M.K. HOME TUITION

A LITTLE MATHEMATICAL MAGIC

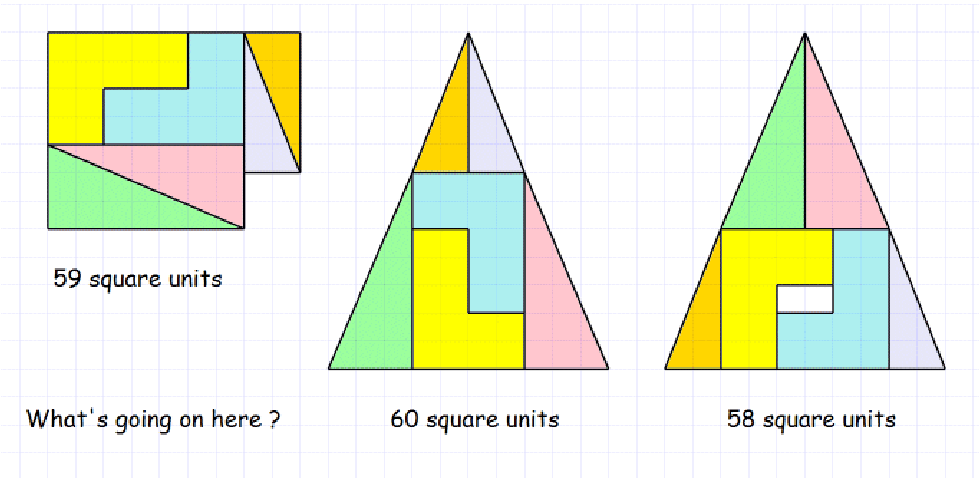
37216
41023
58976
28147
71852
———
237214

$617 \times 143 =$

$617{,}617 \div 7 =$

88,231

1	8
2	7
3	15
4	22
5	37
6	59
7	96
8	155
9	251
10	406
TOTAL	1056



A Little Mathematical Magic.

This section is not meant to be a serious revision guide, but shows some examples of apparent mathematical magic, whether in arithmetic or geometry.

The "tricks" shown here are good for parties (and impressing at school !)

The "Sum of Ten Numbers" Trick.

For this we need a conjuror and one (or two) other players

The conjuror asks each player to give a number between 1 and 10, and then places those numbers in a column. Here, the players have chosen 8 and 7 as the starting numbers.

Term No.	Value
1	**8**
2	**7**
3	
4	
5	
6	
7	
8	
9	
10	
Sum of 10	

Next, the players and the conjuror continue filling in the columns according to this rule: add the two previous numbers to get the next one.

Term No.	Value
1	**8**
2	**7**
3	15
4	22
5	37
6	59
7	96
8	155
9	251
10	406
Sum of 10	**1056**

Just before all 10 numbers have been calculated, the conjuror multiplies the seventh number, 96, by 11 and writes the result, 1056, in the 'sum of 10' entry. The players then check their total, and find to their amazement that the answer is 1056 !

No matter what starting numbers you pick, the sum of the first ten numbers is still 11 times the seventh number provided you follow that rule.

Take the simplest case, starting with the numbers 1 and 1.
The first ten terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 .
This is the "Fibonacci" sequence.

The seventh term of the sequence is 13, and the sum of the first ten terms is 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 + 55 = 143, which is 11 times the seventh term..

Magic Multiplication.

Some 3-figure numbers lend themselves well to lightning multiplication tricks. A conjuror would know of such numbers and choose them as "random" multipliers.

"Magic" number 143.

Conjuror : "Give me any three-figure number."
Player: "617".
Conjuror: "I'll pick a 3-figure number. Mine's 143."

(Conjuror then multiplies the two numbers together mentally)

Conjuror: "617 × 143 = 88,231."

(The astonished player then checks the result on a calculator. The result is indeed true. 617 × 143 = 88,231.)

How did the conjuror work the sum out ?

The conjuror's choice of 143 was not a "random" one at all. It was chosen because 1001 ÷ 7 = 143.

To multiply a three-digit number by 1001, you just repeat the three digits, so 617 × 1001 = 617,617.

The conjuror would have mentally divided 617,617 by 7.

For two-digit numbers, we must add an extra zero as per example:
68 × 143 = 68,068 ÷ 7 = 9,724

Variations:

To multiply by 286: double after multiplying by 143.
Example: 513 × 286 = 513,513 ÷ 7 × 2 = 73,359 × 2 = 146,718

To multiply by 715: stick zero on end and halve after multiplying by 143.
Example: 362 × 715 = 362,362 ÷ 7 × 10 ÷ 2 = 517,660 ÷ 2 = 258,830.

Conjuror's Tip: if the player's number is a multiple of 7, the answer 'stutters', i.e. 714 × 143 = 102,102. To avoid the stutter, check if number is a multiple of 7 and if so, get the player to add a number between 1 and 6 to it.

There are several other three-digit numbers that can be used for similar lightning multiplication tricks.

"Magic" number 667.

Since 667 is 2001 divided by 3, we proceed as follows :

Example: $427 \times 667 = 854{,}427 \div 3 = 284{,}809$.
Note the intermediate number 854.427 : it's twice 427 followed by 427 itself.

Also works for numbers over 500 : $569 \times 667 = 1{,}138{,}569 \div 3 = 379{,}523$.

"Magic" number 334.

Since 334 is 1002 divided by 3, we proceed as follows :
Example: $427 \times 334 = 427{,}854 \div 3 = 142{,}618$.
Note the intermediate number 427,854 : it's 427 followed by twice 427.

For numbers over 500 we must 'carry' 1 into the thousands :
$569 \times 334 = 570{,}138 \div 3 = 190{,}046$.

Variant: To multiply by 167.

Multiply by 334, then halve.

Example: $427 \times 167 = 427{,}854 \div 3 \div 2 = 142{,}618 \div 2 = 71{,}309$.
$569 \times 167 = 570{,}138 \div 3 \div 2 = 190{,}046 \div 2 = 95{,}023$.

"Magic" number 999.

Write down the player's number minus 1 for the first three digits, then subtract each digit from 9 in turn to get the last three digits.

Example: $452 \times 999 = 451{,}548$

$907 \times 999 = 906{,}093$

(Note how the 'halves' of each answer add to 999:
$451 + 548 = 999$; $906 + 093 = 999$)

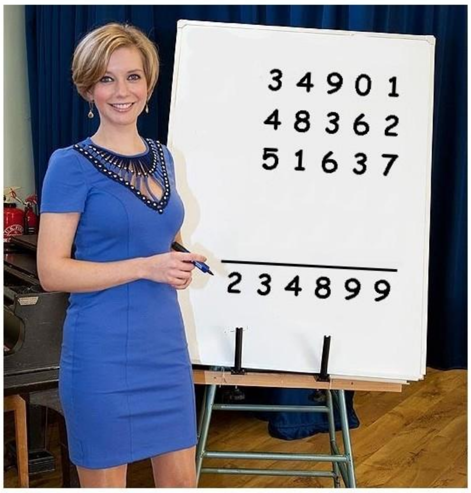
This method is sometimes called taking 9's complements – a fact exploited in the next example.

A Prediction. (Credit to the *Countdown* team, especially Rachel Riley !)

This trick, suitable for families, although simple, is very effective.
You can predict the sum of a list of 5-digit numbers seemingly by magic, like Rachel has done in the pictures !

Rachel : Choose a random 5-digit number.
Nick : 34901.

Rachel : I'll write your number and my predicted total.
Susie, give me another random 5-digit number.
Susie : 48362.



Rachel : There's your 48362, now it's my turn.
My random number will be 51637.
Joe, give us another 5-digit number.
Joe : 85296.

Rachel: There's your 85296 then, and I'll finish off with 14703. Now, check the totals.
All : It all tallies up - what the ????
Rachel : Conjurors never tell their secrets. ;-)

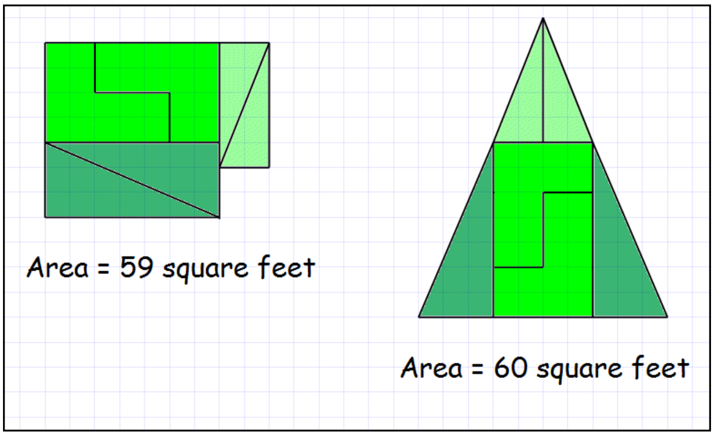
Conjurors never reveal their secrets, but all will be explained in the Key section.

A Try-Angle Problem.

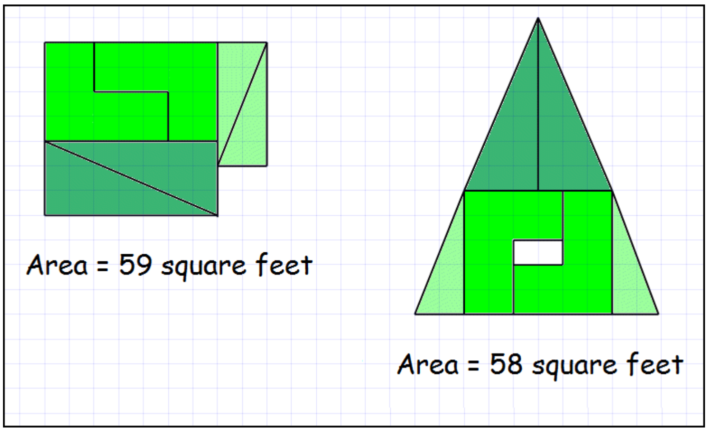
Alan has a small patch of lawn in his front garden, which is a rectangle of 9 ft by 7 ft, but with a 2-foot square section removed from one corner – an area of 59 sq ft. He wants to replace the lawn with shrubs.

In his back garden, there is a gravelled area in the shape of an isosceles triangle of height 12 feet and base 10 feet – an area of 60 sq ft, and Alan wishes to have *that* area replaced with grass.

Alan is a bit of a mathematician, so he cuts the front lawn into six pieces – four triangles and two L-shaped pieces (left), so that he can then rearrange them into an isosceles triangle (right).



According to this plan, Alan seems to gain a square foot of lawn out of nothing – but his landscaper thinks otherwise. He rearranges the pieces wrongly, and as a result the triangular lawn ends up having a 2 sq. ft hole in it, and Alan has lost a square foot of lawn, not gained one !

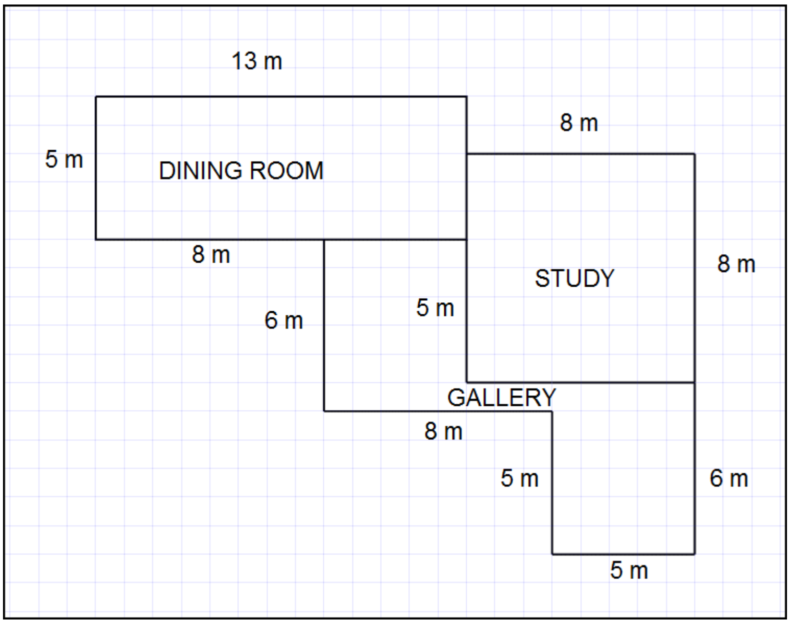


Something is not quite right in either case, but what ?

This problem was originally credited to New York illusionist and mathematician Paul Curry.

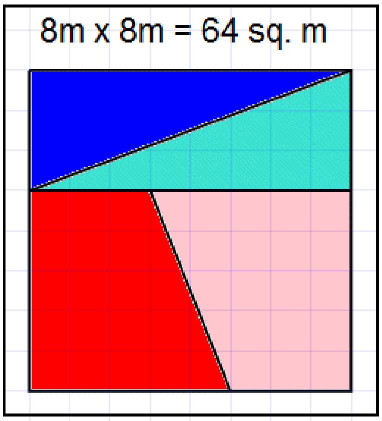
A Carpet Problem.

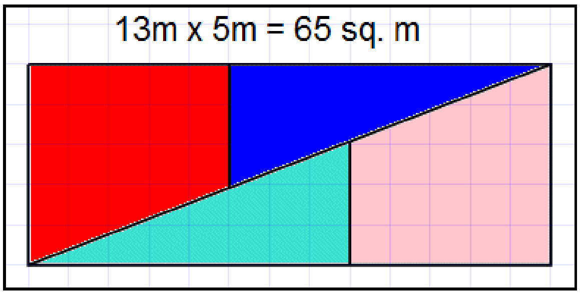
A professor wants to replace the carpet in his study and also buy new carpets for his dining room and gallery to replace worn-out flooring.
The study is square, of side 8m; the dining room measures 13m by 5m. The gallery consists of two viewing rooms of 6m by 5m with a narrow 3m by 1m corridor connecting them.



He expects the carpet in his study to be fitted first, followed by those in the dining room and the gallery.

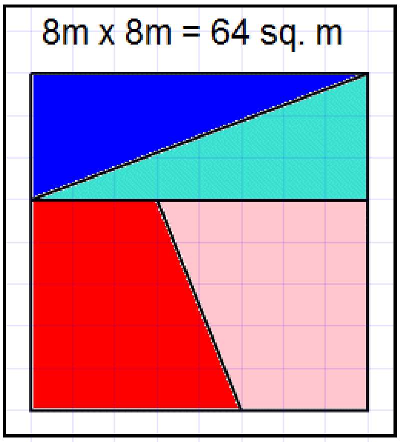
When the new study carpet arrives, he does not simply bin the old carpet, but has a bright idea. He cuts the study carpet into four pieces – two trapezia and two triangles - and then rearranges them in the dining room as a temporary measure until the new carpet is fitted.

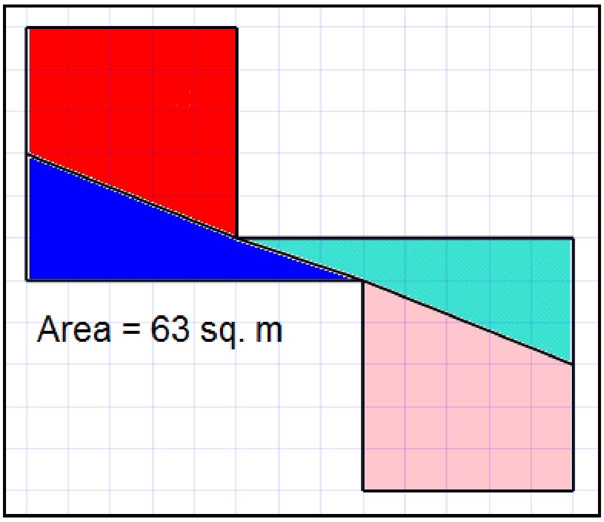




The professor seems to have covered an extra square metre of floor out of nothing ! Something must be amiss, but what ?

Anyhow, a few weeks later, the carpet for the dining room is fitted and the professor decides to re-use the same four pieces of carpet a second time for use in the gallery.





This time, the reverse has happened. A square metre of carpet has seemingly vanished into thin air !

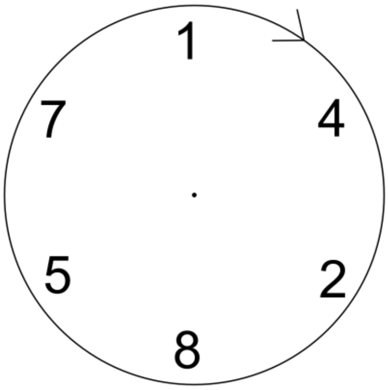
As an activity, try making a model of this dissection very accurately out of card, and see what really has become of the "gained" and "lost" areas.

This dissection has been credited to Sam Loyd, a prolific American puzzle-writer of the 19th century.

More Magic Multiplication.

Some six-digit numbers also lend themselves well to lightning multiplication tricks. One such number is 142,857.

This magic number can be multiplied by 2, 3, 4, 5 and 6 very rapidly by reading clockwise around the circle, but starting at a different point each time.



142,857 × 2 = 285,714
142,857 × 3 = 428,571
142,857 × 4 = 571,428
142,857 × 5 = 714,285
142,857 × 6 = 857,142

The same six digits occur in each result, and in the same cyclic order.

This is because the decimal value of the fraction 1/7 is the recurring decimal 0.142857142857... and so the decimal representations of 2/7, 3/7 ... containing the same digits in the same order, but with a different start point in each case.

Key Section. (or why the magic 'works') .

Sum of 10 Numbers.

Call the two starting numbers p and q.

The table shows the first 10 terms of the sequence, along with their total sum :

Term No.	Value
1	p
2	q
3	$p + q$
4	$p + 2q$
5	$2p + 3q$
6	$3p + 5q$
7	$5p + 8q$
8	$8p + 13q$
9	$13p + 21q$
10	$21p + 34q$
Sum of 10	**$55p + 88q$**

The seventh term is $5p + 8q$: the sum of all ten is $55p + 88q = 11(5p + 8q)$.

Hence the sum of the first 10 terms of any sequence defined by this rule is always 11 times the seventh term.

Magic Multiplication / A Prediction.

Explanations included in the main text.

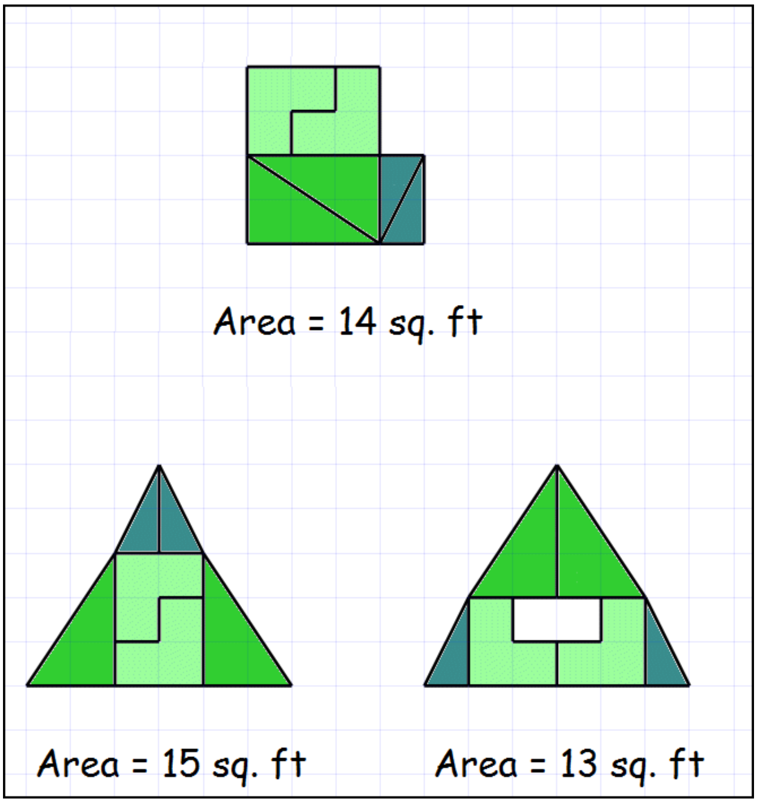
A Try-Angle Problem.

When we check the four right-angled triangles making up the dissection, we find that two of them have a base of 2 feet and a height of 5 feet, and the other two a base of 3 feet and a height of 7 feet. The ratios between the heights and the bases (2 :5 and 3:7) are close but not equal, so the triangles are not similar.

As a result, the "isosceles triangles" formed by rearranging the six pieces are not triangles, but pentagons.

The sloping sides in the "60 square foot triangle" bow inwards, and the sloping sides in the "58 square foot triangle" bow outwards. The amount of bow accounts for the apparent differences in area, but is practically imperceptible to the untrained eye.

The example below uses different figures, but illustrates the fallacy more clearly. The sloping sides of each "isosceles triangle" are obviously bowing inwards (left) and outwards (right).

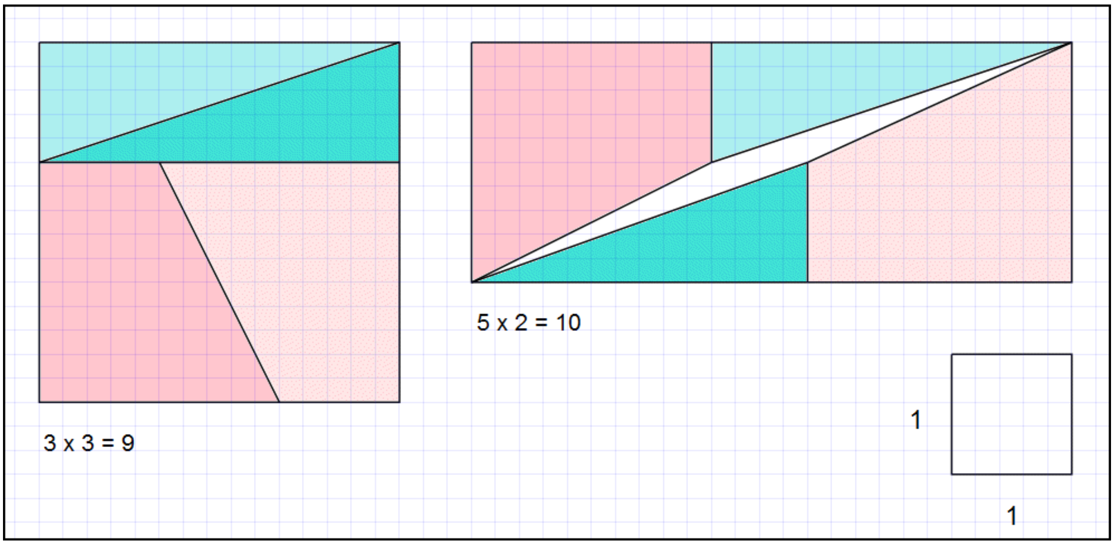


A Carpet Problem.

Again, shapes are not always what they seem. The gradients of the hypotenuse of the triangle, and of the sloping sides of the trapezia, are different. For the triangles, the gradient is 3 in 8, for the trapezia, it is 2 in 5.

As a result, there remains a narrow parallelogram-shaped 'hole' where we have the "diagonal" of the 13m by 5m rectangle. The area of this 'hole' is one square metre – the difference between 64 and 65.

The example below is similar, but shows a 3m square dissected and the parts reassembled (badly) into a 5m by 2m rectangle. The 'hole' is quite obvious – much more so than in the original example.



Conversely, we have a square metre of overlap in the other rearrangement of the four pieces for the gallery carpet.

A Prediction.

Rachel's "random" entries in the rows were anything but !

Nick's number, 34901, had a 2 placed in front and 2 subtracted – the same as adding 199998 to 34901 to get the final total of 234899.

When Susie chose the number 48362, Rachel subtracted each digit from 9 in turn to get 51637.The upshot is that 48362 + 51637 = 99999. What she had done was use 9's complements.

Similarly, when Joe chose 85296, Rachel entered 14703 – the difference between 85296 and 99999. And of course, twice 99999 is 199998 !

Hint: it is best ask the players to make the 5-digit numbers as random as possible by avoiding repeated or ordered digits. Also, if the player's choice begins with a 9, then the conjuror's answer would be a 4-digit number.

If a player were to chose, say, 52223, then the conjuror's answer would be 47776, and the repeated numbers would be rather suspicious.
The same would apply to an ordered 34567, as that would make the conjuror's choice 65432 – again a bit suspicious.