45**
Winter 05-06	31		
		89	**44.5**
Summer 06	58		
		92	**46**
Winter 06-07	34		
		95	**47.5**
Summer 07	61		

We then plot the 2-point moving averages **between** the pairs of seasonal data; thus the first moving average goes between 'Winter 04-05' and 'Summer 05'.



We can see that the bookings trend is slightly upward.

The moving averages follow a **trendline.**

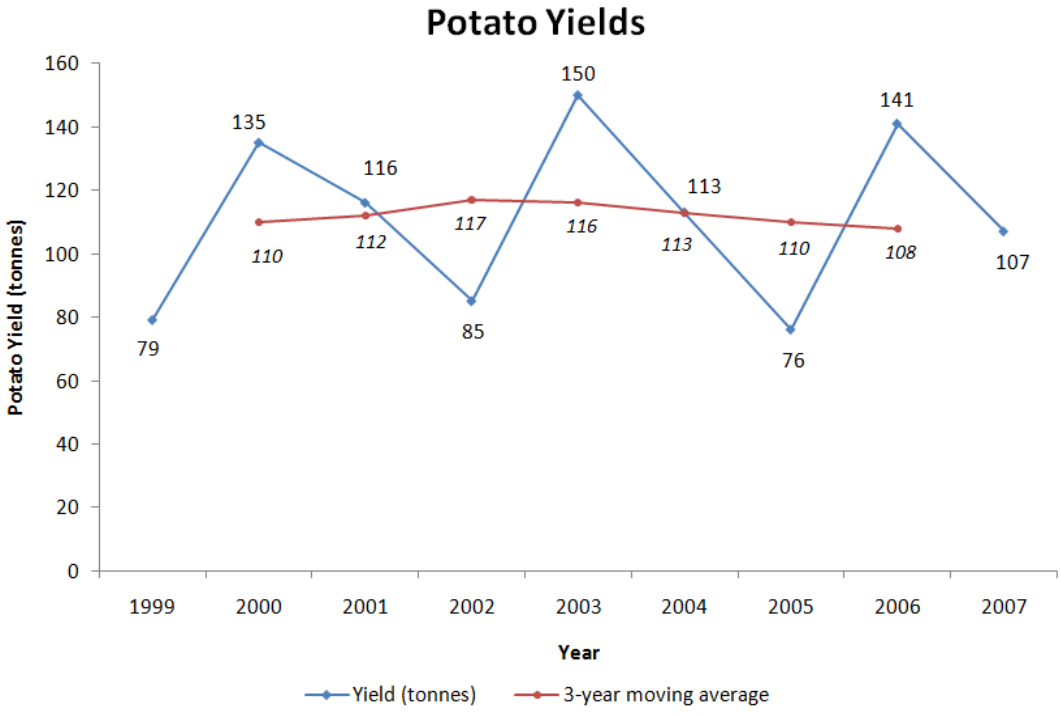
Example (4): A farmer rotates his crops on a three-year cycle, but the fields are unequal in size and therefore the yields for potatoes follow a distinct three-year pattern. Find the moving averages and hence plot both the time series and the trendline.

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007
Yield (tonnes)	79	135	118	85	149	116	74	142	109

We have a time series here with a three-year pattern, so this calls for a three-point moving average.

Year	Yield (tonnes)	**Data to total**	**3-point total**	**3-point moving average**
1999	79	**(Not enough data)**	**N/A**	
2000	135	**1999 + 2000 + 2001**	**332**	**110.7**
2001	118	**2000 + 2001 + 2002**	**338**	**112.7**
2002	85	**2001 + 2002 + 2003**	**352**	**117.3**
2003	149	**2002 + 2003 + 2004**	**350**	**116.7**
2004	116	**2003 + 2004 + 2005**	**339**	**113**
2005	74	**2004 + 2005 + 2006**	**332**	**110.7**
2006	142	**2005 + 2006 + 2007**	**325**	**108.3**
2007	109	**(Not enough data)**	**N/A**	

The first time we can use a moving average here is when we combine the data for 1999 to 2001 to get a 3-year total of 332 tonnes, giving a 3-point moving average of 110.7 after dividing by 3.



We plot the first moving average to coincide with the midpoint of the 3 years, namely 2000.

Next we add the data for 2000 to 2002, find the moving average and plot it to coincide with 2001. We continue in the same way until we reach the period 2005-2007 and plot the moving average under 2006. We cannot put anything under 2007 because we only have two years' data (out of three) centred on that year.

It looks as if the yield of potatoes has been falling over the last few years.

Example (5): A household has been monitoring its electricity consumption over three years as part of an energy efficiency drive.

Year	Qtr	Units Used	4-point Moving Average
2008	1	1236	
	2	716	
			812
	3	452	
			800
	4	844	
			782
2009	1	1188	
			768
	2	644	
			746
	3	x	
			730
	4	756	
			718
2010	1	1124	
			y
	2	596	
			z
	3	372	
	4	700	

Find the missing entries x , y and z in the table and hence plot both the time series line graph and the trendline.

To find x, we need to find a 4-point moving average which includes the third quarter of 2009 as one of its values. The moving average of 746 in the table applies to quarters 1 to 4 of the year 2009, and since this average is a mean, it follows that the total of electricity units used in 2009 was 746×4, or 2984.

We must solve the equation $1188 + 644 + x + 756 = 2984$, which gives $x = 396$. The third quarter consumption for 2009 is therefore **396 units**.

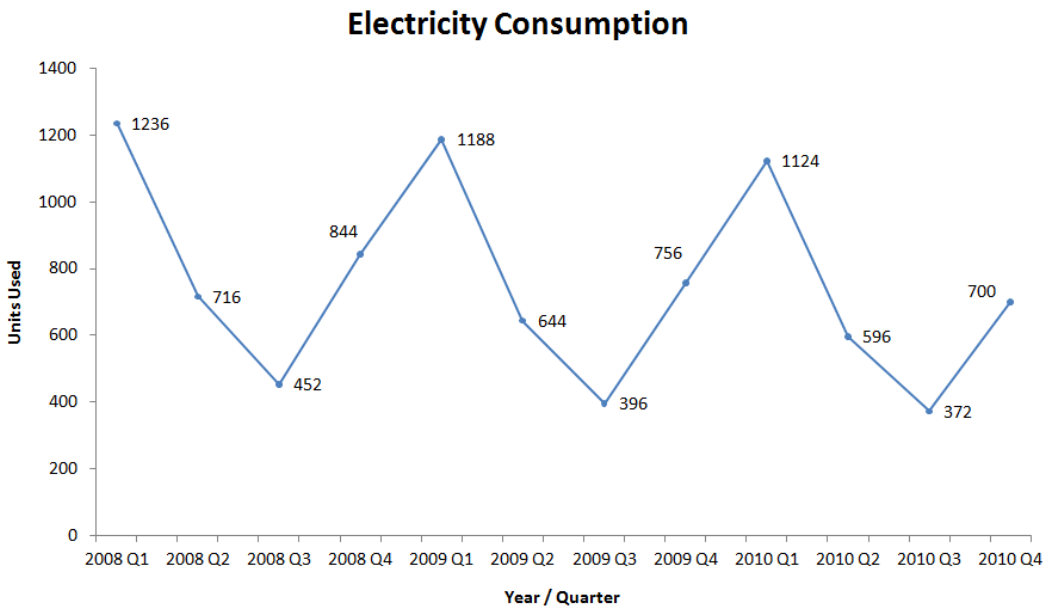
The value of y is the four-point moving average for the period 2009 Q4 to 2010 Q3.
Adding the consumptions of 756, 1124, 596 and 372 gives a total of 2848 units, and dividing by 4 gives a 4-point moving average of $\boldsymbol{y = 712}$ **units**.

We find z in the same way: it is the four-point moving average for the period 2010 Q1 to 2010 Q4.
Adding 1124, 596, 372 and 700 gives a total of 2792 units, and dividing by 4 gives the moving average of $\boldsymbol{z = 698}$ **units**.

The completed table of 4-point moving averages is shown below. The greyed-out entries are shown for completeness, but there is no need to include them as part of the solution to the question.

Year	Qtr	Units Used	Data for totalling	**4-point total**	**4-point Moving Average**
2008	1	1236			
			(Not enough data)		
	2	716			
			2008 Q1 to 2008 Q4	3248	812
	3	452			
			2008 Q2 to 2009 Q1	3200	800
	4	844			
			2008 Q3 to 2009 Q2	3128	782
2009	1	1188			
			2008 Q4 to 2009 Q3	3072	768
	2	644			
			2009 Q1 to 2009 Q4	**2984**	**746**
	3	**396**			
			2009 Q2 to 2010 Q1	2920	730
	4	756			
			2009 Q3 to 2010 Q2	2872	718
2010	1	1124			
			2009 Q4 to 2010 Q3	**2848**	**712**
	2	596			
			2010 Q1 to 2019 Q4	**2792**	**698**
	3	372			
			(Not enough data)		
	4	700			

The line graph is shown below, without the trendline.



The line graph below includes the trendline.

Note how the first entry on the trendline appears halfway between the actual data for the second and third quarters of 2008. We can also see that the householders have slowly been cutting down on their electricity usage !